With Buoyancy

The simulation shows a circle falling through a fluid in a 2-d space. Terminal velocity can be derived from the balance of forces acting on the circle. The following equations will only account for velocity in the y-direction.

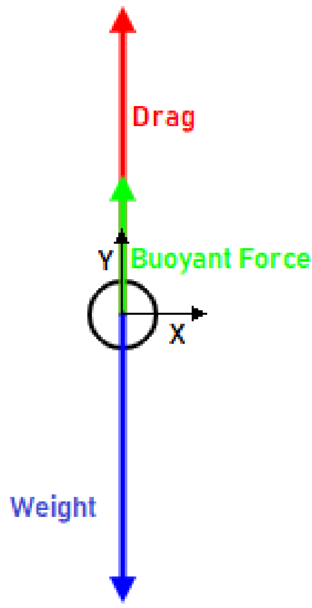

Figure 1: Free Body Diagram of Circle including Buoyant Forces

From the free body diagram the following equation is created

$$D + F_b = W \tag{1}$$

Where D (Drag Force) and F_b (Buoyant Force) is balanced with W (Object Weight) the object is at terminal velocity.

Drag force is defined to be

$$D = \frac{1}{2} c_d \rho v^2 A$$

where c_d is the object's drag coefficient, ρ denotes the fluid density (kg/m^3), v is the speed of the object (m/s), and A is the cross sectional area normal to the falling direction (m).

Buoyant Force is defined to be

$$F_b = \rho g V_s$$

where ρ is the fluid density (kg/m^3), g denotes the acceleration due to gravity (m/s^2), and V_s is the volume displacing the fluid (m^3). In our case $g = -9.81 m/s^2$ and $V_s = \pi r^2$ due the circle always being submerged in the fluid.

Weight is defined to be

$$W = \rho_s g V$$

where ρ_s is the object's density (kg/s^3), g is acceleration due to gravity (m/s^2), and V is the volume of the object (m^2).

The full equation is now

$$\frac{1}{2} c_d \rho v^2 A + \rho g V_s = \rho_s g V \tag{2}$$

The circle will always be fully submerged in the fluid therefore the area submerged is equal to the total area of the circle ($V_s = V$).

$$\frac{1}{2} c_d \rho v^2 A + \rho g V = \rho_s g V \tag{3}$$

Solving for velocity results in

$$v = \sqrt{\frac{2gV(\rho_s - \rho)}{c_d \rho A}} \tag{4}$$

For our purposes the given equation is not suited for a 2-d environment. To convert to a 2-d environment the following changes will be made. A refers to area of the circle (m^2), L refers to the length of the object normal to the falling direction (m), and ρ will now be in (kg/m^2).

$$v = \sqrt{\frac{2gA(\rho_s - \rho)}{c_d \rho L}} \tag{5}$$

A circle has the following properties

$$A = 2r \qquad\qquad V = \pi r^2$$

where r is the radius of the circle(m).

Including the properties results in the complete equation

$$v = \sqrt{\frac{\pi r g(\rho_s - \rho)}{c_d \rho}} \tag{6}$$

Without Buoyancy

As previously shown the forces can be balanced to find terminal velocity.

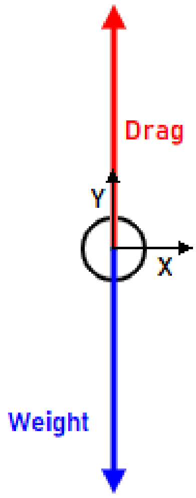

Figure 2: Free Body Diagram of Circle without Buoyant Forces

From the free body diagram the following equation is created

$$D = W \tag{1}$$

The full equation is given as

$$\frac{1}{2} c_d \rho v^2 A = \rho_s g V \tag{2}$$

where c_d is the drag coefficient of the object, ρ is fluid density (kg/m^2), v is the speed of the object (m/s), A is the area of the cross-section normal to the falling direction of the object (m^2), ρ_s is the object's density (kg/m^3), g is acceleration due to gravity ($-9.81m/s^2$), and V is the volume of the object (m^3).

Solving for velocity results in

$$v = \sqrt{\frac{2\rho_s g V}{c_d \rho A}} \tag{3}$$

$$\tag{4}$$

As previously mentioned, changes are made to the equation for the 2-d environment the simulation is in. A refers to area of the circle (m^2), L refers to the length of the object normal to the falling direction (m), and ρ will now be in (kg/m^2).

$$v = \sqrt{\frac{2\rho_s g A}{c_d \rho L}} \tag{5}$$

A circle has the following properties

$$L = 2r \qquad\qquad A = \pi r^2$$

where r is the radius of the circle (m).

Including the properties results in the complete equation

$$v = \sqrt{\frac{\pi r \rho_s g}{c_d \rho}} \tag{6}$$