Poisson, Polynomials, Philately -
a theory of non-statistical linkages

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We shall investigate three conjectures of interest:

1) The ‘small world’ hypothesis;

2) We all owe everything to Siméon Poisson;

3) There are some elements of mathematical trivia unknown even to GPHS.

Some genealogy - DPW, GPHS and others. Use the .mht file and the entire width of the display.

- Hilbert (1900) published a list of 23 problems, all unsolved at the time.

- 10th: Find an algorithm to determine if a given polynomial Diophantine equation with integer coefficients has an solution in (non-negative) integers.

- Yuri Matiyasevich (1970) showed that no such algorithm can exist; he did this by showing that 'A set of integers is Diophantine iff it is recursively enumerable':

  – S is <u>Diophantine</u> if there is a polynomial P, with integer coefficients and variables $\alpha, x_1, ..., x_m$, such that

  $$\alpha \in S \Leftrightarrow \text{the equation } P(\alpha, x_1, ..., x_m) = 0 \text{ is solvable in integers.}$$

  – S is <u>recursively enumerable</u> if there is an algorithm (i.e., a Turing machine) which will list its elements.

- The 'only if' – that a Diophantine set is recursively enumerable, is easy: enumerate the vectors $(\alpha, x_1, ..., x_m)$ by the diagonal method and work through them one by one; if there is a solution $P(\alpha, x_1, ..., x_m) = 0$ it will <u>eventually</u> show up. **If there isn't one you won't ever learn that, in this way.**

- There is no algorithm which will decide whether or not a set is recursively enumerable (Gödel), thus none which will decide whether or not a set is Diophantine. Thus

Hilbert's tenth problem is unsolvable.

- For a long while prior to Matiyasevich's breakthrough, it was known (Martin Davis, Julia Robinson, Hilary Putnam) that the final stone would be in place if one could show the existence of a set of numbers, with exponential growth, which was Diophantine, i.e. a sequence $S = \{f_1, f_2, ...\}$, increasing at an exponential rate, and a polynomial P, such that

  $f \in S \Leftrightarrow P(f, x_1, ..., x_m) = 0$ is solvable in integers.

- Matiyasevich showed that the Fibonacci numbers have this property. Equivalently, the Fibonacci numbers are the positive range of the polynomial

  $$f\left(1 - P^2(f, x_1, ..., x_m)\right),$$

  as the variables range over the integers.

- A consequence is that many other interesting sequences are Diophantine:

  - Primes (Jones, Sato, Wada, Wiens 1976)

  - Mersenne primes (Wiens, MSc thesis, 1974)

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2004

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Click on the hat pin to open the Mersenne prime representing polynomial