Ideas in Science



Quantum Reference Frames: Physics through Relations

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Outline

1. Introduction and motivations
2. Quantum Reference Frames
3. Problem of Time in Quantum Gravity
4. Application to Black Holes

Reference frames

- Physics is always described **respect to a Reference Frame**.
- Reference Frame = collection of **rods** and **clocks**.
- Usually, Reference Frames are treated as **abstract** mathematical entities, e.g. coordinates $\Longrightarrow$ In "real life"-**experiments**, rods and clocks are *always* **physical systems**, i.e. physical objects.
- Ultimately, *any* physical object (should) be described by **Quantum Mechanics** $\Longrightarrow$ Reference Frames should have **quantum properties** too, e.g. superposition.

Fundamentals of Quantum Mechanics

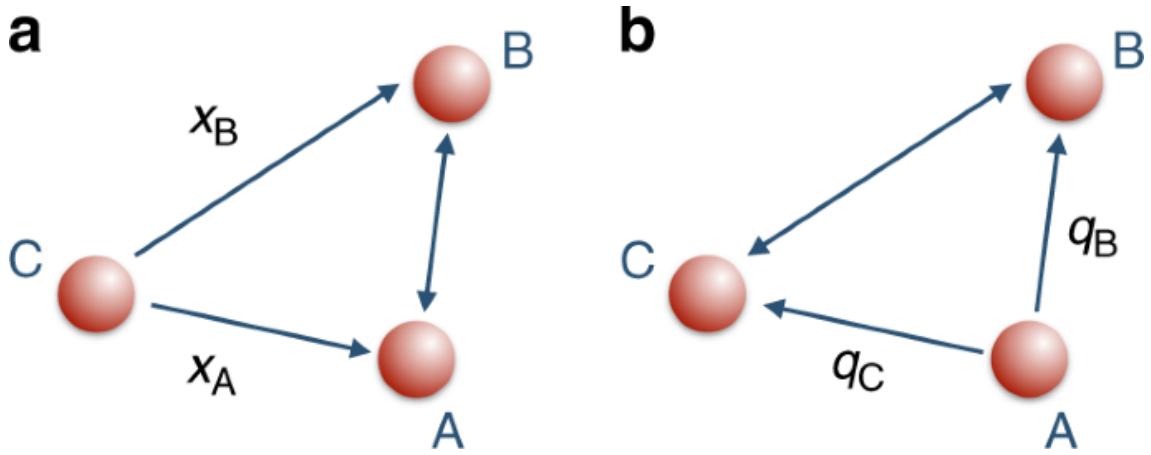
- **Quantum Mechanics** is manifest at very **small scales**, e.g. atom, particles, etc.
- **Fundamental uncertainty** in Nature, e.g. we can *never* know *exactly* the position x and the velocity p of a particle $\Longrightarrow$ **Heisenberg's uncertainty principle**: $\Delta x \Delta p \geq \frac{\hbar}{2}$.
- **Superposition**: *before* measurement, a particle can be in x_1 and x_2 *at the same time*. *After* measurement, it **randomly** appears in x_1 or x_2.
- **Entanglement**: measuring a particle A influences **instantly** a particle B, even far apart:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|x_1\rangle_A |y_1\rangle_B + |x_2\rangle_A |y_2\rangle_B \right) \tag{1}$$

Quantum Reference Frames

- Each **QRF** describes its own "**rest of the world**", including the other reference frames and the observed systems.
- **Example**: 3 particles A, B and $C \Longrightarrow$ 1) From the **point of view of** A: $\mathcal{H}_B \otimes \mathcal{H}_C$; 2) From the **point of view of** C: $\mathcal{H}_B \otimes \mathcal{H}_A$.
- To **shift** from A to C perspective, we consider a **spatial translation** $\Longrightarrow$ in standard QM is **generated** by $\hat{U}_{x_0} = e^{\frac{i}{\hbar} x_0 \hat{p}}$.
- For a QRF, what is the value of x_0? If A and C are in **relative superposition**, x_0 can get many values $\Longrightarrow$ we **promote** x_0 to an **operator** $\hat{x}$!

Spatial Translations



$$\begin{cases} \hat{x}_A \to -\hat{q}_C \\ \hat{x}_B \to \hat{q}_B - \hat{q}_C \end{cases} \tag{2}$$

Quantum operator $\hat{S}_x$

- We are going to define a **unitary** operator $\hat{S}_x$ such that:

$$\begin{cases} \hat{S}_x : \mathcal{H}_A^{(C)} \otimes \mathcal{H}_B^{(C)} \longrightarrow \mathcal{H}_C^{(A)} \otimes \mathcal{H}_B^{(A)} \\ \boxed{\hat{S}_x = \hat{\mathcal{P}}_{AC}\ e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}} \end{cases} \tag{3}$$

- The operator $e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}$ shifts B in space of an amount x_A which **depends on the quantum state** of the new QRF A.
- $\hat{\mathcal{P}}_{AC}$ is the **parity swap** operator, defined as:

$$\begin{cases} \hat{\mathcal{P}}_{AC} : \mathcal{H}_A^{(C)} \longrightarrow \mathcal{H}_C^{(A)} \\ \hat{\mathcal{P}}_{AC}\Psi_A(x) = \Psi_C(-x) \end{cases} \tag{4}$$

Action on quantum state

If the **initial state** is $\boxed{|\Psi\rangle_{AB}^{(C)} = |\phi\rangle_A |\psi\rangle_B}$, with $|\phi\rangle_A = \frac{1}{\sqrt{2}}\left(|x_1\rangle_A + |x_2\rangle_A\right)$ and $|\psi\rangle_B = |x_0\rangle_B$, the operator $\hat{S}_x$ will give:

$$\hat{\mathcal{P}}_{AC}\, e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B} |\phi\rangle_A |\psi\rangle_B = \hat{\mathcal{P}}_{AC}\, \frac{e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}}{\sqrt{2}}\left(|x_1\rangle_A + |x_2\rangle_A\right)|x_0\rangle_B$$

$$= \hat{\mathcal{P}}_{AC}\, \frac{1}{\sqrt{2}}\Big[\,|x_1\rangle_A |x_0 + x_1\rangle_B + |x_2\rangle_A |x_0 + x_2\rangle_B\,\Big] \qquad (5)$$

$$= \boxed{\frac{1}{\sqrt{2}}\Big[\,|-q_1\rangle_C |q_0 - q_1\rangle_B + |-q_2\rangle_C |q_0 - q_2\rangle_B\,\Big] = |\Psi\rangle_{CB}^{(A)}}.$$

This is an **entangled** state! $\Longrightarrow$ Entanglement and Superposition **depend** on the QRF.

Problem of Time in Quantum Gravity

- Our two fundamental theories of the universe: **General Relativity** (big things) and **Quantum Mechanics** (small things).
- GR + QM $\rightarrow$ **Time disappears!**
- **GR** has a Symmetry: *the laws of physics are independent on coordinates (or observers)* $\rightarrow$ total Hamiltonian H is a constraint, i.e. $H = 0$.
- In **QM**, H generates *time evolution*:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi.$$

- When $\hat{H} = 0$, then $\boxed{\frac{\partial \psi}{\partial t} = 0}$ $\rightarrow$ **Frozen Universe!**

Page-Wootters mechanism

- Don Page (UofA) and W. Wootters ('83): **Time evolution** *can be recovered from* **relations** *between subsytems*.
- e.g.: System = particle A + particle B $\rightarrow$ A can evolve w.r.t. B, and viceversa. We "build" a (quantum) clock out of the degrees of freedom of B (or A).
- Suppose H is **separable**: $H = H_A + H_B = 0$. Physical states satisfy: $\boxed{\hat{H}\psi_{\text{ph}} = \left(\hat{H}_A + \hat{H}_B\right)\psi_{\text{ph}} = 0}$.
- In general, (gauge) **symmetries** in quantum gravity *call* for a **relational** description of reality. $\hat{H}\left|\Psi\right\rangle_{\text{phys}} = 0$ expresses **independence on coordinates** and contains all the possible **relational points of view** at once $\rightarrow \mathcal{H}_{\text{phys}}$ is **Perspective-Neutral**.

Page-Wootters mechanism

- B is the clock $\rightarrow$ "build" Time from its **conjugate** $[\hat{T}_B, \hat{H}_B] = i\hbar \rightarrow \hat{T}_B \left|\tau\right\rangle_B = \tau \left|\tau\right\rangle_B$.
- From $\left|\tau\right\rangle_B$, we can build the **conditional states**:
$$\boxed{\left|\psi_{A|B}(\tau)\right\rangle := \left\langle\tau|\psi_{\text{ph}}\right\rangle}.$$
- We can recover a **Relational Schrodinger equation**:

$$\begin{aligned}
&\hat{H}_A \left|\psi_{A|B}(\tau)\right\rangle = \hat{H}_A \left(\left\langle\tau|\psi_{\text{ph}}\right\rangle\right) = \left\langle\tau|\hat{H}_A|\psi_{\text{ph}}\right\rangle = \left\langle\tau|\hat{H} - \hat{H}_B|\psi_{\text{ph}}\right\rangle \\
&= \left\langle\tau|\hat{H}|\psi_{\text{ph}}\right\rangle - \left\langle\tau|\hat{H}_B|\psi_{\text{ph}}\right\rangle = -\left\langle\tau|\hat{H}_B|\psi_{\text{ph}}\right\rangle = -i\hbar\frac{\partial}{\partial\tau}\left(\left\langle\tau|\psi_{\text{ph}}\right\rangle\right) \qquad (6) \\
&\longrightarrow \boxed{i\hbar\frac{\partial}{\partial\tau}\left|\psi_{A|B}(\tau)\right\rangle = -\hat{H}_A \left|\psi_{A|B}(\tau)\right\rangle}.
\end{aligned}$$

Black Holes

- Black Holes are prediction of GR $\longrightarrow$ their H should be 0 too. *How do we recover time evolution inside of a Black Hole?*
- Consider a generic metric inside a Black Hole:

$$ds^2 = N(\tilde{t})^2 \frac{a(\tilde{t})}{b(\tilde{t})} d\tilde{t}^2 - \frac{b(\tilde{t})}{a(\tilde{t})} dx^2 - a(\tilde{t})^2 d\Omega^2 \tag{7}$$

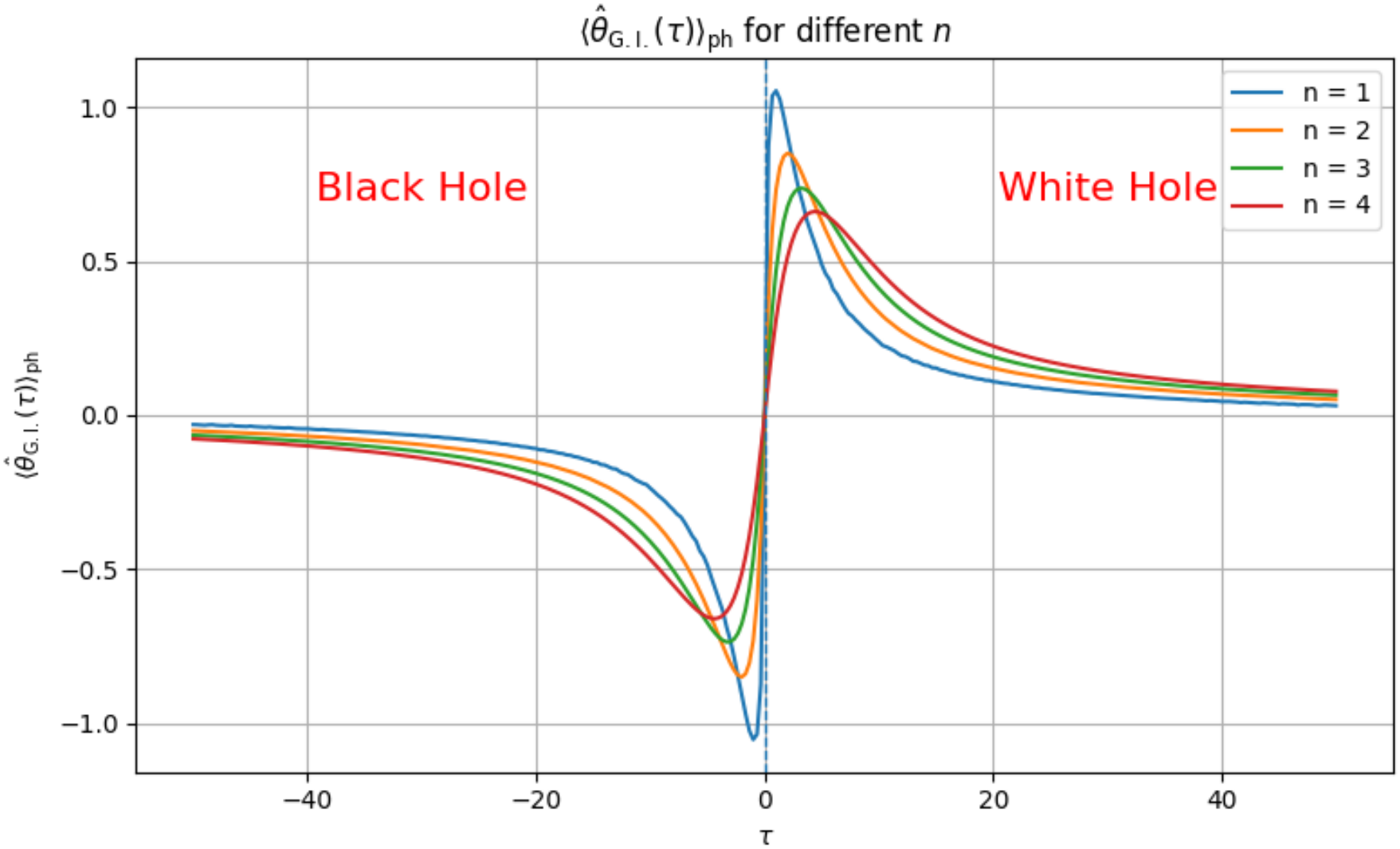
where $\tilde{t}$ is time-like, and x is space-like.

- Notice that $p_a = -\alpha \frac{\dot{b}}{N}$ and $p_b = -\alpha \frac{\dot{a}}{N}$. Hamiltonian is:

$$H = -\frac{N}{\alpha}\left(\alpha^2 + p_a p_b\right) \rightarrow \boxed{H = \bar{N}\mathcal{H} = \bar{N}\left(\frac{\alpha^2}{p_a} + p_b\right)}, \tag{8}$$

where $\bar{N} = -\frac{N}{\alpha} p_a$. **Separable**!

White Holes


$\langle\hat{\theta}_{G.I.}(\tau)\rangle_{ph}$ for different n
Black Hole
White Hole
n = 1
n = 2
n = 3
n = 4
1.0
0.5
0.0
−0.5
−1.0
−40
−20
0
20
40
τ
$\langle\hat{\theta}_{G.I.}(\tau)\rangle_{ph}$