PROBLEM CORNER

Gábor J. Székely, Column Editor

Problem No. 19: Maximize the Expected Work Force

A company hires n people. It declares a holiday on the birthday of any employee. What value of n maximizes the expected work force per year?

A one-year (extension of your) subscription to *Chance* will be awarded to up to two correct solutions received by the Column Editor by January 12, 2001. If there are more than two correct solutions, then the winners will be selected randomly. As an added incentive, a picture and short biography of each winner will be published in a subsequent issue. Please mail or email your solution to Gabor J. Szekely, Problem Corner Editor, Department of Mathematics and Statistics, Bowling Green State University, Bowling Green, OH 43403-0221, USA; email: *gabors@bgnet.bgsu.edu*, by January 12, 2001. Please note that winners to the problem contest in any of the three previous issues will not be eligible to win this issue's contest.

Solution to Problem No. 17: Chance of Having a Saddle-point, *Chance*, Vol. 13, No. 2, Spring 2000, p. 60.

An n by m matrix A has independent, identically distributed entries from an arbitrary continuous distribution. Find the probability that A has a saddle-point (= an entry which is the both the smallest in its row and the largest in its column).

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Solution by Robert Dawson (a similar solution was submitted by Douglas P. Wiens and Menggang Yu)

First, note that there can be at most one saddle-point. Suppose, for a contradiction, a_{ij} and a_{kl} to be saddle-points; then $a_{ij} > a_{il} > a_{kj} > a_{ij}$. The absence of a saddle-point and the presence of one in column 1, column 2,..., column n thus form a mutually exclusive and exhaustive set of events. The probability of a saddle-point in a specified column is equal to the probability that each of the $n-1$ entries sharing a row with the smallest element of the column is yet smaller. That is—if we assign n − 1 "small" numbers at random within the $m + n - 1$ locations on the union of the row and column, what is the probability that they are all in the $n-1$ locations not in the column? Clearly the answer is

$$\frac{(n-1)!\,m!}{(m+n-1)!};$$

summing this over n columns, we obtain

$$P(\text{saddle-point}) = \frac{n!\,m!}{(m+n-1)!}.$$

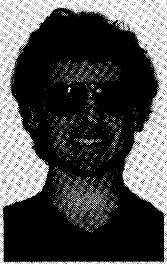
Editorial Comments: Further correct and nice solutions were submitted by two former problem corner winners, Michael P. Cohen (National Center for Education Statistics, 1900 K Street NW #9060, Washington, DC 20006-1103, email: *mcohen@inet.ed.gov*) and Bob Agnew (currently he has an independent consulting firm called Agnew Analytics, and is the editor of PARAMETER, newsletter of the Chicago Chapter ASA; his address: 840 Deere Park Court, Deerfield, IL 60015-3007, email: *raagnew@aol.com*).

Further correct solutions were submitted by Enkelejd Haskova (*enkelejd.haskova@stat.unibe.ch*, University of Bern), Kathleen E. Lewis (*lewis@oswego.edu*, Department of Mathematics, SUNY Oswego, NY 13126), Glenn Hofmann (*glenn@gauss.cfm.udec.cl*), Aidan Palmer (*aidan.palmer@gte.net*), and Zhiwei Zhang (*zhzst5@pitt.edu*, graduate student, Department of Biostatistics, University of Pittsburgh).

Additional Comment on Problem No. 16

Professor H.A. David (Iowa State University) called my attention to an incorrect remark on the solution to Problem No. 16, Vol. 13, No. 3, p. 51. Based on a letter from some problem solvers I claimed the following: The covariance of the sample mean and an arbitrary ordered sample element is always $1/n$ for general distributions. (The correct solution of the proposed problem proved that the covariance of the normal sample mean and sample median is $1/n$.) The above mentioned generalization "led me up the garden path." A simple counterexample is the standard exponential parent distribution where the covariance of the sample mean and the smallest ordered sample element is $1/n^2$.

Puzzle Corner Winners



Lawrence Schwartz is an Associate Director of Statistics for the Pharmaceutical Division of Bayer Corporation in West Haven, Connecticut. When not spending time with his primary interest (his wife and two sons), he can be found exploring his hobbies of sports, games, numbers, and reading. His address is: Bayer Corporation, Pharmaceutical Division, 400 Morgan Lane, West Haven, CT 06516, USA; email: *lawrence.schwartz.b@Bayer.com*.

Nathan Wetzel is an Assistant Professor in the Department of Mathematics and Computing at the University of Wisconsin – Stevens Point. His primary interests are statistical graphics and the teaching of statistics in the undergraduate curriculum. He enjoys crossword puzzles, gardening, and origami. His address is: Department of Mathematics and Computing, University of Wisconsin-Stevens Point, Stevens Point, WI 54481, USA; email: *nwetzel@uwsp.edu*.

Problem Corner Winners

Robert Dawson teaches mathematics at Saint Mary's University and has also taught first- and second-year statistics on various occasions. His hobbies include making wine and beer and playing the recorder. He lives with his wife (Bridget, meteorologist), their dog (Silkie, telephone pole inspector), cat (Dusty, rodent control specialist), and two sons (Alex and Ian, entropy sources). His address is: Department of Mathematics and Computing Science, Saint Mary's University, Halifax, NS, Canada, email: *rdawson@stmarys.ca*.

Douglas P. Wiens is a Professor of Statistics in the Statistics Centre, Department of Mathematical Sciences, at the University of Alberta in Edmonton, Alberta. He has been there since 1987, prior to which he was with Dalhousie University in Halifax, Nova Scotia. He received his Ph.D. (Statistics) in 1982 from the University of Calgary, where he previously studied mathematical logic before switching to statistics. He works in the general field of robustness, in particular robust methods in the design of experiments. He and his wife have two children (10 and 23); his hobbies include bicycle touring and cross-country skiing. His address is: Department of Mathematical Sciences, University of Alberta, Edmonton, AB, CANADA T6G 2G1, email: *doug.wiens@ualberta.ca*.

Menggang Yu is currently a Ph.D. student at the Department of Biostatistics, University of Michigan. He graduated from Bowling Green State University in 1999, where he majored in applied statistics. His hobbies include playing pool, table tennis, and tennis. He loves Michael Jackson. His address is: Department of Biostatistics, University of Michigan, 1420 Washington Heights, Ann Arbor, MI 48109–2029, USA; email: *menggang@m.imap.itd.umich.edu*.