Inherent transversely isotropic elastic parameters of over-consolidated shale measured by ultrasonic waves and their comparison with static and acoustic in situ log measurements

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Received 17 May 2007
Accepted for publication 4 January 2008
Published 30 January 2008
Online at stacks.iop.org/JGE/5/103

Abstract

In terms of elastic anisotropy, sedimentary shales may be considered to have transverse symmetry. Experimentally determining the five independent elastic constants required for this case remains challenging. This paper proposes the use of ultrasonic waves in determination of the five constants. Arrays of specially constructed transducers with different modes of vibration were mounted on samples trimmed from natural cores to measure ultrasonic $P$-waves and $S$-waves along the horizontal, vertical and $45^\circ$-inclination axes on the samples. The elastic constants calculated from these wave velocities were compared to those determined from static tests and acoustic in situ logs. The static tests included drained triaxial compression and confined torsion tests. The acoustic logs were conducted in an open hole using monopole compressional and dipole shear transmitters and receivers. It was found that the elastic constants determined by the static tests were much lower than those determined from the ultrasonic tests and acoustic logs. Discrepancies among the elastic constants determined from these three different methods are discussed and explained.

Keywords: shale, transversely isotropic elasticity, ultrasonic wave, static test, acoustic log

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Natural soils or rocks exhibit two common types of anisotropy in stiffness: inherent and stress induced. Inherent isotropy is a physical intrinsic characteristic in the material and independent of the external loads. This type of anisotropy is attributed to the depositional environment and mineral fabric of the soil or rock mass (Casagrande and Carrillo 1944). Strong particle orientation with cementation plays a major role in this type of anisotropy. Stress-induced anisotropy is predominant in granular soil mass when the particle reorientation and rearrangement occurs under stress rotation (Oda et al 1985). Anisotropic in situ stresses could also produce stress-induced anisotropy in a deposited soil or rock mass. This paper focuses on the study of inherent anisotropy in stiffness in a clay shale at small strains up to 0.01%.

Clay shales are often deposited over areas of large lateral extent. The primary vertical deformations they experience
subsequent to deposition are essentially one dimensional, in the absence of horizontal tectonic loading. In addition, clay shales are predominantly made up of clay particles which are primarily oriented parallel to the horizontal plane forming pronounced laminations or beddings (figure 1(a)). Such structured materials possess a single vertical rotational axis of symmetry (Z-axis; figure 1(a)). The elastic properties along the horizontal directions X and Y are the same (isotropic) regardless of azimuth. Then, the choice of the X and Y co-ordinate axes is arbitrary. However, the properties within the vertical plane vary depending on the direction with respect to the axis of symmetry. This hexagonal symmetry gives rise to a common type of anisotropy called transverse isotropy (TI).

Because of its preferential particle orientation in the horizontal direction, the over-consolidated clay shale studied in this paper displays the stiffness in the horizontal direction higher than that in the vertical direction.

There are numerous works on the mathematical derivation of elastic parameters of TI geomaterials from both the geophysical (e.g., Hearmon 1961,Musgrave 1970, Auld 1973, Thomsen 1986, Cheadle et al 1991) and geotechnical (e.g., Barden 1963, Pickering 1970, Graham and Houlsby 1983) communities. These theoretical investigations illustrated that five independent elastic constants are required to completely describe the stiffness matrix of a TI medium. Experimental determination of all five independent TI elastic parameters in natural materials is limited. In the past, dynamic tests (such as resonant column tests and wave propagation tests) and static tests (such as triaxial tests) were used to determine dynamic and static stiffness values, respectively. Because of different strain magnitudes, strain rates and stress paths used in dynamic and static tests, it has been an engineering practice to treat dynamic and static stiffness values separately. A number of researchers (e.g., Jones and Wang 1981, Johnston and Christensen 1995, Vernik and Liu 1995, Wang 2002) used ultrasonic techniques (∼1 MHz) to provide the complete set of necessary elastic properties on indurated shales. Other researchers combined results from the quasi-static stress-path triaxial tests with the dynamic bender-element tests to determine all of the five independent elastic parameters in soils (e.g., Hoque et al 1996, Kuwano et al 1999, Lings et al 2000, Kuwano and Jardine 2002) and rocks (Homand et al 1993). The stress-path triaxial tests determine only two independent elastic parameters, $E_v$ and $v_{vh}$, and a combination of $E_h$ and $v_{hh}$. The bender element tests yield the other two parameters, $G_{vh}$ and $G_{hh}$, by passing a horizontally polarized shear wave propagating along the vertical (axial) and horizontal core axes, respectively. Results from these two types of tests (triaxial and bender element) are combined to determine all five independent elastic parameters of TI material. However, such a combination of static (from triaxial test) and dynamic (from bender element test) moduli may lead to some inconsistencies in the strain levels and frequencies (quasi-static to ∼1 MHz) used in the tests (Pennington 1999). Tatsuoka et al (2001) reported that the strain-rate dependence of stiffness of geomaterials at strains less than about 0.001% is very low. They also pointed out that it is not necessary to distinguish between dynamically and statically measured elastic stiffness values, in particular for fine-grained soils, when the measurements are conducted under the same testing conditions (stress path, stress history and drainage).

There have been a number of studies in rock materials in which the differences between the static and dynamic measurements of the elastic constants are reported. The dynamic modulus typically exceeds the static modulus by a substantial fraction. Simmons and Brace (1965) observed that the ratio of static modulus to dynamic modulus increases from 0.5 at low confining pressures (close to atmospheric) to unity at high confining pressures of 300 MPa on granite samples. Cheng and Johnston (1981), King (1983) and Murphy (1984) found a similar difference between static and dynamic moduli in studies on sandstone, limestone, oil shale and crystalline rock samples. Such a trend extends to differences in the apparent moduli under the extreme strain rates encountered in shock wave conditions (Bless and Ahrens 1977). Most recently, Tigrek et al (2005) developed an empirical relationship between the static and dynamic moduli of two sandstones from simultaneous pseudo-static and ultrasonic velocity measurements in a triaxial cell under the assumption that the ultrasonic measurements were representative of seismic band measurements. One additional complication is that there can be a large dispersion between high frequency (∼1 MHz) laboratory measurements on cores and low frequency (∼10 to 100 Hz) seismic band velocities in saturated fluid-filled rocks (Winkler 1986). Fluid saturation will further separate the static and dynamic values relative to the dry condition. Absorption of water to grains lowers the free surface energy between adjacent mineral grains reducing their static modulus while the compressibility of the same liquid results in an increase of the dynamic modulus (e.g., King 1983, 2002). Domenico (1995) conducted ultrasonic tests on dry and saturated Ottawa sand samples. He observed that compressional waves (P-wave) travelled much slower in dry samples than in saturated samples. At an effective

Figure 1. (a) A transversely isotropic body in which the X–Y plane is the plane of isotropy (v and h denote vertical and horizontal directions, respectively). (b) Scanning electron micrograph images of the horizontal X–Y plane (top panel) and the vertical x–z plane (lower panel). The scale bar is 10 µm.
confining stress of 4 MPa, the $P$-wave velocity decreased from 6200 m s$^{-1}$ in brine-saturated samples to 2500 m s$^{-1}$ in gas-saturated samples. There was no difference in $S$-wave velocity in brine- and gas-saturated samples. The calculated Poisson’s ratio decreased from 0.10 (at 35 MPa) to 0.08 (at 4 MPa) with decreasing confining stress in gas-saturated samples whereas the ratio increased from 0.36 (at 35 MPa) to 0.42 (at 4 MPa) with decreasing confining stress in brine-saturated samples. Similar trends between Poisson’s ratio and confining stress in gas- and brine-saturated unconsolidated samples were also observed by Mukerji et al. (2002).

There has been, in the past, a significant amount of work on understanding the elastic anisotropic behaviour of soils and rocks. Experimental data on the complete determination of the five elastic constants in over-consolidated clay shales are limited. In this paper, the five elastic parameters are independently determined on an over-consolidated clay shale using ultrasonic and static tests. These results are then compared against the elastic parameters of the same material obtained from acoustic and density logs carried out in an open borehole. Discrepancies among the results measured from three different methods are explored in detail and explained with reference to different measurement techniques and testing conditions imposed on the tested samples.

2. Theoretical background

2.1. Transverse isotropy in elastic stiffness

Before describing the elastic behaviour of TI materials, it is useful to briefly review that for idealized isotropic materials. Only two independent elastic constants are required in such an idealized case. A variety of combinations are used depending on the application. They may be Young’s modulus $E$ and Poisson’s ratio $v$, or the bulk $K$ and shear $G$ moduli, or the Lame parameters $\lambda$ and $G$, or the elastic parameters $C_{11}$ and $C_{44}$ in reduced Voigt notation (Nye 1985). Two elastic body waves exist in isotropic materials, i.e., a longitudinally polarized $P$-wave and a transversely polarized $S$-wave. Both wave velocities are independent of the direction of propagation through the material and are related to the elastic parameters via (Bourbie et al. 1987)

$$
P = \sqrt{\frac{K + \frac{4}{3}G}{\rho}} = \sqrt{\frac{\lambda + 2G}{\rho}} = \sqrt{\frac{G(4E - G)}{(3G - E)\rho}} \tag{1}
$$

$$
S = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{3K - \lambda}{2\rho}} = \sqrt{\frac{3KE}{9K - E}\rho} \tag{2}
$$

where $P$ and $S$ represent the velocities of the longitudinally and transversely polarized waves, respectively. As such, any assortment of the elastic parameters may be obtained from the measurements of $P$- and $S$-wave velocities together with the knowledge of the bulk density $\rho$.

In contrast, the $P$- and $S$- wave velocities in anisotropic materials vary with the directions of both wave propagation and particle motion polarization. Generally, there are one $P$ (longitudinal polarization) and two distinct $S$ (transverse polarization) velocities in any direction. The polarizations of the two independent $S$ waves are orthogonal and are controlled by the symmetry of the material. The level of anisotropy is controlled by the symmetry beginning with complete symmetry for isotropic materials (e.g., silicate glass) requiring two elastic constants through to the most complex case of triclinic (e.g., plagioclase crystal) requiring 21 independent constants. In studies of uncracked composite earth materials, however, considerations of the symmetries introduced during sedimentation and simple metamorphic deformation usually may be limited to isotropic, hexagonal (i.e., transversely isotropic) and orthorhombic. These last two symmetry classes require five and nine independent elastic constants, respectively. The study presented in this paper assumes that the clay shale material is transversely isotropic.

The constitutive law for TI shale can be expressed in terms of five independent parameters: $E_x$, $E_y$, $v_{xy}$, $v_{zh}$ and $G_{zh}$ (Gautam 2004, Gautam and Wong 2006), and the corresponding matrix becomes

$$
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\gamma_{yz} \\
\gamma_{zx}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_x} & \frac{v_{zh}}{E_x} & \frac{v_{zh}}{E_x} \\
\frac{v_{zh}}{E_x} & \frac{1}{E_x} & \frac{v_{zh}}{E_x} \\
\frac{v_{zh}}{E_x} & \frac{1}{E_x} & \frac{v_{zh}}{E_x} \\
\frac{1}{G_{zh}} & \cdot & \cdot \\
\cdot & \frac{1}{G_{zh}} & \cdot \\
\cdot & \cdot & \cdot
\end{bmatrix}
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\tau_{yz} \\
\tau_{zx}
\end{bmatrix} + \begin{bmatrix}
\sigma_{xz}' \\
\sigma_{yz}' \\
\sigma_{zx}' \\
\tau_{xy}'
\end{bmatrix} \tag{3}
$$

where $\varepsilon_{ii}$, $\gamma_{ij}$ = normal and shear strains, respectively ($i, j = x, y, z$); $\sigma_{ij}'$, $\tau_{ij}'$ = effective normal and shear stresses, respectively ($i, j = x, y, z$); $E_x$ = Young’s modulus measured along the vertical axis of the rotational symmetry (Z-axis); $E_h$ = Young’s modulus measured at any direction lying within the horizontal (X-Y) plane; $v_{zh}$ = Poisson’s ratio for the deformation along the horizontal X- or Y-direction upon the vertical loading along the Z-direction; $\tau_{zh}$ = Poisson’s ratio for the deformation along the horizontal X-direction upon the horizontal loading along the Y-direction; and $G_{zh}$ = shear modulus within the X-Z plane.

For wave propagation applications, Hooke’s law of (3) may be rewritten in terms of strains according to the Voigt
where $C_{ij}$ are elastic parameters, ($i, j = x, y, z$). For TI materials, $C_{22} = C_{11}, C_{55} = C_{44}, C_{12} = C_{11} - 2C_{66}$ and $C_{23} = C_{13}$.

Equation (4) also contains five independent elastic constants: $C_{11}, C_{33}, C_{44}, C_{66}$ and $C_{13}$. These five constants can be related to the elastic body wave velocities using Christoffel equations (Cheadle et al. 1991, Mah and Schmitt 2001a). Thus, determination of 5 $C_{ij}$ constants requires five independent wave measurements. The study in this paper proposes to use three compressional waves ($P_{xx}$, $P_{zz}$ and $P_{xz}$), and two shear waves ($S_{xz}$ or $S_{zx}$ and $S_{xy}$). The first subscript (index) refers to the wave propagation direction. $P_{xx}$ represents a P-wave propagating along the X-axis with polarization parallel to this axis. $S_{xz}$ corresponds to a vertically polarized shear wave propagating also in the X-direction. For the study in this paper, the Christoffel equations become as follows:

$$
\begin{pmatrix}
\sigma_{x x} \\
\sigma_{y y} \\
\sigma_{z z} \\
\tau_{x y} \\
\tau_{x z} \\
\tau_{y z}
\end{pmatrix} =
\begin{pmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{pmatrix}
\begin{pmatrix}
\epsilon_{x x} \\
\epsilon_{y y} \\
\epsilon_{z z} \\
\gamma_{x y} \\
\gamma_{x z} \\
\gamma_{y z}
\end{pmatrix}.
$$

(4)

where $C_{ij}$ are elastic parameters, ($i, j = x, y, z$). For TI materials, $C_{22} = C_{11}, C_{55} = C_{44}, C_{12} = C_{11} - 2C_{66}$ and $C_{23} = C_{13}$.

3. Experimental program

3.1. Material

Shale samples used in the ultrasonic wave measurement and static tests were recovered at a depth of 294–295 m from an exploration well (Western Canadian Survey Co-ordinates: LSD 13-09-66-04) near Cold Lake, Alberta, Canada. These samples belong to the Westgate Formation of the Upper Cretaceous Colorado Group (Beaumont 1984, Stelek and Koke 1987, Leckie and Reinson 1993). The Westgate (WG) Formation in the Cold Lake area is about 50 m thick. It is characteristically heterolithic, comprising of dark finely interbedded siltstone and shale laminae along with minor bentonite layers. The shale material has a water content of about 16–17%, liquid limit of 69–80%, plastic limit of 30–31% and unit weight of 20.15–20.93 kN m$^{-3}$. The results from hydrometer tests and clay separation methods show that the shale material contains 95% less than 0.1 mm and 57–60% less than 2 µm (clay fraction). X-ray diffraction data show that the major component of clay minerals is a mixed layer illite/smectite and illite with kaolinite and chlorite to a lesser extent. Scanning electron microscopy (SEM) conducted on selected WG formation shale samples show that most of the clay particles are in face-to-face contact forming a ‘bookhouse’ type of structure. Such parallel bedding alignment of clay particles results in distinctly different fabrics along the horizontal and vertical directions (figure 1). This textured layering makes shale a TI medium. The horizontal X–Y plane lies parallel to the bedding (figure 1(a)). The Z-direction is perpendicular to the bedding (figure 1(b)) and in this case parallel to the drilling direction.

A conventional double barrel coring technique was used in the core drilling and recovery. In the field, the cores recovered were immediately tightly sealed inside Teflon shrink tubes which were then encased in aluminium tubes filled with mineral oil. The main purpose of such sealing with shrink tubing and mineral oil was to provide some confinement and to prevent moisture loss from the core samples.

3.2. Ultrasonic (dynamic) tests

3.2.1. Sample preparation. Shale samples of 89 mm in diameter and about 90 mm in length were trimmed from cores for the ultrasonic tests. Mounting transducers on the sample required preparation of flat surfaces on the shale sample. Because of the weakly cemented nature of the shale material, manual cutting with a miter box and a hack saw was used in the surface trimming. The trimmed surface was smoothed using a piece of fine sand paper. The shale was water saturated, and subjected to moisture loss when exposed to the atmosphere. When the sample was not being used, it was placed in a sealed plastic tank with its base filled with water-saturated drierite crystals. When the crystals were saturated, they would turn from blue to pink in colour indicating the level of humidity in the tank. During the sample preparation, only the side that was being worked on was exposed with the remaining sides sealed in paraffin wax. It was found that the moisture loss during the sample preparation was insignificant when comparing the moisture contents of the test samples with those of the fresh intact samples.

3.2.2. Transducer preparation. Transducers used in this study are identical to those developed by Mah and Schmitt
Inherent transversely isotropic elastic parameters of over-consolidated shale

Horizontally Polarized S-wave

Vertically Polarized S-wave

Figure 2. Photograph showing transducers for measurements of \( P_{xx} \), \( S_{xy} \) and \( S_{xz} \).

Figure 3. Schematic showing transducer placement and polarization of various piezoelectric ceramics on sample for which velocities are determined (note that, strictly, \( S_{45x} \) should be denoted \( S_{45y} \) as drawn but as these velocities are the same in a TI medium \( X \) has been retained in order to signify that the polarization is parallel to the layering plane).

Figure 4. Experimental setup for the ultrasonic wave measurements (jacketed samples are subject to isotropic confining stress).

The confining pressure system shown in figure 4 consists of electrical feedthroughs in the confining pressure cell connecting the piezoelectric transducers to the pulser and the oscilloscope. The transmitter was activated with a 200 V step function with 5 ns rise time. The receiver voltages were sampled at 100 M samples/second for 150 \( \mu \)s. The shale sample was subjected to cyclic hydrostatic compression inside the cell. The confining stress was varied from 0 to 10 MPa with the waveforms acquired approximately at every 1 MPa stress increment. No back pressure system was used to saturate the shale sample in the ultrasonic measurement test.

Transit times were picked from the first extreme value of the waveform on the raw records when possible, and checked after FFT bandpass filtering to remove low frequency bias (0–0.05 MHz) and high frequency noise (1.2–1.7 MHz). Values of \( P = 2160 \) m s\(^{-1} \) and \( S = 700 \) m s\(^{-1} \) were used to account for the small delays introduced by the lead electrodes and the co-axial cable lengths. Velocities were calculated as the ratio of the transit time to the length of the propagation path. The sample deformation occurring during the cyclic hydrostatic compression was small (about 0.3%) and was not considered in the velocity calculation directly, but was

(2001a, 2001b) where the methodology was evaluated on isotropic soda-lime glass and orthorhombic composite of laminated fibre mats in phenolic epoxy. Details on the transducer construction and characterization and the level of experimental errors induced in the ultrasonic tests are found in the above references, and a brief description is given herein.

\( P \)-wave (circular disc of 25 mm in diameter) and \( S \)-wave (square plate of 22 mm in length) transducers were constructed from piezoelectric crystals made from lead ziconate titanate material. Both transducers have a thickness of 2 mm, and a 1 MHz primary mode resonance. The orientation of the \( S \)-wave transducers controls their polarization. When the direction of the particle motion is parallel to the bedding, a horizontally polarized shear wave (\( S_{xy} \)) is produced. When the direction of the particle motion is perpendicular to the direction of the bedding, a vertically polarized shear wave (\( S_{xz} \)) is produced. Piezoelectric transducers of finite sizes could result in the co-production of a parasitic \( P \)-wave which may not provide reliable transit times.

A thin copper strip was bonded to the top surface of the transducer using silver paint. Once the silver paint has solidified, a thin layer of quick setting epoxy was applied over the top surface of the transducer to secure the copper strip to the transducer and to protect the top surface electrically. A piece of lead foil (5 \( \mu \)m thick) was attached to the underside of the transducer using the silver paint. This acted as a ground for the transducer. The transducers were then bonded to the surfaces of the shale sample using the quick setting epoxy (figure 2). Two wires were soldered to the transducer, one to the lead foil as the ground and the other to the copper strip as the positive end. The sample was then sealed in silicon rubber to protect the transducers. The sample was finally encased in a heat shrink tube with both ends sealed with urethane liquid (Flexane\textsuperscript{TM} 80 Liquid) to prevent communication of the sample with the confining (pressure) fluid.

Five pairs of piezoelectric transducers were mounted on the trimmed shale sample to measure the \( P_{xx} \), \( P_{zz} \), \( P_{45} \), \( S_{xz} \), or \( S_{zx} \) and \( S_{xy} \) modes as shown in figure 3. The horizontal and vertical distances were about 74 mm, and the distance along the diagonal path was about 77 mm.
3.3. Static tests

Three test series were conducted to determine the five independent elastic constants at strains of about 0.01% and an effective confining stress of 6 MPa. In the first test series, drained triaxial compression tests on full-diameter cores (89 mm in diameter and 90 in length) were carried out to measure $E_v$ and $\nu_{vh}$ directly. The second test series included drained triaxial compression tests on horizontal core plugs of 51 mm in diameter drilled from the full-diameter cores to measure $E_h$ and $\nu_{vh}$ directly. The third test series involved confined torsion tests on full diameter cores (89 mm in diameter and 150 mm in length) for determination of $G_{vh}$. Details of the sample preparation, testing equipment and procedures can be found in Gautam (2004) and Gautam and Wong (2006).

In the drained triaxial compression test, the sample was allowed to consolidate at an effective confining stress of 6 MPa. Then, an axial stress was applied to the sample in increments of 20 kPa, and the axial and radial strains in mutually perpendicular directions were measured directly using local deformation transducers of 0.001 mm resolution (figure 5(a)).

For the confined torsion tests, a pressure cell of 10 MPa capacity similar to a standard triaxial cell was designed and fabricated. The modified triaxial cell (torsion test cell; figure 5(b)) comprises a specially designed bottom flange and top platen to accommodate a full diameter core sample and for application of a torque to the test sample. In the confined torsion tests, the sample was allowed to consolidate at an effective confining stress of 6 MPa. Shear strains in increments of 0.001% were applied to the sample, and the torque was measured.

All triaxial and torsion tests were conducted with a back pressure of 3 MPa, i.e., the top and bottom drainage valves were open to the back pressure. To ensure the full dissipation of excess pore pressure induced by axial loading/unloading, the shale sample was allowed to consolidate for 30 min to reach its equilibrated state at each stress increment/decrement. This was achieved by monitoring the changes in the axial/radial strains continually until the equilibrated state. It was also
validated that the strain rates applied in the triaxial and torsion tests were less than the critical strain rate for full drainage conditions recommended by Bishop and Henkel (1969) (detailed calculations shown in appendix A).

3.4. Acoustic in situ logs

Acoustic logs were run in the same borehole from which the cores were recovered for the ultrasonic and static laboratory tests. A shear sonic tool was used in the open hole (150 mm in diameter) filled with drilling mud. The tool (about 75 mm in diameter and 10 m in length) uses a downhole weight and centralized springs travelling downward into the hole. The tool is equipped with one monopole compressional and one dipole shear transmitter and a semi-rigid receiver string with three shear receivers and four compressional receivers, allowing both compressional and shear data to be recorded during a single tool deployment. In addition, the azimuth of the dipole shear transmitter in the borehole is recorded using an orientation-module with each waveform, documenting the differences when anisotropy is encountered. Each acoustic wave could penetrate into the shale formation within a depth of about 230 mm. The monopole compressional transmitter (∼30 kHz) and receivers measure the elastic parameters in the vertical direction or Z-direction whereas the dipole shear transmitter (∼4 kHz) and receivers compare the shear wave propagation along the two vertical planes mutually perpendicular to each other.

4. Results

4.1. Ultrasonic (dynamic) tests—elastic wave velocity determination

It is important to note that equations (5) given for the conversion of the observed wave velocities to the elastic stiffness parameters assume ‘phase’ velocities, i.e., the velocity that would be observed for a plane wave. The phase velocity differs from the group (or ray) velocity through the material (Kebaili and Schmitt 1997). It is important to make this distinction because these two velocities would not necessarily be the same at a given direction in an anisotropic material. However, the travel times measured in laboratory experiments should represent phase velocities if the ratio of core-sample height to transducer width is less than 3 (Dellinger and Vernick 1994). Since the ratios used in our tests were within a range of 2.96–3.36, we are confident that phase velocities were measured in our tests.

Ultrasonic tests were conducted on two shale samples. Both tests produced very consistent behaviour, and thus results obtained from one sample are presented herein. The ultrasonic P- and S-waves obtained in the test are shown in figures 6 and 7, respectively. The first arrival versus confining stress curves show a weakly nonlinear shape because the elastic properties vary with the confining stress. The dynamic Young’s modulus of the shale material increases with increasing confining stress, and thus the first arrival time decreases with increasing confining stress. It is also important to note that little or no signal is recorded at the lowest confining stress. This problem might be associated with the amount of coupling that the transducers have with the samples. Even when they were mounted to the shale sample by high-strength epoxy, there might be still too much attenuation of the wave energy to achieve a good return signal at low confining stress. As the confining stress increases there is a vast improvement in the coupling and a great gain in amplitude.

There is one other, possibly important, observation upon closer examination of the waveforms in figures 6 and 7. The predominant frequencies of the P and S wave modes are ∼1 MHz and ∼0.1 MHz, respectively, despite the fact that the predominant frequencies of both types of transmitting and receiving piezoelectric crystals are close to 1 MHz. This effect may best be seen in figure 7 where the higher frequency parasitic P-wave appears, for example, as the sharp peak superimposed on a broader 20 µs width swell at about 38 ms
Suite of $S_{zx}$ and $S_{xy}$ waveforms simultaneously obtained versus isotropic confining pressure (arrows highlight travel time picks).

while the slower $S$-wave does not appear until 70 µs. The ‘swell’ at the arrival of the $P$-wave is an artefact of the FFT bandpass filtering of the signal used to remove a much lower frequency trend. Both the higher frequency $P$ and lower frequency $S$ arrivals are apparent in the raw data but with noisy traces. This large change in frequency caused difficulties in the initial analysis of the data, and until more results were obtained it was thought that the $S$ arrival was not that of the direct $S$ body wave but of a parasitic surface wave mode. However, additional measurements on the samples that took into account the variations in travel paths could not reasonably be explained by a parasitic surface wave mode leaving the conclusion that it must be the $S$ body wave; this mode is consequently highly attenuated such that the higher frequencies are completely lost. The reason for this is not known. It may be due to either a combination of the actual physical behaviour of this material or due to issues in coupling of the $S$-wave crystals to the material. Additional work will be required to address this issue.

The $P$-wave and $S$-wave travel at an increasing velocity with increasing confining stress as expected (figure 8) although the increases in velocity are small ranging from 1% to 5% over the range in confining pressures of 1 to 10 MPa. The velocity behaviour of the samples is similar to many other observations of TI media with $P_{45} > P_{45} > P_{zz}$. At some low confining stresses, the $S$-waves were not detected because of high attenuation in shale or loss of coupling the contact between the transducer and shale.

With the measured velocities shown in figure 8, (5) and (6) were used to determine the relationships among the dynamic Young’s moduli, Poisson’s ratios and confining stress, and the calculated results are plotted in figure 9. The dynamic Young’s moduli and Poisson’s ratios are more or less invariant with the confining stress. The small increasing trend is reasonable because the shale is heavily over-consolidated, i.e., the deformations occurring during the hydrostatic compression are small. It is important to note that no back pressure was applied to the shale sample in the ultrasonic measurement tests, and the sample was encased inside the urethane membrane without any drainage port. The stress relief due to core drilling could cause desaturation or gas evolving in the sample. The re-application of in situ stress to the sample might restore its saturation slightly below the full saturation state, and thus would not cause any excess pore pressure. Some unknown excess pore pressure might be induced above the 6 MPa confining stress. Hence, the confining stresses above the in situ stress (about 6 MPa) shown in figures 8 and 9 are not true effective confining stresses. Five independent acoustic wave measurements at the in situ effective confining stress of 6 MPa are $P_{xx} = 2435, P_{zz} = 2246, P_{45} = 2290, S_{xy} = 1099$ and $S_{zx} = 988$ m s$^{-1}$. These five readings were inverted to determine the five elastic constants, and were included in table 1 under the
4.2. Static tests

This section presents some representative static test results for illustration, and detailed results of all the static tests can be found in Gautam (2004) and Gautam and Wong (2006). Figure 10 shows the stress–strain response measured in a triaxial test conducted on a horizontally oriented core sample (TRIAXH 16). During the testing, the incremental cyclic axial stresses were applied while keeping the effective radial stress constant at 6 MPa. For the first cycle, an increment of 100 kPa axial stress was applied at 20 kPa steps and then was unloaded following the same steps. For the second and third cycles increments of 300 kPa and 900 kPa axial stresses were applied, respectively. For both of these cases, the stresses were applied at 100 kPa steps both during loading and unloading.

As clearly seen from figure 10, some plastic deformation was observed after each cycle. When the sample was unloaded from the first cycle after reaching an axial strain of 0.013%, the behaviour was nonlinear and almost hysteretic but still with a small irrecoverable strain. The amount of plastic deformation was found to be increasing with increasing strain level to which the sample was loaded.

The modulus was determined from the slope of the raw stress–strain curve of figure 10 at different incremental stress levels. It was found that the modulus remains constant—at very small strain (up to 0.006%) and then the modulus decreases with increasing axial strain during loading. This constant modulus or stiffness parameter is considered as an elastic parameter in this study.

Summarizing the stress–strain responses observed in static triaxial and torsion tests, nonlinear elastic behaviour was detected even at small strain levels of 0.01% during the primary loading. Non-detectable radial strain was monitored in the axial compression tests of strains up to 0.01% yielding Poisson’s ratios of a very small value or zero. Transverse isotropy was confirmed by comparing the radial deformations in mutually perpendicular directions. Linear elasticity was only observed during the unload–reload path. The secant (average) modulus and Poisson’s ratio values at an axial strain of 0.01% along the primary loading path were estimated and included in table 1, along with the shear modulus at 0.01% strain measured in the torsion tests.

4.3. Acoustic in situ logs

Results from acoustic logs show that the dipole shear waveforms in two vertical planes were consistent and almost identical. This implies that the in situ shale mass possesses a single vertical rotational axis of symmetry as that shown in figure 11(a). The P- and S-wave velocities measured at the depth where the cores were recovered from fall in ranges of 1960 to 2105 and 655 to 736 m s⁻¹, respectively. The ratio of P-wave velocity to S-wave velocity lies in a range of 2.6 to 3.0. The in situ log-derived velocities are found to be lower than those detected in the laboratory samples.

<table>
<thead>
<tr>
<th>Static testsa</th>
<th>Laboratory ultrasonic tests (Gassman’s model)b</th>
<th>Laboratory ultrasonic testsa</th>
<th>In situ acoustic logsd</th>
</tr>
</thead>
<tbody>
<tr>
<td>(drained)</td>
<td>(undrained) (drained)</td>
<td>(undrained) (drained)</td>
<td></td>
</tr>
<tr>
<td>$E_b$ (MPa)</td>
<td>1150</td>
<td>6694</td>
<td>$E = 3280$–$3780$</td>
</tr>
<tr>
<td>$E_v$ (MPa)</td>
<td>675</td>
<td>6099</td>
<td></td>
</tr>
<tr>
<td>$\nu_{bh}$</td>
<td>0</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td>$\nu_{vh}$</td>
<td>0</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>$G_{bh}$</td>
<td>431</td>
<td>1913</td>
<td></td>
</tr>
<tr>
<td>$G_{vh}$</td>
<td>2100</td>
<td>1822</td>
<td></td>
</tr>
<tr>
<td>$E_b/E_v$</td>
<td>1.70</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>$G_{bh}/G_{vh}$</td>
<td>1.33</td>
<td>1.24</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Comparison of five elastic constants determined from static tests, ultrasonic wave tests and in situ acoustic logs at a confining stress of 6 MPa.

a Measured at strains of 0.01%.

b Compressibility values of water and solid quartz = $4.5 \times 10^{-4}$ and $2.6 \times 10^{-5}$ MPa⁻¹, respectively; porosity = 0.30; density = 2100 kg m⁻³; assuming linear elastic isotropy.

c Measured at strains of 0.0001% and 0.0005% for P- and S-waves (1 MHz), respectively.

d Assuming linear elastic isotropy; P-wave (30 kHz), S-wave (4 kHz).
density logs recorded an average density of 2150 kg m$^{-3}$. Assuming linear elastic isotropy in the shale material, Young’s modulus and Poisson’s ratio derived from the acoustic velocity measurements lie in ranges of 3.28–3.78 GPa and 0.41–0.44, respectively. These ‘undrained’ parameters are included in table 1.

5. Discussion

Table 1 compares the values of five elastic constants measured or determined from ultrasonic and static laboratory tests along with results derived from acoustic in situ logs. Dynamic tests yield much higher values than static tests. The following subsections attempt to explain the discrepancies observed in these methods.

5.1. Measurement techniques

This current study investigates the anisotropic behaviour of Colorado shale using static and ultrasonic tests separately. The resolution of the strain measurement and the determination procedures are different in two methods. The accuracy in strain measurement in the static tests is limited by the accuracy of the local deformation transducers which is 0.001 mm. The static tests used in this study measure the five elastic constants directly from five independent direct measurements (i.e., \(E_v\), \(v_{vh}\), \(E_h\) and \(v_{hh}\) from two triaxial compression tests and \(G_{vh}\) from the confined torsion test), but with three different samples. Even though the level of difficulty involved with preparing and testing of horizontally drilled core samples is tremendous, the possible inconsistency that could arise due to combining data from stress-path triaxial tests with the bender element test for determination of \(E_h\) is greatly reduced. It can be argued that the sample heterogeneity and sample disturbance may yield some variation in measurement of elastic constants. To reduce the degree of sample heterogeneity, X-ray and computed tomography images were made on cores inside the core PVC tubes for the sample selection.

In the ultrasonic tests, the accuracy depends on the measurement of the travel time and distance. The ultrasonic tests measure the three \(P\)-waves and two \(S\)-waves in a same shale sample, and the five elastic constants are estimated using (5) and (6). It can be seen that only the shear modulus \(G_{vh}\) is directly a function of \(S_{13}\) whereas the other four elastic constants are functions of five ultrasonic wave measurements. Figures 11 and 12 present the results of a sensitivity study on the interdependence between the four elastic constants (\(E_v\), \(v_{vh}\), \(E_h\) and \(v_{hh}\)) and five wave measurements. To reduce the number of variables, normalized plots were used. For the case of transverse isotropy in over-consolidated shale, the \(P_{45}\) wave velocity lies within the range between \(P_{tt}\) and \(P_{lz}\) and \(P_{xx}\) is larger than \(P_{zz}\). Thus, all calculated data were normalized with \(P_{zz}\). Figure 11 illustrates that the modulus ratio (\(E_h/E_v\)) increases with the increasing ratio (\(P_{45}/P_{zz}\)). For a given \(P_{45}/P_{zz}\) and \(P_{xx}/P_{zz}\) ratios (say 1.1 for both), the modulus ratio (\(E_h/E_v\)) increases with decreasing ratio \(S_{13}/P_{zz}\) from 0.48 to 0.38 (figure 11(a)). Similarly, for a given \(P_{45}/P_{zz}\) and \(S_{13}/P_{zz}\) ratios (say 1.1 and 0.43, respectively) the modulus ratio (\(E_h/E_v\)) increases with decreasing ratio \(P_{xx}/P_{zz}\) from 1.2 to 1.1. Comparison of figures 11(a)–(c) shows that...
the modulus ratio \(E_h/E_v\) increases with increasing ratio \(S_{xy}/P_{zz}\) from 0.43 to 0.55. From figure 12, Poisson’s ratio \(\nu_{vh}\) exhibits similar nonlinearity with anisotropy response, whereas Poisson’s ratio \(\nu_{hh}\) exhibits a linear relationship with increasing \(P_{45}\)-wave velocity. For TI media, Poisson’s ratios could be greater than 0.5. Lings (2001) showed that for TI materials there are bounds on the values of Poisson’s ratios \((-1 \leq \nu_{vh} \leq 1; \frac{\nu_{hh}}{2} \leq 1 - \nu_{vh} \leq 2\nu_{hh} \geq 0\) because of the thermodynamic requirement that the strain energy function be non-negative in an elastic material. From the above results, the calculated elastic constants are sensitive to the measured wave velocities. Thus, the determination of the five elastic constants in TI media requires very accurate measurements in travel time and distance; otherwise the errors could be significant. In addition, ultrasonic transducers are frequency dependent. If the shale sample has some visco-elastic properties, the elastic constants could be frequency dependent.

5.2. Drained parameters versus undrained parameters

Static tests were conducted under drained conditions at which the effective stress was maintained at a constant value by allowing dissipation of any excess pore pressure induced by the axial stress applied to the sample. Thus, the elastic constants or parameters measured from the static tests are referred to as ‘drained’ parameters. They are measures of mechanical properties of the shale matrix, independent of pore fluid compressibility. However, in ultrasonic tests, waves were triggered by vibration of the transducers so that the waves travelled in a very short period of time, in terms of micro-seconds. Excess pore pressure induced during the wave propagation did not have sufficient time for complete dissipation. The measured elastic parameters, referred to as ‘undrained’, depend on not only the shale matrix, but also the pore fluid compressibility.

The samples used in this study were close to its in situ saturated condition. No ultrasonic test was conducted on dry samples because shale physical properties are very sensitive to interaction between the water and clay particles. Drying could disrupt the clay shale interlaminar structure (Jones and Wang 1981). Hence, the elastic constants estimated from the ultrasonic wave measurements are under an ‘undrained’ condition. To compare these constants with ‘drained’ parameters measured in the static tests, we have to account for the effect of pore fluid compressibility. Gassmann (1951) derived a relationship between the drained and undrained bulk moduli for porous isotropic rocks. Using average \(P\)-wave and \(S\)-wave velocities of 2300 and 1100 m s\(^{-1}\) at 6 MPa, respectively, the isotropic elastic parameters were calculated with the assumed water and solid quartz compressibility values of \(4.5 \times 10^{-6}\) and \(2.6 \times 10^{-5}\) MPa\(^{-1}\), respectively, and were included in table 1. The undrained Young’s modulus and Poisson’s ratio are higher than the drained values whereas the shear modulus remains the same in both conditions.

The model proposed by Gassmann (1951) only considers wave propagation in a saturated isotropic medium. Wave propagation in the TI medium is more complex, and thus we cannot use Gassmann’s model to estimate the effect of pore fluid on the wave velocity in the TI medium. In this study, a simplified model was used to account for the effect of pore fluid on the wave velocity. The model assumes that the fluid saturation affects the \(P\)-wave propagation only, and the matrix suction if developed in the test samples is relatively small as compared to the high confining effective stress. Thus, it is reasonable to assume that all measured \(P\)-wave (including \(P_{as}\), \(P_{zz}\), and \(P_{45}\)) velocities decreases with decreasing saturation in an equal rate. At full saturation and confining stress of 6 MPa, \(P_{as} = 2435, P_{zz} = 2246, P_{45} = 2290, S_{sy} = 1099\) and \(S_{sx} =\)
988 m s⁻¹, and the corresponding five elastic constants are listed in table 1. Accounting for decreasing saturation, the dynamic Young’s moduli and Poisson’s ratio at different saturations were calculated by reducing the P-wave velocities. The results of the five elastic constants as a function of decreasing P-wave velocity were plotted in figure 13. The decreasing P-wave velocity accounts for the saturation decreasing in water in the shale sample. This figure shows that the dynamic Young’s moduli and Poisson’s ratios decrease with decreasing P-wave velocity or saturation. Since very small strains are induced during the ultrasonic wave propagation, Poisson’s ratio should not drop below zero. In addition, the static tests yield a very small Poisson’s ratio under drained conditions. Thus, it is inferred that Poisson’s ratio of zero corresponds to the dry or drained condition. From figure 13, the P-wave velocity in the dry condition is about 65% of the P-wave velocity in the saturated condition. The dynamic $E_v$ and $E_h$ values will be reduced by a magnitude of about 2 GPa, and the Poisson’s ratios may be reduced to values close to zero. At this dry condition, the elastic constants are referred to as drained parameters, and are included in table 1 for comparison. The drained Young’s moduli and Poisson’s ratio are much lower than the undrained ones. Since saturation has no effect on shear wave propagation, the shear moduli $G_{vh}$ and $G_{hh}$ remain unchanged.

In summary, comparison of the drained parameters measured in static test, ultrasonic tests and acoustic in situ logs (table 1) consistently indicates that the stiffness parameters measured in the static tests are much lower than those measured in the ultrasonic tests or acoustic logs.

The phenomenon of the difference in elastic parameters measured under ‘drained’ and ‘undrained’ conditions mentioned above is also referred to as the total velocity dispersion (TVD) between zero frequency and any measurement frequency (Winkler 1986). TVD is defined as the discrepancy between the calculated zero-frequency velocities and the measured ultrasonic velocities. At high or ultrasonic frequencies (∼1 MHz), the pore pressure in the porous medium does not have time to equilibrate during one half-cycle of the acoustic wave. A local-flow absorption/dispersion mechanism could develop in the porous medium. At low or seismic frequencies (∼10–100 Hz), the pore pressure does equilibrate. In such cases, Gassmann’s model can be used to predict wave velocities in the saturated porous medium. Since ultrasonic velocities are affected by the local-flow mechanism and seismic velocities are probably not, ultrasonic measurements may not accurately reflect velocities at seismic frequencies. Winkler (1986) observed TVD in a range of 2 to 25% in rocks depending on rock type and saturation conditions. He also suggested that poor consolidation, the presence of gas and low porosity would all tend to reduce the magnitude of TVD. In this study, the ultrasonic velocities measured in the shale samples were at frequencies of ∼1 MHz and the acoustic logs were conducted at frequencies of ∼30 and ∼4 kHz for the P-wave and S-wave, respectively. That the P-wave and S-wave velocities measured in the ultrasonic laboratory tests are higher than those measured in the acoustic in situ logs could be due to the difference in frequencies used or the dispersion effect. The desaturation in the shale samples due to core drilling could also reduce the dispersion effect.

5.3. Strain level dependence

The strains applied to the shale samples in the static tests were much larger than those in the ultrasonic tests. In static tests, the elastic constants were estimated by averaging the stress–strain responses over the axial strains up to 0.01% during the primary loading path. In the ultrasonic tests, the waves were generated by oscillation of the transducers. Based on the dimensions and piezoelectric properties of the transducers and the excitation voltage, the maximum strains induced in P-wave and S-wave vibrations are approximately 0.0001% and 0.0005%, respectively (detailed calculations shown in appendix B). Thus, Young’s moduli and Poisson’s ratios were not measured at the same strain amplitude as those in the static tests. Georgiannou et al (1991) experimentally showed that there was a continuity between dynamic and static shear moduli in over-consolidated clay samples. The shear modulus remained constant a high value at strain levels of 0.0001 to 0.01%, and started to decrease drastically when the applied strain exceeded 0.01%. However, they did not report on the effect of strain level on the Young’s modulus and Poisson’s ratio. This study shows that the strain level impacts on all the latter two parameters as well (table 1).

5.4. Transverse isotropy and linear elasticity with zero Poisson’s ratio

The shale samples used in this study display transverse isotropy in deformation and ultrasonic isotropy in static tests (table 1). In static tests, transverse isotropy was confirmed by comparing the radial deformations in mutually perpendicular directions. That the stiffness parameters in the horizontal plane are isotropic was confirmed from measurements of shear ($S_{xy}$ and $S_{xz}$) waves in the ultrasonic tests. The dipole shear wave measurement in acoustic in situ logs also shows no anisotropy in the horizontal direction. Table 1 indicates that the stiffness parameters in the vertical direction are lower than those in the horizontal direction. Anisotropy ratios have been used in
characterizing the anisotropic behaviour of soils. Jovicic and Coop (1998) reported a value of 1.5 as an anisotropy ratio on the shear stiffness \((G_{bh}/G_{vb})\) of London clay based on the shear modulus data determined from bender element tests. Similarly, Pennington et al (1997), Pennington (1999), Lings et al (2000) and Lings (2001) have reported anisotropy ratios of 3.96 for Young’s moduli \((E_b/E_v)\) and 2.25 for shear moduli \((G_{bh}/G_{vb})\) of stiff Gault clay. They combined data from stress-path triaxial tests and bender element tests. Since the two ratios are independent, these two ratios should not be necessarily the same (Graham and Houlshby 1983). Data of table 1 show that the anisotropic ratios increase with increasing strain amplitude. This degradation in stiffness was also reported by Hardin and Black (1966).

If the Colorado shale samples are truly linear elastic, the elastic constants should not be a function of strain level as shown in table 1. In static tests, nonlinear stress–strain response along with a small plastic deformation was observed even at a strain level of 0.01% during the primary loading path. Linear elasticity was only detected during the unload–reload path.

The static tests revealed very low or zero Poisson’s ratio measured on the shale samples. Measurement of such low values in the static tests might be due to the fact that the local deformation transducers were not sensitive to detect the induced radial strain at such low strain level even though the sensor could detect displacements of 0.001 mm. Kosar (1989) also observed Poisson’s ratio values close to zero at axial strains of up to 0.1% in shale samples in conventional triaxial compression tests. Pennington et al (1997) and Lings et al (2000) conducted conventional triaxial tests on natural over-consolidated Gault clay and observed that the vertical Poisson’s ratio was zero and the horizontal Poisson’s ratio was small but negative at small strains of 0.001%–0.01%. It is of no doubt that the over-consolidated soils or shales possess anisotropic stiffness. However, whether they are linear elastic is still in controversy. Apparent zero Poisson’s ratios in both the vertical and horizontal directions allude to the transversely isotropic compliance constitutive matrix of (3) being symmetrical. The condition of symmetry is one of the major requirements for elasticity.

Lings et al (2000) showed that shear modulus reduced from a plateau with increasing strain amplitude. Though there is a distinct onset in shear strain (Cuccovillo and Coop 1997) beyond which the modulus starts to decrease, there is some doubt if this onset marks the limit of elastic behaviour (Jardine 1992). However, the effects of strain amplitude on Young’s moduli and Poisson’s ratios have not been reported. From table 1, there are significant differences in Young’s and shear moduli at strain levels of 0.0001% (ultrasonic tests) and 0.01% (static tests). There might exist an elastic limit between these two strain levels which could not be quantified in this study. However, it appears that Poisson’s ratio is insensitive to the strain level. The static tests yield very small or zero values where the Poisson’s ratios at very small strains measured by the ultrasonic method could be zero if the effect of pore fluid on wave propagation was accounted for (figure 13 and table 1).

### 6. Conclusions

Ultrasound wave measurements were conducted on Colorado shale samples to determine the transversely isotropic stiffness parameters of the material. The shale samples display that the dynamic elastic moduli along the horizontal direction are higher than those along the vertical direction because of the preferred clay fabric orientation. The dynamic elastic moduli determined from the ultrasonic method (~1 MHz) are higher than those measured in the acoustic logs (~4–30 kHz), which could have resulted from the drainage condition or dispersion effect. A model was developed to account for the effect of pore fluid on the modulus so that the dynamic elastic moduli determined from the ultrasonic method after calibration could be compared against those measured in the static tests. Under the drained conditions, the dynamic moduli are much higher than the static moduli, about 4–5 times. The differences between the drained dynamic and static moduli could be due to the difference in strain levels induced in the two tests. The stiffness moduli decrease with increasing strain amplitude. Both the ultrasonic and static methods yield small or zero Poisson’s ratios. Anisotropic stiffness was clearly detected in both tests. The degree of anisotropy in stiffness increases with increasing strain amplitude. Due to the limited data and strain measurement capability, this study was unable to define the regime of truly linear elasticity in shale during the primary loading path in the static tests (which must be smaller than the axial strain of 0.01%). Linear elastic stress–strain response was only observed in the static tests during the unload–reload path. However, very small or zero Poisson’s ratios in both the vertical and horizontal directions apparently allude to the elasticity because of the apparent symmetry in the constitutive matrix.

### Acknowledgment

The authors appreciate the funding provided by the Alberta Department of Energy, Natural Science and Engineering Research Council, Imperial Oil Canada, and The University of Calgary.

### Appendix A. Determination of the critical strain rate for full drainage conditions in triaxial and torsion tests

Bishop and Henkel (1969) proposed the minimum strain rate to allow full dissipation of excess pore pressure in the triaxial test, which is given as follows:

\[
\dot{\varepsilon} = \frac{\dot{\varepsilon}_f}{t_f} \tag{A.1}
\]

\[
t_f = \frac{20D^2}{\eta c_v}, \tag{A.2}
\]

where \(\dot{\varepsilon}\) is the critical strain rate; \(\dot{\varepsilon}_f\) = strain at failure; \(t_f\) = time for failure; \(D = \) minimum drainage path; \(\eta = \) constant depending on drainage condition; and \(c_v = \) coefficient of consolidation.
The $\varepsilon_f$ and $c_v$ values of Westgate Formation shale are about 2\% and $5 \times 10^{9}$ m$^2$ s$^{-1}$, respectively (Gautam 2004). For the top and bottom drainage condition, $\eta = 30$ and $D =$ 45 mm. The calculated critical strain rate is about 0.03\% h$^{-1}$. In the triaxial and torsion tests, the maximum strain in each stress/strain increment was about 0.01\% and the time for dissipation was about 30 min. Thus, the strain rate was about 0.02\% h$^{-1}$, less than the critical value.

Appendix B. Calculation of strains in ultrasonic wave measurement

For the circular $P$-wave piezoelectric transducer, the normal displacement, $\delta_n$, induced by the voltage excitation, $V$, is given as ($\text{www.piezotest.com}$)

$$\delta_n = d_{33} V,$$

where $d_{33} = $ longitudinal charge coefficient of the piezoelectric crystal ($400 \times 10^{-12}$ m V$^{-1}$). For $V = 200$ V and $L$ (sample length) = 73 mm, the normal strain exerted on the shale sample is equal to 0.0001\% assuming that the normal deformation induced by the transducer is distributed along the full length of the sample.

For the square $S$-wave piezoelectric transducer, the transverse displacement, $\delta_t$, induced by the voltage excitation, $V$, is given as

$$\delta_t = d_{31} V \left( \frac{W}{t} \right),$$

where $d_{31} = $ transverse charge coefficient of the piezoelectric crystal ($175 \times 10^{-12}$ m V$^{-1}$) $W$, $t = $ width and thickness of the piezoelectric transducer.

For $V = 200$ V, $W = 22$ mm and $t = 2$ mm and $L$ (sample length) = 73 mm, the transverse (shear) strain exerted on the shale sample is equal to 0.0005\% assuming that the shear deformation induced by the transducer is distributed along the full length of the sample.

References


Barden L 1963 Stress and displacement in a cross anisotropic soil *Geotechnique* 13 198–210


Cucullovito T and Coop M R 1997 The measurement of local axial strains in triaxial testing using LVDTs *Geotechnique* 47 167–71

Dellinger J and Vernick L 1994 Do travel times in pulse-transmission experiments yield anisotropic group or phase velocities? *Geophysics* 59 1774–9


Gautam R 2004 Anisotropy in deformation and hydraulic properties of Colorado Shale *PhD Thesis* Department of Civil Engineering, University of Calgary, Canada


Jardine R J 1992 Some observations on the kinematic nature of soil stiffness *Soils Found.* 32 111–24


Kosar K M 1989 Geotechnical properties of oil sands and related strata *PhD Thesis*, Department of Civil and Environmental Engineering, University of Alberta, Canada


Kuwano R and Jardine R J 2002 On the applicability of cross-anisotropic elasticity to granular materials at very small strains *Geotechnique* 52 727–49


Lings M L 2001 Drained and undrained anisotropic elastic stiffness parameters *Geotechnique* 51 555–65
Lings M L, Pennington D S and Nash D F T 2000 Anisotropic stiffness parameters and their measurement in a stiff natural clay Geotechnique 50 109–25
Mah M and Schmitt D R 2001a Experimental determination of the elastic coefficients of an orthorhombic material Geophysics 66 1217–25
Simmons G and Brace W F 1965 Comparison of static and dynamic measurements of compressibility of rocks J. Geophys. Res. 70 5649–57
Thomsen L 1986 Weak elastic anisotropy Geophysics 51 1954–66
Vernik L and Liu X 1995 Velocity anisotropy in shales; a petrophysical study Geophysics 62 521–32
Winkler K W 1986 Estimates of velocity dispersion between seismic and ultrasonic frequencies Geophysics 51 183–9