

STAT 312 Lab 8

1. Recall 2(b) of Lab 7. Obtain the mean of this geometric random variable, with $P(X = k) = p(1 - p)^k$ for $k = 0, 1, 2, \dots$, by first obtaining the p.g.f. $E[z^X]$. [Hint: If this p.g.f. has $\rho > 1$ it can be differentiated term-by-term at $z = 1$. What is the result of this? Is $\rho > 1$?]
2. Obtain the bound (26.1) on the error in the approximation of $\sqrt[3]{9}$. [Hint: Find an upper bound on $f'''(x)$, valid in the range over which x varies.]
 - (a) Define what we mean by the ‘cumulants’ of a distribution.
 - (b) Show that the second cumulant of a distribution (which has an m.g.f.) is the variance.
3. Let X represent the number of failures before the first success, in a sequence of independent Bernoulli experiments with $P(\text{success}) = p$.
 - (a) What is the m.g.f.?
 - (b) Suppose that X_1, \dots, X_r are independent, each having the same distribution as X . What is the m.g.f. of their sum?
 - (c) Use (b) and appropriate results from the lectures (then no calculations will be necessary) to obtain the distribution of the sum. Could this have been foreseen merely from the definitions of these r.v.s.? How?
4. In the preceding question, consider the distribution of the number of remaining failures to the first success, *given that there have already been m failures without a success*. This is

$$P(X = m + n | X \geq m) = \frac{P(X = m + n)}{P(X \geq m)}.$$

(The equality follows (how?) from $P(A|B) = P(A \cap B) / P(B)$.) Continue this by explicitly evaluating these probabilities. Interpret the result as a ‘memoryless’ property – what does it say about the chance of a gambler ending a ‘losing streak’?

...over

5. Suppose that the continuous r.v. T , with d.f. F and density f , represents the age at failure of a certain component. The *failure rate* $h(t)$ at time t is defined as the ‘instantaneous’ probability of failure in the next (very small) Δt units of time, given survival up to time t : $h(t)\Delta t \approx P(T \leq t + \Delta t | T > t)$. More precisely,

$$h(t) = \lim_{\Delta t \rightarrow 0} \frac{P(T \leq t + \Delta t | T > t)}{\Delta t}.$$

- (a) Show that

$$h(t) = \frac{f(t)}{1 - F(t)}.$$

- (b) Suppose that T has the exponential distribution, with $F(t) = 1 - e^{-\lambda t}$, $t \geq 0$. Show that the hazard rate is constant.
- (c) Show that a r.v. with a constant hazard rate *must* have an exponential distribution.