Newton's Cradle - Impact Calculations for Collisions between Balls

To calculate the velocities of two balls after a collision, we combine conservation of momentum and coefficient of restitution equations. In a real newtons cradle, the balls may have both vertical and horizontal components of velocity during impact, so much more math is involved. However, for this calculation, we will assume that the vertical component of the balls' velocity is 0 at the time of impact and the line of impact is horizontal.

We can start the derivation using the following conservation of momentum and coefficient of restitution equations in the *x* direction (because $v_{i\text{ay}}$ = $v_{ib\text{y}}$ = 0):

$$m_a v_{ia\text{x}} + m_b v_{ib\text{x}} = m_a v_{fa\text{x}} + m_b v_{fb\text{x}}$$

$$e = \frac{|v_{fb\text{x}} - v_{fa\text{x}}|}{|v_{ib\text{x}} - v_{ia\text{x}}|}$$

where m_a is the mass of the first ball, m_b is the mass of the second ball, $v_{ia\text{x}}$ is the initial velocity of the first ball in the *x* direction, $v_{ib\text{x}}$ is the initial velocity of the second ball in the *x* direction, v_{fax} is the final velocity of the first ball in the *x* direction, $v_{fb\text{x}}$ is the final velocity of the second ball in the *x* direction and e is the coefficient of restitution.

We can then rearrange the equations to get the following:

$$v_{fa\text{x}} = \frac{m_a v_{ia\text{x}} + m_b v_{ib\text{x}} - m_b v_{fb\text{x}}}{m_a} \text{ m/s}$$

$$v_{fb\text{x}} = e\,(v_{ib\text{x}} - v_{ia\text{x}}) + v_{fa\text{x}} \text{ m/s}$$

Now, the second equation is substituted into the first and the variables $v_{fa\text{x}}$ and $v_{\text{f}bx}$ are isolated as shown below:

$$v_{fax} = \frac{m_a v_{iax} + m_b v_{ibx} + m_b e\,(v_{ibx} - v_{iax})}{m_a + m_b} \text{ m/s}$$

$$v_{\text{f}bx} = \frac{m_a v_{iax} + m_b v_{ibx} + m_\text{a} e\,(v_{\text{ia}x} - v_{\text{ib}x})}{m_a + m_b} \text{ m/s}$$

Using these equations, we can calculate the velocities of any two balls after impact. However, for the first collision after releasing the leftmost ball, we know that $v_{ib\text{x}}$ = 0. Using this information, we can simplify the equation even further by substituting 0 in for $v_{ib\text{x}}$ as shown below.

$$v_{fax} = \frac{m_a v_{iax} - m_b e\, v_{iax}}{m_a + m_b} \text{ m/s}$$

$$v_{\text{f}bx} = \frac{m_a v_{iax} + m_\text{a} e\, v_{iax}}{m_a + m_b} \text{ m/s}$$

Finally, we get the equations shown in the output section of the simulation. For an explanation of how velocities after 2-D collisions are calculated, refer to the "Impact Calculations for Collisions between Balls" section in the Pool Simulation PDF.