STAT 312 Lab 7

1. Suppose that, after doing a regression of a variable Y on some independent variables, and plotting the residuals against the fitted values, you decide that it seems as though the variance of Y is proportional to the square of the mean: $\sigma_Y^2 \propto \mu_Y^2$. Suggest a transformation of Y that should stabilize the variance.

2. Suppose that a random variable X has the geometric distribution, with $P(X = k) = p\,(1-p)^k$ for $k = 0, 1, 2, ...$ This is the distribution of the number of 'failures' before the first 'success' in a sequence of Bernoulli trials, on each of which the probability of 'success' is $p \in (0, 1)$.

   (a) Show that the mean exists, i.e. that the sequence of partial sums of the series $\sum kP(X = k)$ converges.

   (b) Sum this series to obtain $E[X] = (1-p)/p$.

3. The government sometimes embarks on 'stimulus spending' – they inject a certain amount, say 'c' dollars, into the economy and count on a 'multiplier effect'. This means that a certain proportion p of this money will be spent by the recipients in the first month, and a proportion p of that spending will be re-spent in the second month, etc. How much has been spent after one year? Derive an equation which you would solve (by computer) in order to determine the number of months required in order that the entire initial amount be re-spent.

4. Suppose that $\{a_i\}_{i=1}^{\infty}$ is such that $s = \sum_{i=0}^{\infty} a_i$ exists. Show that then 'the tail of the series goes to zero': $\sum_{i=n+1}^{\infty} a_i \to 0$ as $n \to \infty$.

5. Uniqueness of power series: suppose that $\sum_{n=0}^{\infty} a_n x^n$ and $\sum_{n=0}^{\infty} b_n x^n$ both represent a function $s(x)$, for $|x| < \rho$ ($\rho > 0$). Show that then $a_n = b_n$, $n = 0, 1, 2, ...$

6. Prove: If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are sequences such that $b_n \to b \neq 0$ and $\frac{a_n}{b_n} \to 0$, then $a_n \to 0$.

7. Obtain a series expansion of $(1-x)^{-3}$, valid for $|x| < 1$. [*Hint*: What do you get when you differentiate $(1-x)^{-1}$?]