

## STATISTICS 368

### A Close Look at the Effects of Different Factors on the Bouncing of Tennis Balls.

(April 18, 2005)



“Science is simply common sense at its best; that is, rigidly accurate in observation, and merciless to fallacy in logic.”

(Thomas Henry Huxley – English Biologist)

## **1. Preliminary Description of the Experiment**

In this experiment, our objective was to determine which factors affect the bounce height of tennis balls. The factors of interest were the ball brands, the type of packaging used for the balls – regular or vacuum-sealed – as well as the balls themselves within a certain brand.

We obtained four brands of balls, all of which varied in price: “Ashley Scott” (\$1.32), “Cooper” (\$1.39), “Wilson” (\$3.76), and “Athletic Works” (\$1.97) (see page 3). The Ashley Scott and Cooper balls were packaged in regular bags, while the Wilson and Athletic Works balls were vacuum-sealed. To measure the bounce height of the balls, we initially planned to roll the balls down a ramp apparatus, and allow it to bounce and contact a sheet of carbon paper, at which point we could measure the height of contact. After obtaining carbon paper, however, we found that the bouncing balls did not contact the paper with enough force to leave visible marks. We thus decided to rather drop the balls. We taped a tape measure, extended to 5 feet, to the wall. We also decided to take 2 sets of measurements; the bounce heights from dropping the ball from a height of 5 feet, and from a height of 2.5 feet (see page 3).

The response variable we were interested in was ‘bounce height’, the maximum height the ball reached after its drop and initial contact with the floor. We thought that the easiest way to obtain measurements was to simply ‘eye’ the ball against the tape measure, and record our estimates in inches. To control for the nuisance factor of human error (the speed and precision at which the human eye can follow the ball may be subjective and inconsistent), we had two people (Dan and Angie) observe the ball, and used the average of the two estimates for our ‘y’ value (bounce height). Additionally, we had only one person (Aaron) drop the ball in every trial, to account for the error that might arise in drop height, since we did not have an apparatus to ensure that the ball was dropped at precisely 5 feet or 2.5 feet in the trials (however the existence of the measuring tape helped with maintaining consistency in the height level). Another nuisance factor was the fact that in some trials, both Dan and Angie did not follow the ball quickly enough to accurately observe a bounce height; thus the ball had to be dropped twice or three times for bounce heights to be ascertained and recorded. This occurred in both early and late trials.

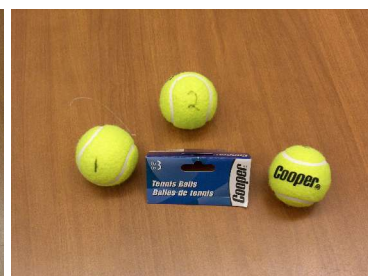
## **2. Design and Data Collection**

We initially considered running the experiment as a  $2 \times 2 \times 3$  factorial (2 levels of packaging, 2 brand price levels, and 3 different balls in each brand). With this model we would use 12 different balls (3 of each brand) and label them ‘1’, ‘2’ and ‘3’. We would be making inferences about the effect of brands, the different packaging types and the effect of the balls. We also considered the height levels as a potential factor, but after some discussions we decided that height would be an obviously deterministic factor on the bounce height and so it didn’t present too much interest in studying. We still



In this picture you can see Aaron getting ready to drop one of the balls from a height of 5 feet. This picture was taken for illustrative purposes (otherwise Dan and Angie would appear in it too, recording the bounce of the ball).

In this picture you can see the actual bouncing of the ball from the ground, which “sadly” was the climax of our experiment.



And these are the “victims”:

conducted the experiment for both height levels but we realized that we would ultimately be using the observations at one of the two levels.

Meet: Ashley Scott, Athletic Works, Cooper and Wilson.

To randomize the order of the balls, we put the labeled balls into a bag and pulled one out randomly for each run. After one run was recorded for a ball, say “Ashley Scott”

number 2, for instance, we would replace the ball (put it back into the bag). Upon the second appearance of the same ball (the second replicate), we would remove the ball from the bag. The process would continue until no balls were left in the bag. We thus gathered a total of 48 observations (24 at each height level).

When we actually started thinking about the questions we wanted answered, we realized that a different kind of experiment would be a lot more useful. (At the time we started the experiment we were not very aware of what nested designs were about). We realized that the brand 1 (or for that purpose brand price 1) within regular packaging and vacuum-sealed packaging was not the same, and that therefore, the effect of brands within packaging would be a factor of interest for us. We also noticed that the balls were not the same throughout the 4 brands and therefore another factor of interest would be the difference between the balls within a brand within a certain packaging type. We were in effect interested if there are any differences in the manufacturing process of balls from a certain brand. Hopefully this was not to be the case, but the question presented quite a bit of interest to us. We thus concluded that the best-suited design for our experiment would be a 3 stage-nested design.

The data collected throughout the experiment appears in the table from below.

**Data**  
**(24 observations recorded at height - 5 feet)**

Ball	V		V		R		R	
	Wilson		Ath. Works		Cooper		A. Scott	
1	39	39	31.5	31.5	33.5	33	28	27.5
2	38	37.5	32	30.5	34	33.5	28.5	27.5
3	39	38.5	32	31.5	33	33.5	27.5	28.5

Also the table corresponding to the data that contains the order in which the measurements in the table above were recorded is given below. As it can be seen from this table the randomization is complete, but this does not create any problems with our nested design, which requires randomization of the balls within the brands within a certain packaging type.

Ball	V		V		R		R	
	Wilson		Ath. Works		Cooper		A. Scott	
1	4	7	10	20	18	21	6	17
2	11	22	15	24	1	9	14	16
3	3	13	2	23	5	19	8	12

As it can be seen in the two tables, each of the 12 balls has two replicates. After conducting the experiment and the data gathering we decided that replicates should also be a factor of interest in the model that would maybe account for the potential inconsistency in the reading of the bounce heights. While it is true that at each run we recorded the average bounce as observed by Dan and Angie, the inclusion of the

replicates in the model would give us a sense of the consistency (or lack of it) of the readings.

### 3. Statistical Model

The effects model for the data is:

$$y_{ijkl} = \mu + \tau_i + \beta_{j(i)} + \gamma_{k(ij)} + \alpha_l + \epsilon_{ijkl}, \text{ where:}$$

1.  $\tau_i$  is the effect due to packaging type  $i$ ;
2.  $\beta_{j(i)}$  is the effect of the  $j$ -th brand (or brand price) within  $i$ -th packaging type;
3.  $\gamma_{k(ij)}$  is the effect of the  $k$ -th ball within the  $j$ -th brand, within the  $i$ -th packaging type;
4.  $\alpha_l$  is the effect of the  $l$ -th replicate;
5.  $\epsilon_{ijkl}$ , is the random error for the  $l$ -th run with  $l=1,2$  in the cell  $(i,j,k)$ ;

In the above model,  $\mu$  is the overall mean bounce height, irrespective of the other effects. Also since any average effect can be incorporated into the overall mean we can impose the usual constraints that:

$$\sum \tau_i = 0 \ (i=-1 \text{ to } 1); \sum \sum \beta_{j(i)} = 0 \ (i=-1 \text{ to } 1, j=-1 \text{ to } 1); \sum \sum \sum \gamma_{k(ij)} = 0 \ (i=-1 \text{ to } 1, j=-1 \text{ to } 1, k = 1 \text{ to } 3); \sum \alpha_l = 0 \ (l=1 \text{ to } 2);$$

As it can be seen from the model from above there are no interactions present in the model. That is because in the case of nested design experiments interactions do not make a lot of sense.

### 4. Data Analysis

The data as entered in R is given in the appendix, found at the end of the project. The table contains columns pertaining to packaging, brand (or brand price), balls, replicate and bounce height; however, an additional column, which contains the order in which the measurements were recorded when performing the experiment is present.

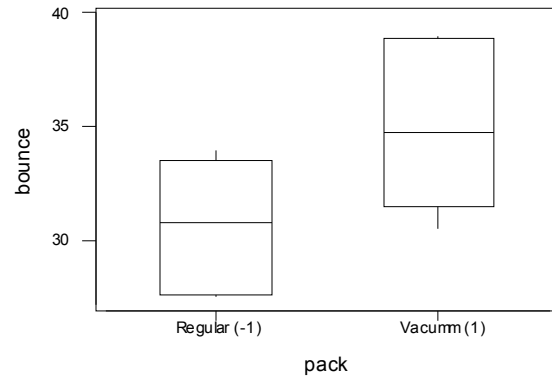
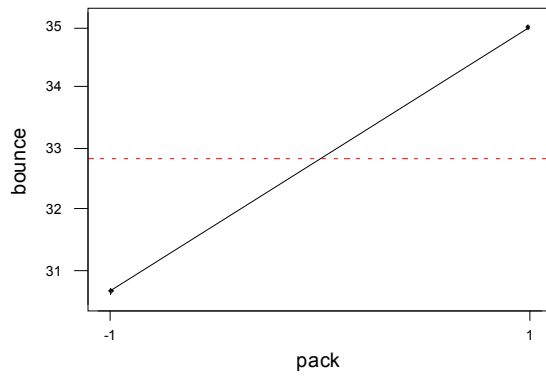
The least square estimates of the main effects in the model (explicitly stated in part 3) are:

- $\mu(\text{hat}) = y(\text{bar})_{....} = 32.83$
- $\tau_{\text{Regular}}(\text{hat}) = \tau_{-1}(\text{hat}) = y(\text{bar})_{-1...} - y(\text{bar})_{....} = 2.17$
- $\tau_{\text{Vacuum}}(\text{hat}) = \tau_1(\text{hat}) = y(\text{bar})_{1...} - y(\text{bar})_{....} = -2.17$
- $\beta_{\text{AshleyScott}}(\text{hat}) = \beta_{-1(-1)}(\text{hat}) = y(\text{bar})_{-1-1..} - y(\text{bar})_{....} = -4.91$
- $\beta_{\text{Cooper}}(\text{hat}) = \beta_{1(-1)}(\text{hat}) = y(\text{bar})_{-1+1..} - y(\text{bar})_{....} = 0.59$
- $\beta_{\text{AthleticW}}(\text{hat}) = \beta_{-1(1)}(\text{hat}) = y(\text{bar})_{1-1..} - y(\text{bar})_{....} = -1.33$
- $\beta_{\text{Wilson}}(\text{hat}) = \beta_{1(1)}(\text{hat}) = y(\text{bar})_{11..} - y(\text{bar})_{....} = 5.67$
- $\gamma_{\text{ballAS1}}(\text{hat}) = \gamma_{1(-1-1)}(\text{hat}) = y(\text{bar})_{-1-1\ 1.} - y(\text{bar})_{....} = -5.08$
- $\gamma_{\text{ballAS2}}(\text{hat}) = \gamma_{2(-1-1)}(\text{hat}) = y(\text{bar})_{-1-1\ 2.} - y(\text{bar})_{....} = -4.83$
- $\gamma_{\text{ballAS3}}(\text{hat}) = \gamma_{3(-1-1)}(\text{hat}) = y(\text{bar})_{-1-1\ 3.} - y(\text{bar})_{....} = -4.83$
- $\gamma_{\text{ballC1}}(\text{hat}) = \gamma_{1(-1+1)}(\text{hat}) = y(\text{bar})_{-1+1\ 1.} - y(\text{bar})_{....} = 0.42$
- $\gamma_{\text{ballC2}}(\text{hat}) = \gamma_{2(-1+1)}(\text{hat}) = y(\text{bar})_{-1+1\ 2.} - y(\text{bar})_{....} = 0.92$
- $\gamma_{\text{ballC3}}(\text{hat}) = \gamma_{3(-1+1)}(\text{hat}) = y(\text{bar})_{-1+1\ 3.} - y(\text{bar})_{....} = 0.42$
- $\gamma_{\text{ballAW3}}(\text{hat}) = \gamma_{1(+1-1)}(\text{hat}) = y(\text{bar})_{+1-1\ 1.} - y(\text{bar})_{....} = -1.33$
- $\gamma_{\text{ballAW2}}(\text{hat}) = \gamma_{2(+1-1)}(\text{hat}) = y(\text{bar})_{+1-1\ 2.} - y(\text{bar})_{....} = -1.58$
- $\gamma_{\text{ballAW3}}(\text{hat}) = \gamma_{3(+1-1)}(\text{hat}) = y(\text{bar})_{+1-1\ 3.} - y(\text{bar})_{....} = -1.08$
- $\gamma_{\text{ballW1}}(\text{hat}) = \gamma_{1(+1+1)}(\text{hat}) = y(\text{bar})_{+1+1\ 1.} - y(\text{bar})_{....} = 6.17$
- $\gamma_{\text{ballW2}}(\text{hat}) = \gamma_{2(+1+1)}(\text{hat}) = y(\text{bar})_{+1+1\ 2.} - y(\text{bar})_{....} = 4.92$
- $\gamma_{\text{ballW3}}(\text{hat}) = \gamma_{3(+1+1)}(\text{hat}) = y(\text{bar})_{+1+1\ 3.} - y(\text{bar})_{....} = 5.92$

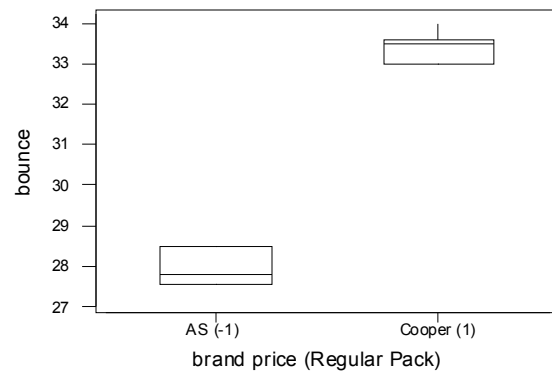
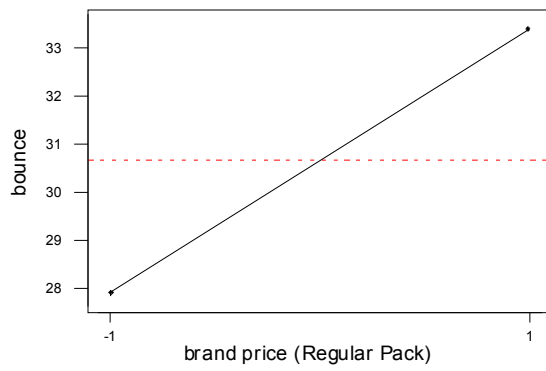
As we can see from all the effects calculated above the regular packaging seems to have a negative effect while vacuum has a positive one on the bounce height. It remains to be seen if this effect is significant. It also appears that the Wilson balls perform very well, while the balls of brand Ashley Scott under perform when we look at the overall mean. Their effects are quite large compared to the other 2 brands. Once again more analysis is needed in order for one to be able to draw appropriate conclusions.

To get a better idea of what is going on we have to examine the box plots and interaction plot of different factors. Let us have a look at the impact of packaging on bounce.

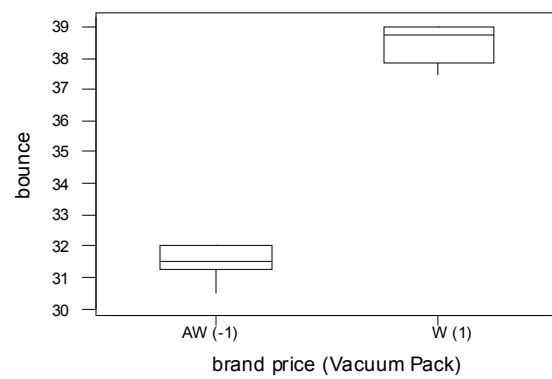
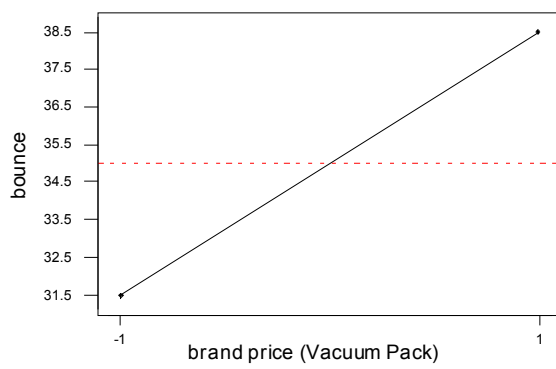
Main Effects Plot - Data Means for bounce



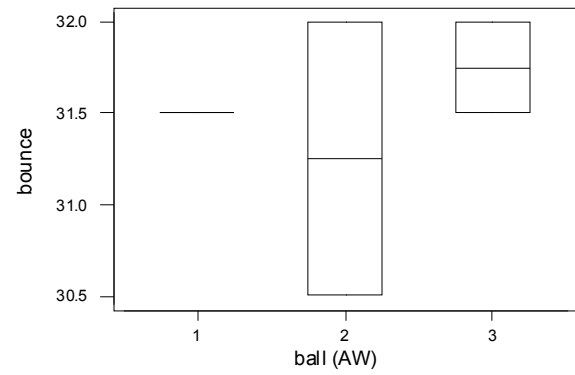
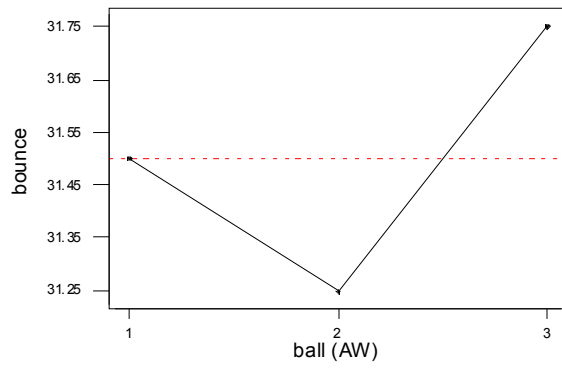
Main Effects Plot - Data Means for bounce



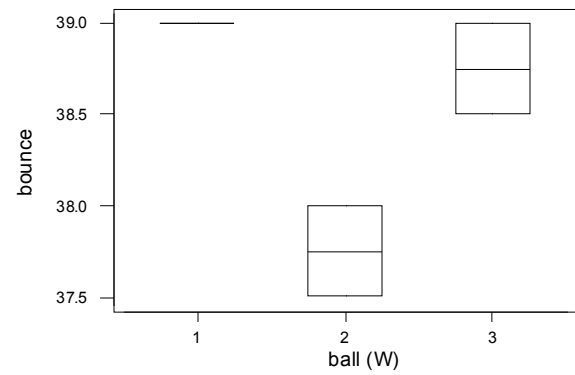
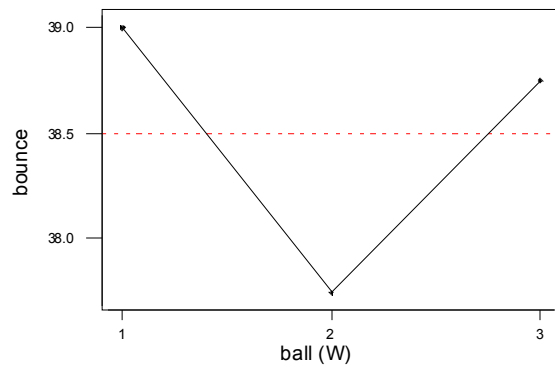
Main Effects Plot - Data Means for bounce



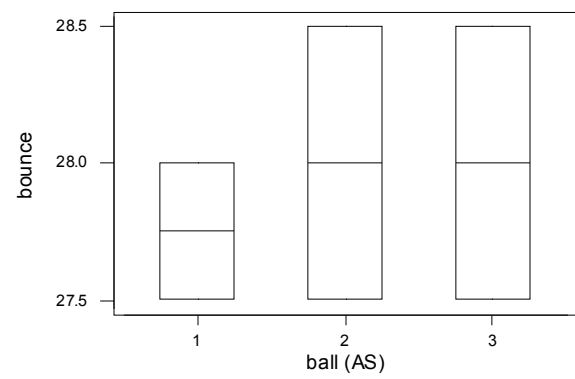
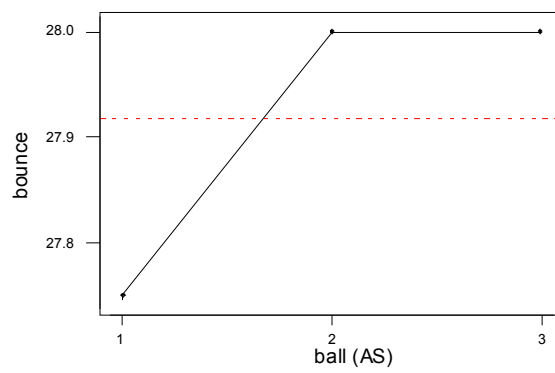
Main Effects Plot - Data Means for bounce



Main Effects Plot - Data Means for bounce

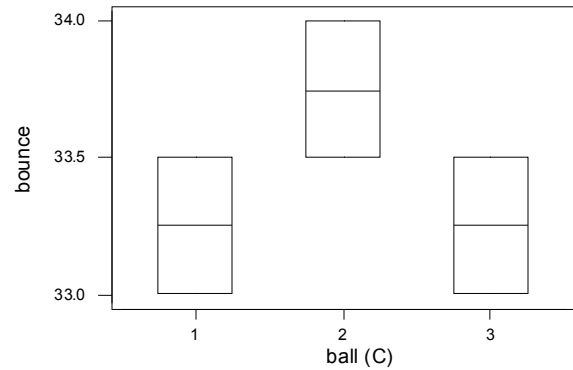
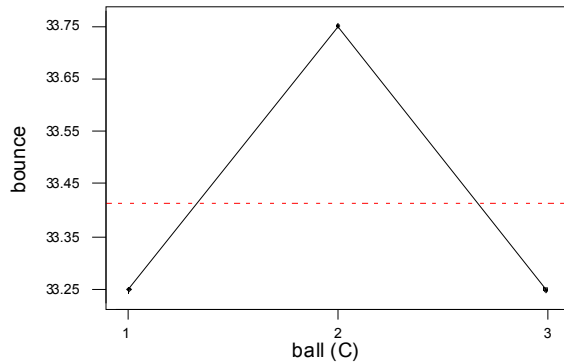


Main Effects Plot - Data Means for bounce



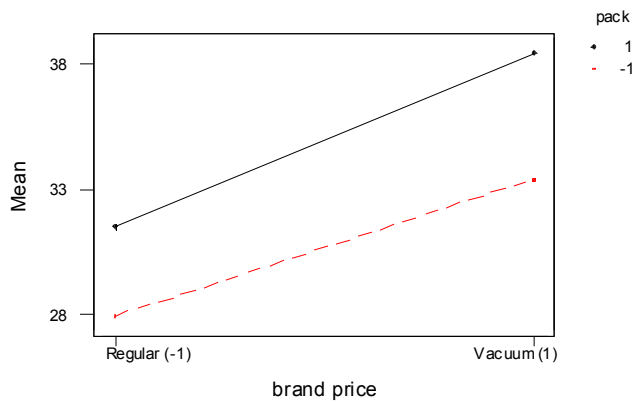


Main Effects Plot - Data Means for bounce

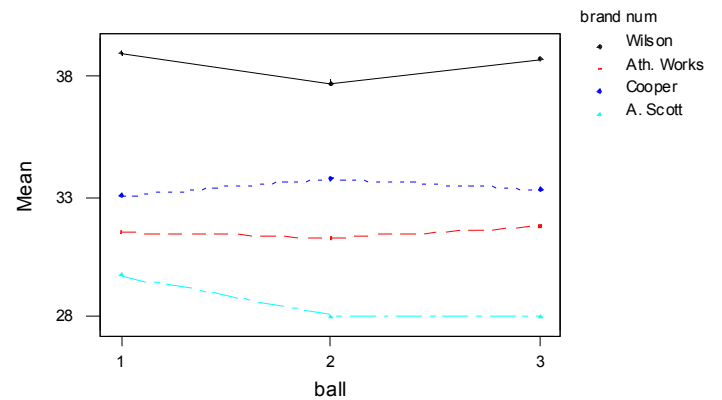


The plots from above show that the mean bounce heights are higher for the vacuum-sealed packaging than for the regular one. Also within regular packaging higher mean values for the bounce are registered for the Cooper brand than for the Ashley Scott brand. Similarly when looking at the regular packaging the Wilson balls seem to bounce higher, on average than the Athletic Works balls. The two interaction plots from below, confirm the results from above. There also doesn't seem to be that much variation between the balls within a brand (judging by the second interaction plot given below).

Interaction Plot - Data Means for bounce



Interaction Plot - Data Means for bounce



## 5. Analysis of Variance and Residual Analysis

The ANOVA analysis was run in R and the results that were found are as follows:

#### Analysis of Variance Table

Response: bounce

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
pack	1	112.667	112.667	531.1429	1.162e-10 ***
replicate	1	0.667	0.667	3.1429	0.1039
pack:brand_price	2	237.750	118.875	560.4107	8.548e-12 ***
pack:brand_price:ball	8	2.417	0.302	1.4241	0.2869
Residuals	11	2.333	0.212		

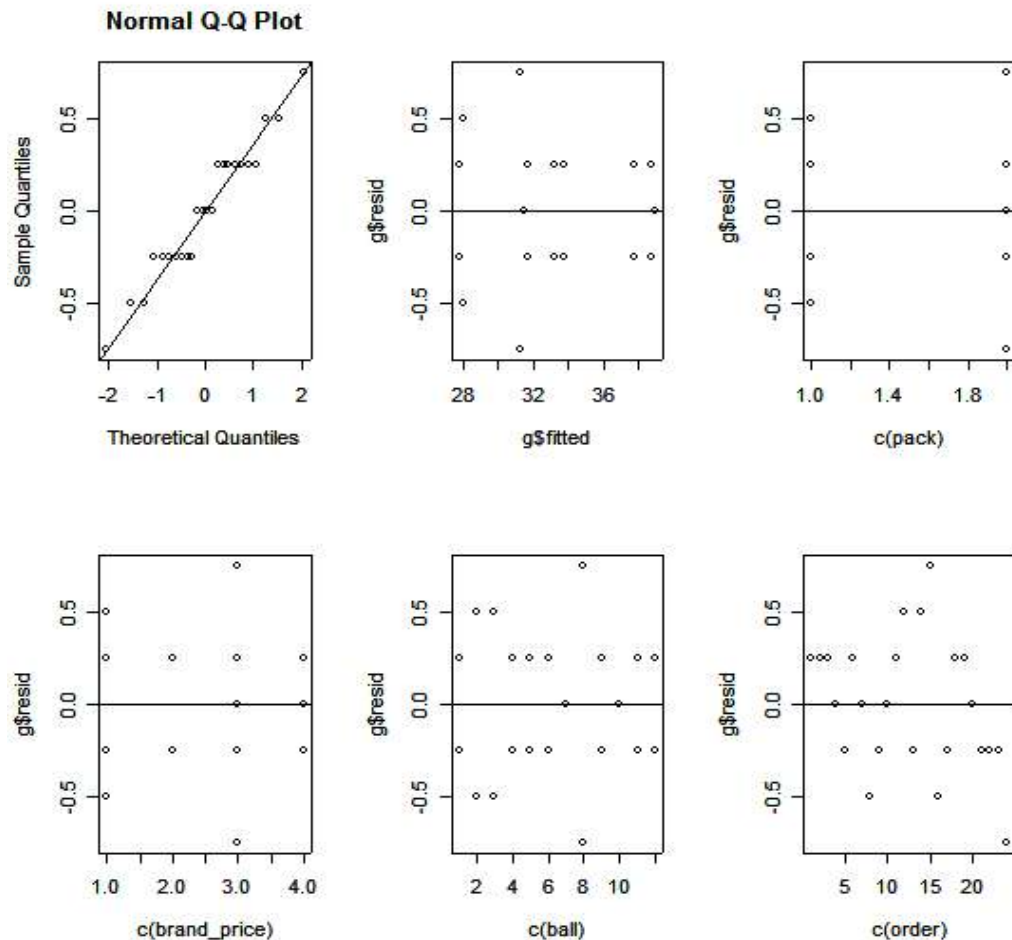
It appears that packaging and the brand (or brand price) within packaging are very significant factors when it comes to bounce height. The balls within a brand within a certain packaging type do not seem to be significant, which is not a surprising result. After all we would expect that the manufacturing process by which the balls are created would be very consistent and there would not be any significant differences among the balls. It also appears that the replicates are not a significant factor (at  $\alpha=0.05$ ), which shows that the observers (Dan and Angie) were also relatively consistent when recording bounce heights throughout the experiment. So let us remove the replicates from the model and redo the ANOVA table.

#### Analysis of Variance Table

Response: bounce

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
pack	1	112.667	112.667	450.6667	6.945e-11 ***
pack:brand_price	2	237.750	118.875	475.5000	3.744e-12 ***
pack:brand_price:ball	8	2.417	0.302	1.2083	0.3703
Residuals	12	3.000	0.250		

The validity of our conclusions cannot yet be certified before the analysis of the residuals is performed. The assumptions of normality and constant variance have to be verified before these results can be accepted. The QQ normality plot, the residual versus fitted values plot and residual versus different factor plots are given below.



From the plots given above there seem to be no major problems with the normality assumption or the homoscedasticity assumption. Since pack and brand-price are such significant factors we just wanted to double check the constant variance assumption by performing Bartlett's test (we is quite reliable given that the normality assumption holds). The results are as follows:

Bartlett test for homogeneity of variances  
data: bounce and pack  
Bartlett's K-squared = 0.6127, df = 1, p-value = 0.4338

Bartlett test for homogeneity of variances  
data: bounce and brand\_price  
Bartlett's K-squared = 1.2449, df = 1, p-value = 0.7423

As it can be seen the p-values are large in both cases and thus we cannot reject  $H_0$ . Thus, we conclude that the constant variance assumption holds, and so the conclusions from our ANOVA table regarding the significance of the model factors are perfectly valid.

## **Summary**

The overall conclusions for our experiment are that the most important factors that influence the bounce height of the balls are the packaging (regular vs. vacuum) and the brand (or brand\_price) within a certain packaging type. The brand Wilson seems to lead to the highest bounce height measurements, while the Ashley Scott brand balls “underperform”. It is now the “duty” of the tennis ball player to decide which of these brands is best (or most appropriate) to be used. What matters is that whichever brand he/she chooses he/she should expect no inconsistency (no significant differences) between the balls within that particular brand.

## APPENDIX

DATA:

	pack	brand	price	ball	replicate	order	bounce
1	-1	-1		1	1	6	28.0
2	-1	-1		2	1	14	28.5
3	-1	-1		3	1	8	27.5
4	-1	-1		1	2	17	27.5
5	-1	-1		2	2	16	27.5
6	-1	-1		3	2	12	28.5
7	-1	1		1	1	18	33.5
8	-1	1		2	1	1	34.0
9	-1	1		3	1	5	33.0
10	-1	1		1	2	21	33.0
11	-1	1		2	2	9	33.5
12	-1	1		3	2	19	33.5
13	1	-1		1	1	10	31.5
14	1	-1		2	1	15	32.0
15	1	-1		3	1	2	32.0
16	1	-1		1	2	20	31.5
17	1	-1		2	2	24	30.5
18	1	-1		3	2	23	31.5
19	1	1		1	1	4	39.0
20	1	1		2	1	11	38.0
21	1	1		3	1	3	39.0
22	1	1		1	2	7	39.0
23	1	1		2	2	22	37.5
24	1	1		3	2	13	38.5

Legend:

Pack – “-1” = regular; “+1” = vacuum sealed;

Brand\_Price: “-1” -> Ashley Scott (if pack = -1);

Athletic Works (if pack=+1);

“+1” -> Cooper (if pack=-1);

Wilson (if pack=+1);

Ball -> “1” – if ball 1 in whichever the pack;

“2” – if ball 2 in whichever the pack;

“3” – if ball 3 in whichever the pack;

Replicate -> “1” – if first replicate;

“2” – if second replicate for the same ball used in replicate 1;

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