System Identification for Nonlinear Aeroelastic Models

C. A. Popescu*, Y. S. Wong*and B.H.K. Lee†

Keywords:freeplay, hysteresis, structural non-linearity identification, non-parametric estimations.

1 Introduction

Woolston, Runyan and Byrdson first reported a theoretical study of aeroelastic systems with structural freeplay in 1955. Recently, there has been considerable interest in the study of nonlinear aeroelastic systems using experimental investigation and numerical simulations ([1],[3], [5]).

Nonlinearities in aeroelastic systems can result due to the structural and aerodynamic forces. Typically, real structures may have structural nonlinearities such as freeplay and hysteresis. In an effort to reduce the uncertainty in nonlinear aeroelastic models, it may be of interest to identify the specific type of structural nonlinearity in an aeroelastic response. The more accurate information of the aeroelastic characteristic may be useful in the development of aeroelastic models, which in turns leads to the improved design of aircraft safety and performance.

In this paper, we continue our early work reported in [7], and we propose a methodology which is capable to identify the specific type of structural nonlinearities such as freeplay and hysteresis from a given aeroelastic response. The aeroelastic data can arise from an experiment or a ground test. Moreover, our proposed technique also estimates the locations of the switch-

*Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, Alberta, Canada, T6G 2G1, e-mail:popescu@sirius1.math.ualberta.ca

†Institute for Aerospace Research, National Research Council, Ottawa, Ontario, Canada, K1A 0R6

ing points in the freeplay and hysteresis. The switching points define the changes of the system dynamics in the corresponding linear sub-domains.

In [7], we have reported a procedure based on the successive calculations of the difference between consecutive observations. The method have been successfully tested for a numerically simulated aeroelastic signal with freeplay. However, if the signal is corrupted with noise, the method can not be applied. In this paper, we propose a methodology based on a statistical non-parametric method to identify freeplay or hysteresis for a given response data. To the best of our knowledge, no other work has been reported on the identification of the structural nonlinearity directly using the aeroelastic response. The developed methodology is capable of dealing with noisy aeroelastic signals. It has been tested to aeroelastic responses arising from polynomial spring, freeplay and hysteresis. In a typical experiment, we are given response signals representing the pitch angle, plunge displacement and the flap rotational motion of an aeroelastic system with a control surface. Without knowing that a freeplay structural nonlinearity is imposed to the control surface, using our method we successfully identify the freeplay in the flap degree-of-freedom by testing the aeroelastic response signals (in pitch, plunge and flap motions). Moreover, the switching points are accurately located.

So far, our study is limited to having a freeplay nonlinearity in one-degree-of-freedom in the aeroelastic system. It is of interest to investigate if there are two freeplays in the aeroelastic system. We expect the proposed technique could still identify the freeplay individually, but we need to obtain or to simulate the aeroelastic data to confirm this remark. The technique, however, could deal with aeroelastic signals resulting with both structural and aerodynamic nonlinearities. Moreover, notice that after the structural nonlinearity is determined, we can apply system identification methods to reconstruct the corresponding aeroelastic model.

2 Aeroelastic systems with structural non-linearities

The concentrated structural non-linearities can be generally classified as polynomial springs, freeplay, or hysteresis. A polynomial stiffness may be observed in the large bendings of wings or propeller blades. Mathematical this non-linearity can be expressed by a polynomial function

$$M(x) = a_n x^n + \ldots + a_1 x + a_0. \tag{1}$$

For example, for the cubic spring displayed in Fig.1 (a), $M(x)$ is given by $M(x) = a_1 x + a_3 x^3$.

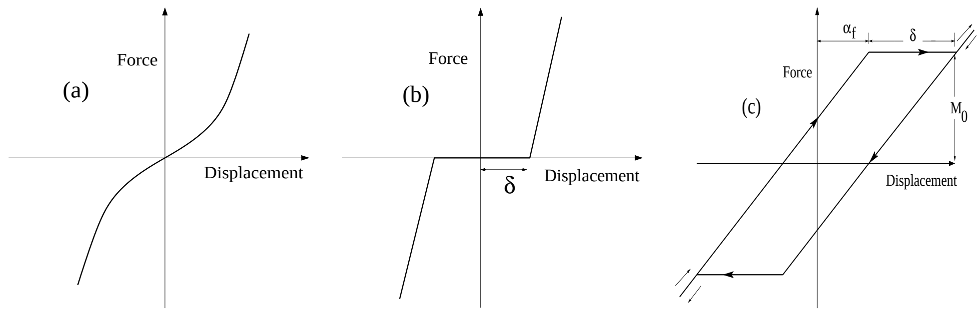

Figure 1:General sketch:(a) cubic spring (b) freeplay (c) hysteresis

In this paper we are mainly concerned with non-linearities represented by piece-wise linear functions, such as freeplay or hysteresis. A backlash in loose or worn control surface linkages or hinges may be the cause of a freeplay nonlinearity. Mathematically, we have a continuous piece-wise linear function

$$M(x) = \begin{cases} M_0 + x + \alpha_f & \text{if } x < \alpha_f \\ M_0 + M_f(x - \alpha_f) & \text{if } \alpha_f \leq x \leq \alpha_f + \delta \\ M_0 + x - \alpha_f + \delta(M_f - 1) & \text{if } x > \alpha_f + \delta, \end{cases} \tag{2}$$

where M_0 is the preload, δ is the freeplay and α_f is the beginning of the freeplay. A general sketch of a freeplay without a preload ($M_0 = 0$) is given in Fig.1(b).

When friction affects linkage dynamics, the aeroelastic system may exhibit a hysteresis non-linearity. If $\uparrow$ and $\downarrow$ denotes that the motion is increasing or decreasing in the x direction, a hysteresis nonlinearity can be expressed as

$$M(x) = \begin{cases} x - \alpha_f + M_0 & \text{if } x < \alpha_f \uparrow \\ x + \alpha_f - M_0 & \text{if } x > -\alpha_f \downarrow \\ M_0 & \text{if } \alpha_f \leq x \leq \alpha_f + \delta \uparrow \\ -M_0 & \text{if } -\alpha_f - \delta \leq x \leq -\alpha_f \downarrow \\ x - \alpha_f - \delta + M_0 & \text{if } x > \alpha_f + \delta \uparrow \\ x + \alpha_f + \delta - M_0 & \text{if } x < -\alpha_f - \delta \downarrow . \end{cases} \tag{3}$$

Similarly as defined in equation (2), M_0 is the preload, δ is the freeplay and α_f is the beginning of the freeplay. The function $M(x)$ corresponding to a hysteresis is illustrated in Fig.1(c).

Many mathematical models for aeroelastic systems with structural non-linearities can be formulated as systems of ordinary differential that can be expressed in the state-space form:

$$X_t^{'} = AX_t + F(X_t), \tag{4}$$

where A is a matrix containing the system coefficients, F is a non-linear function, and $'$ denotes the real time derivative.

A two-degree-of-freedom airfoil oscillating in pitch and plunge can be modeled by the following system of equations [4]:

$$\begin{aligned} &\xi^{''} + x_\alpha \alpha^{''} + 2\zeta_\xi \frac{\tilde{\omega}}{U^*}\xi^{'} + \left(\frac{\tilde{\omega}}{U^*}\right)^2 G(\xi) = -\frac{1}{\pi\mu}C_L(t) \\ &\frac{x_\alpha}{r_\alpha^2}\xi^{''} + \alpha^{''} + 2\frac{\zeta_\alpha}{U^*}\alpha^{'} + \frac{1}{U^{*2}}M(\alpha) = \frac{2}{\pi\mu r_\alpha^2}C_M(t) \end{aligned} \tag{5}$$

Here $G(\xi)$ and $M(\alpha)$ are the nonlinear plunge and pitch stiffness terms, respectively. $C_L(t)$, $C_M(t)$ are the lift and pitching moment coefficients, and they are expressed by integral terms for the subsonic flows. By introducing four new variables ω_1, ω_2, ω_3, ω_4, the integro-differential system (5) can be reformulated as in equation (4) with $X = [\alpha, \alpha^{'}, \xi, \xi^{'}, \omega_1, \omega_2, \omega_3, \omega_4]^t$. Depending on the type of the structural non-linearity, the functions $G(\xi)$ and $M(\alpha)$ can be expressed as in formulas (1), (2) or (3).

For an airfoil with a freeplay control surface, the mathematical model can be written as:

$$M_s X'' + B_s X' + K_s X = F. \tag{6}$$

Here the matrices M_s, B_s and K_s are given in [5], and $X = [h, \alpha, \beta]^t$, where h is the plunge displacement, α is the pitch angle, and β is the flap rotation. With a structural freeplay in flap the nonlinearity is given by the following formula

$$M(\beta) = \begin{cases} \beta + \delta & \text{if } \beta < -\delta \\ 0 & \text{if } -\delta \leq \beta \leq \delta \\ \beta - \delta & \text{if } \beta > \delta. \end{cases} \tag{7}$$

The model (6) can also be expressed in a state-space form [1].

3 Nonlinearity identification

In this paper we propose a method to determine the presence of the structural nonlinearities such as freplay and hysteresis, using only the noisy aeroelastic response. Moreover, we find the freeplay parameters δ and α_f. The same problem was solved in [7] for simulated clean data, by analysing the differences between consecutive observations. However, this method can not be extended to noisy data because of the commonly non-differentiable nature of the noise.

3.1 Non-parametric estimation

Here, we propose a robust method similar to the methods used to find the thresholds for self-exciting autoregressive models (SETAR) [10]. Let $\{t_0, t_1, \ldots, t_l\}$ denote the thresholds, i.e. a linearly ordered subset of real numbers, such that $t_0 < t_1 < \ldots < t_l$, where $t_0 = -\infty$ and $t_l = +\infty$. A self-exciting threshold autoregressive model of order $(l; p, \ldots, p)$ or SETAR $(l; p, \ldots, p)$ where p is repeated l times, is a univariate time series $\{X_n\}$ of the form

$$X_n = a_0^{(j)} + \sum_{i=1}^{p} a_i^{(j)} X_{n-i} + e_n, \quad t_{j-1} < X_{n-d} \leq t_j, \tag{8}$$

for $j = 1, 2, \ldots l$, where d is a fixed integer belonging to $\{ 1, 2, \ldots, p\}$, and $\{e_n\}$ is a Gaussian, independent, identically distributed white noise sequence. If for $j = 1, 2, \ldots, l$, we have $a_i^{(j)} = 0$ for $i = p_j + 1, p_j + 2, \ldots, p$, then $\{X_n\}$ is known as a SETAR$(l; p_1, p_2, \ldots, p_l)$ model. Hence, a SETAR $(1, p)$ model is equivalent to a linear autoregressive (AR) model of order p.

If we define the vector $Y_n = (X_n, \ldots, X_{n-p+1})^t$, then to a SETAR$(l; p, p, \ldots, p)$ model corresponds a discrete state-space form:

$$Y_n = f(Y_{n-1}) + E_n, \tag{9}$$

where $E_n = (e_n, 0, \ldots, 0)^t$, $f(Y_{n-1}) = (h(Y_{n-1}), X_{n-1}, \ldots, X_{n-p+1})^t$, and $h(Y_{n-1})$ is given in the right side of the equation (8).

The most difficult task for these models is to determine the thresholds and the delay parameter d. In [10], Chapter 7.2.3, Tong suggests to use exploratory data analysis, and to study the non-parametric lag regression estimates. If $m_j(x) = E(X_n | \, X_{n+j})$, a non-parametric kernel estimate $\hat{m}_j(x)$

for $m_j(x)$ is given by

$$\hat{m}_j(x) = \sum_{i=-j+1}^{N} X_i \delta_N(x - X_{i+j}) / \sum_{i=-j+1}^{N} \delta_N(x - X_{i+j}), \tag{10}$$

for $j = -s, \ldots, -1$, where s is a positive integer much smaller than the size N of the data set (see page 218 in [10] and Chapter 4.1.5 in [11]). Here, $\delta_N(\cdot)$ is a function defined by

$$\delta_N(z) = \begin{cases} (1 - |z|/h_N)/h_N & \text{if } |z| \leq h_N \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

where h_N is chosen such that $h_N \to 0$ as $N \to \infty$. A similar formula can be written for the non-parametric estimates $\hat{v}_j(x)$ of the variance $v_j(x) = VAR(X_n|X_{n+j})$. Analysing the plots of $\hat{m}_j(x)$ and $\hat{v}_j(x)$ for several values of j, we can determine the thresholds and the delay parameter. In addition to the simple kernels $\delta_N(\cdot)$, there are also other possible choices for the kernels (see [11], page 139), e.g the Gaussian or the Epanechnikov kernels. The bandwidth h_N can be chosen using the leave-on-out cross-validation ([11], page 141). Asymptotic properties of $\hat{m}_j(x)$ are given in [9].

It should be noted that there are many similarities between the discrete state-space model given in (9) and the aeroelastic model stated in (4). The techniques used to determine the thresholds can in fact be extended to identify the structural nonlinearity in aeroelastic response data. In the next subsection we present the results obtained using the simple kernels given in (11). A more detailed study and discussion will be presented in the full paper.

3.2 Case studies

Figs.2 and 3 display the pitch motions of an oscillating airfoil. The data[1] presented in Fig. 2 are obtained from an experimental wind-tunnel testing performed at the Texas A&M University, in which a polynomial structural nonlinearity is imposed in the pitch-degree-of-freedom. The corresponding mathematical model ([2]) is similar with (5). Fig. 3 illustrates a pitch motion

[1] The data are from the file DN04J.dat available online at http://aerounix.tamu.edu/aeroel.

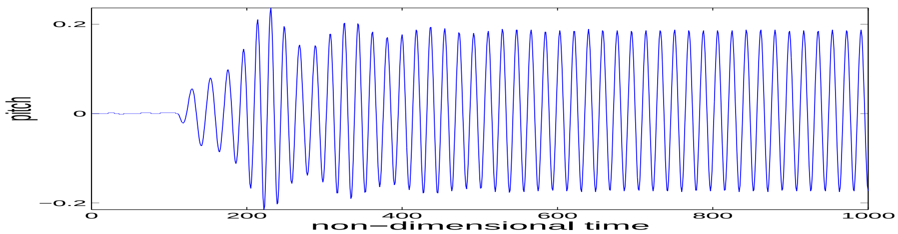

Figure 2: Aeroelastic response with polynomial nonlinearity: pitch angle

from a numerical time-integration method applied to the two-dimensional aeroelastic model given in (5). The numerical data (solid red lines) are contaminated with additive Gaussian noise such that the signal-to-noise ratio is 3 (dash dotted blue line). Here, a hysteresis nonlinearity is imposed in the pitch degree-of-freedom, with $\delta = 1$, and $\alpha_f = 0$.

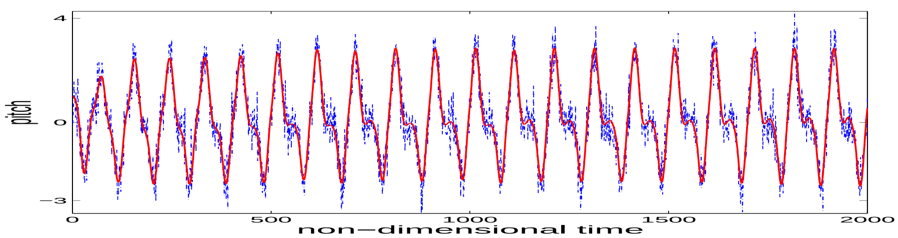

Figure 3: Aeroelastic response with hysteresis: pitch angle

The response signal shown in Fig. 4, corresponds to the flap rotation motion with a freeplay nonlinearity. The data (solid red lines) are generated numerically using the model given in (6). The freeplay is $\delta = 1$, and the signal contains an additive white Gaussian noise with signal-to-noise ratio=3.

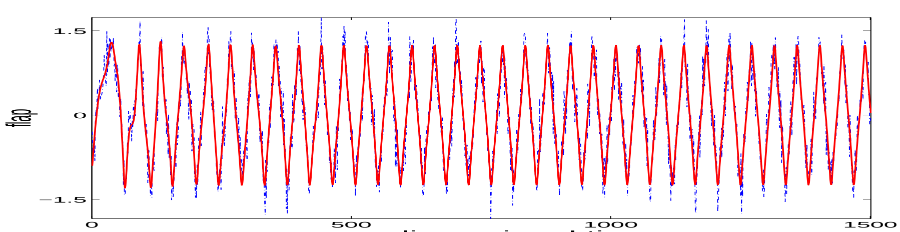

Figure 4: Aeroelastic response with freeplay: flap displacement

From the aeroelastic responses shown in Figs. 2 - 4, it is impossible to identify which signal is resulted from a freeplay or hysteresis. Now, by applying the non-parametric technique presented in the previous subsection, we can study the conditional means $E(x_t|x_t+j)$ and variances $Var(x_t|x_t+j)$ of the response signal for various value of j.

Figs. 5 and 6 shows the results corresponding to the data given in Fig.2. The estimates of the means are well approximated by a straight line. For the variances, we obtain simple concave curves. Here we present only the results for j=-1, -10 and -30, since the profile of the curves obtained for values of j between -80 and 0 are similar. Since we know that the experimental data displayed in Fig. 2 do not exhibit any piece-wise non-linearity, we can use them as a control data set.

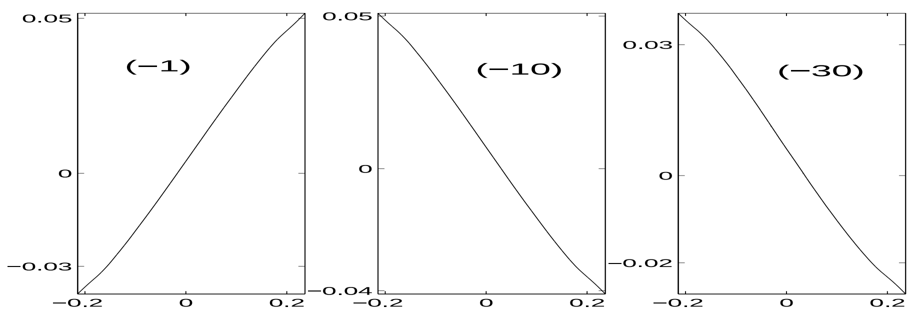

Figure 5: Polynomial aeroelastic response: the estimates of the mean

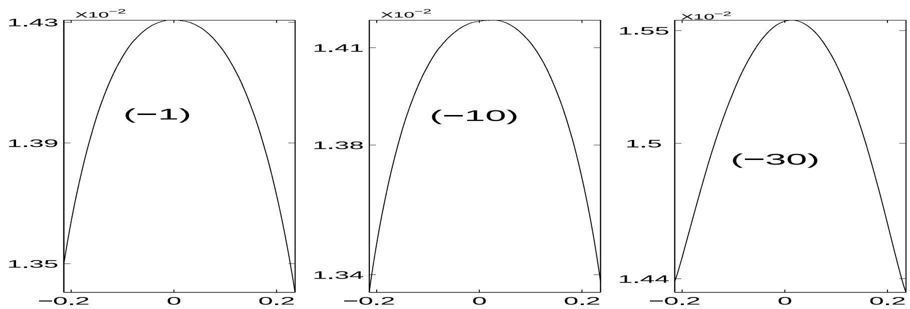

Figure 6: Polynomial aeroelastic response: the estimates of the variance

Now we study the non-parametric estimates for the data in Fig. 3. The results presented in Fig. 7 indicate the existence of three inflection points in the mean. This provides a clear identification of the hysteresis nonlinearity, with the parameters near $\delta = 1$ and $\alpha_f = 0$. The plots of the corresponding variances (see Fig. 8) further confirm this observation, where the curves comprise of three extrema located around -1, 0 and 1.

Finally, the plots of the means corresponding to the flap motion are shown in Fig. 9, and a freeplay nonlinearity is expected from the two inflection

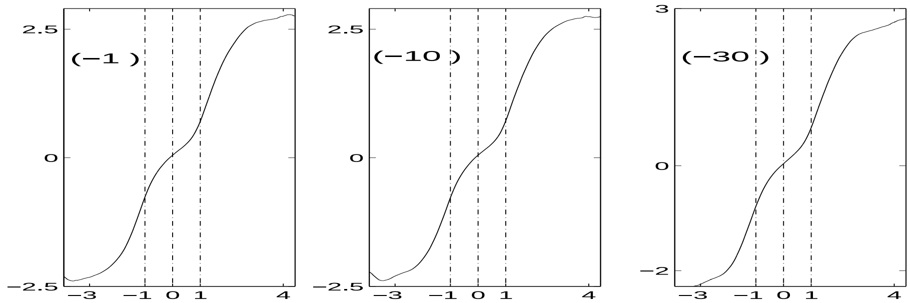

Figure 7: Aeroelastic response with hysteresis: the estimates of the means

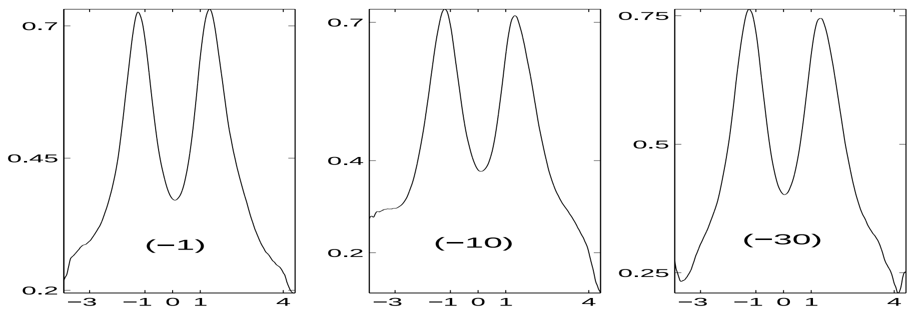

Figure 8: Aeroelastic response with hysteresis: the estimates of the variances

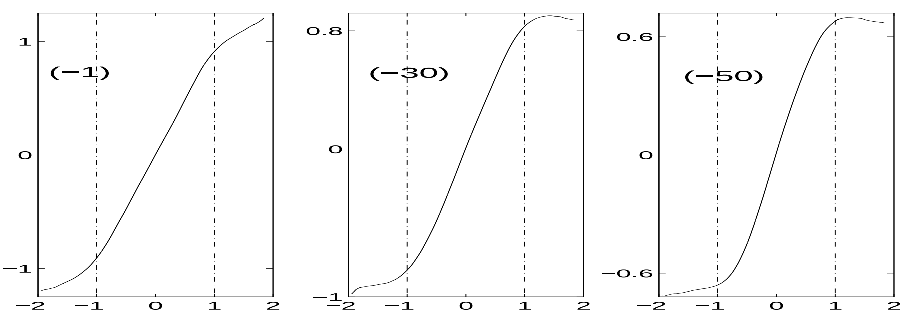

Figure 9: Aeroelastic response with freeplay: the estimates of the means

points near -1 and 1.

4 Aeroelastic system identification

Knowing the structural nonlinearity, is very helpful in order to perform system identification for an aeroelastic model. For example, the state space form corresponding to a two-degree-of-freedom airfoil characterized by the model (5) with a freeplay in pitch is as follows:

$$\begin{aligned} X_t' &= AX_t + F_1 \text{ if } X_t(1) < \alpha_f, \\ X_t' &= BX_t + F_2 \text{ if } \alpha_f \leq X_t(1) \leq \alpha_f + \delta, \\ X_t' &= AX_t + F_3 \text{ if } X_t(1) > \alpha_f + \delta, \end{aligned}$$

where $X(1) = \alpha$ is the first component of the eight dimensional vector $X = [\alpha, \xi, \alpha', \xi', \omega_1, \omega_2, \omega_3, \omega_4]^t$. Hence, we have three linear systems that can be solved analytically. With a sufficiently small sampling step τ, the solution can be expressed as

$$\begin{aligned} X_{t+\tau} &= A_1(\tau)X_t + b_1(\tau) \text{ if } X_t(1) < \alpha_f, \\ X_{t+\tau} &= A_2(\tau)X_t + b_2(\tau) \text{ if } \alpha_f \leq X_t(1) \leq \alpha_f + \delta, \\ X_{t+\tau} &= A_1(\tau)X_t + b_3(\tau) \text{ if } X_t(1) > \alpha_f. \end{aligned} \quad (12)$$

Since in practice only α and ξ can be measured, we associate with the system (12) the following linear discrete switching state-space system:

$$x_{k+1} = A_{S_{k+1}} x_k + b_{S_{k+1}} + v_{k+1}, \quad (13)$$

$$y_k = Cx_k + w_k, \quad (14)$$

where S_k is a discrete random variable given by

$$S_{k+1} = \begin{cases} 1 & \text{if } x_k(1) < \alpha_f, \\ 2 & \text{if } \alpha_f \leq x_k(1) \leq \alpha_f + \delta, \\ 3 & \text{if } x_k(1) > \alpha_f. \end{cases}$$

Here, A_i, $i = 1, 2, 3$ are 8×8 matrices, $A_3 = A_1$, b_i, $i = 1, 2, 3$ are eight-dimensional vectors, $y_k = [\alpha, \xi]^t$ is the two-dimensional observation vector, x_k is the eight-dimensional state vector, $v_k \sim N(0, Q_{S_k})$ and $w_k \sim N(0, R)$ are independent Gaussian white noise vectors, Q_i, $i = 1, 2, 3$ are 8×8 matrices, R is a 2×2 matrix, and C is the 2×8 matrix

$$C = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \end{bmatrix}.$$

Suppose that we know α_f and δ (e.g. we determine the thresholds α_f and $\alpha_f + \delta$ using the non-parametric method presented in previous section), thus the values of the switching variable S_k are known. Then we can estimate the system parameters using the Expectation Maximization (EM) algorithm and the Kalman filter [6]. If the system exhibit both polynomial and freeplay nonlinearities, then, after we determine the thresholds, for system identification we can apply the EM algorithm and the extended Kalman filter or the Unscented filter [8]. Applications for aeroelastic system identification will be presented in the full paper.

5 Conclusions

From the results presented here, the proposed procedure is capable to correctly identify the specific type of structural nonlinearities such as freeplay and hysteresis directly for a given aeroelastic response. Moreover, the location of the switching points are also estimated. Once the type of the nonlinearity is determined, we can develop a state-space model for the aeroelastic system, and estimate the system parameters using the EM and the Kalman filtering techniques.

References

[1] M. D. Conner, D. M. Tang, E. H. Dowell, and L.N. Virgin. Nonlinear behavior of a typical airfoil section with control surface freeplay. *Journal of Fluids and Structures*, 11:89–109, 1997.

[2] J. Ko, T. W. Strganac, and A. J. Kurdila. Stability and control of a structurally nonlinear aeroelastic system. *AIAA J. Guid.,Contr., Dynamics*, 21(5):718–725, 1998.

[3] B. H. K. Lee and P. Leblanc. Flutter analysis of a two-dimensional airfoil with cubic nonlinear restoring force. *Aeronautical Report, National Research Council of Canada*, 36(2543), 1986.

[4] B. H. K. Lee, S. J. Price, and Y. S. Wong. Nonlinear aeroelastic analysis of airfoils: bifurcation and chaos. *Progress in Aerospace Sciences*, 35:205–334, 1999.

[5] L. Liping and E. H. Dowell. A harmonic balance approach for an airfoil with a freeplay control surface. *submitted to AIAA Journal*, 2004.

[6] C. A. Popescu and Y. S. Wong. Applications of the EM algorithm for the study of the aeroelastic systems with structural nonlinearities. In *Proceedings of the 3^{rd} International Workshop on Scientific Computing*, Hong Kong, 2003. Nova Science Publisher Inc.

[7] C. A. Popescu and Y. S. Wong. A nonlinear statistical approach for aeroelastic response prediction. *AIAA J. Guid.,Contr., Dynamics*, 26(4):565–572, 2003.

[8] C. A. Popescu and Y. S. Wong. The Unscented and Extended Kalman filter for systems with polynomial restoring forces. In *Proceedings of the 44th AIAA/ ASME/ ASCE/ AHS/ ASC/ Structures, Structural Dynamics, and Materials Conference*, Norfolk, VA, USA, 2003. AIAA Paper 2003-1410.

[9] P.M. Robinson. Non-parametric estimation for time series models. *J. Time Ser. Anal.*, 4:185–208, 1983.

[10] H. Tong. *Nonlinear Time Series.* The Clarendon Press Oxford University Press, New York, 1990.

[11] R.S. Tsay. *Analysis of Financial Time Series.* J. Wiley & Sons, New York, 2002.