52^{nd} *Canadian Operator Symposium*

COSy 2024

Unversity of Alberta, May 27–31, 2024

Abstracts of Talks

Plenary Talks

1. Michael Brannan, *University of Waterloo*

   Quantum graphs, tensor categories, and subfactors

   I will report on joint work with Roberto Hernandez Palomares, where we investigate the theory of quantum (or non-commutative) graphs from the perspective of rigid -tensor categories and their concrete realizations within the framework of subfactors.

2. Alain Connes, *I.H.E.S. (France)*

   From operator algebras to noncommutative geometry

3. Jason Crann, *Carleton University*

   Geometric manifestations of cocycle derivatives and holographic quantum error correction

   Over the past decade, perspectives from quantum information have shed valuable insight into the conjectured anti- de Sitter space/conformal field theory (AdS/CFT) correspondence. In particular, a growing body of evidence suggests that (approximate) quantum error correction could be the mechanism underlying the holographic correspondence: local bulk AdS observables are encoded as operator algebraic quantum error correcting codes on a subspace of the boundary CFT Hilbert space and bulk AdS locality emerges from complementary recovery of the encoding boundary algebras. Motivated by this theory and a recent conjecture of Bousso et. al. on the bulk AdS description of boundary cocycle flow, we characterize (approximate) complementary recovery in terms of (approximate) intertwining of bulk and boundary cocycle derivatives. Our result suggests that, from the operator algebraic perspective, Bousso et. al.s kink transform is precisely bulk cocycle flow, which, in light of the geometry of cocycle derivatives for wedge regions in vacuum AdS, provides additional evidence in support of their conjecture. Time permitting, we will discuss rigorous infinite-dimensional models which admit this error correcting structure. Based on joint work with Monica Jinwoo Kang.

4. Andrew Dean, *Lakehead University*

   Classification questions concerning real structures and gradings on -algebras

   The -algebraic approach to the study of topological insulators provides motivation for the study of real structures and compatible gradings on -algebras. We shall give an introduction to real structures, survey classification results for these objects, and discuss recent progress.

5. George Elliott, *University of Toronto*

   C^*-algebra classification—a birds eye view

   Starting with Glimm sixty five years ago, large classes of separable amenable C^*-algebras have been classified by means of a simple functorial invariant. So far, these have been assumed to be well behaved, in the sense of being Jiang-Su stable, which is not at all automatic (although common), and also to satisfy the Universal Coefficient Theorem (UCT), which may very well be automatic. All simple such algebras have now been dealt with. Very recently, certain simple algebras not in this class have been dealt with by a strengthening of the invariant.

6. Terry Gannon, *University of Alberta*

   Does string theory see the Haagerup subfactor?

   Following Vaughan Jones' seminal work in the early to mid 80s, there has been considerable effort expended on constructing and classifying subfactors. So far at least, exotic subfactors seem quite rare, the most famous being the Haagerup subfactor. But is the Haagerup subfactor relevant? Following Borcherds' seminal work in the late 80s to early 90s, there has been considerable effort expended on constructing and classifying vertex operator algebras (VOAs). So far at least, exotic VOAs seem exceedingly rare. More precisely, there are very few if any examples which are independent of classical math such as Lie algebras or lattices or finite groups. Is this because that is all the VOAs there are? Or are there whole worlds of new families of VOAs, and we are just too dumb (too classical) to find them? The representation theory of VOAs is controlled by structures like subfactors, so it is natural to search for a Haagerup VOA. Indeed, it has been pursued by both mathematicians and physicists (who would call it a Haagerup CFT) for well over a decade. In my talk I'll sketch this story, which has recently taken a new turn.

7. Thierry Giordano, *University of Ottawa* ***Cohomology of free minimal Z^d-actions on the Cantor set.***

   In this talk, I will review properties of cohomology groups H^* (X, ϕ) of free minimal Z^d-actions (X, ϕ) on the Cantor set. In particular, I will recall how to obtain complete invariants of conjugacy, isomorphism and continuous orbit equivalence of Z^d-odometers using their first cohomology groups. In the second part of my presentation I will describe new partial results for free minimal Z^2-actions on the Cantor set beyond the odometer case.

8. Piotr M. Hajac, *IMPAN (Poland)*

   The functoriality of graph algebras

   Motivated by natural examples in noncommutative topology, we study pullbacks of graph C*-algebras. While some pullbacks can be easily understood as coming from pushouts of directed graphs via a contravariant functor assigning C*-algebras to graphs, other pullbacks require both covariant and contravariant functors assigning C*-algebras to graphs considered as objects in two different categories. In the latter case, one cannot explain the pullback structure of a graph C*-algebra in terms of a commuting diagram of underlying graphs in one category of graphs. We solve this problem by introducing a new category of directed graphs, where morphisms are relations rather than maps, and define a covariant functor from this category to the category of C*-algebras. Now, the new functor generalizes both the covariant and contravariant functors used before. Our main result is a pullback theorem stating under which assumptions the new functor maps a commutative diagram of graphs into the pullback diagram of their graph C*-algebras. (Based on recent joint works with Mariusz Tobolski and Gilles Gonalves de Castro.)

9. Matthew Kennedy, *University of Waterloo*

   The ideal structure of reduced crossed products

After providing some motivation and history, I will give an overview of recent results about the ideal structure of reduced crossed products of noncommutative C*-algebras by discrete groups. I will also discuss some open problems. Featuring joint work with Larissa Kroell and Camila Sehnem.

10. Marcelo Laca, *University of Victoria*

    Crystallization of C*-algebras

    Given an almost periodic time evolution σ on a C*-algebra A, we define another C*-algebra A_c –which we call the crystal of (A, σ)– that represents the zero-temperature limit of (A, σ), and we show that the ground states of (A, σ) correspond to the states of A_c. We also show that, under relatively mild assumptions and for sufficiently large inverse temperature β, the KMS$_\beta$-states on A are induced from traces on A_c by means of a generalized Fock module. Motivated by previous work by several authors, we compare the K-theoretic structures of A and A_c, confirming that under favorable conditions A and A_c have (rationally) isomorphic K-groups, but we also show by example that, in general, there is no easy way to relate these groups. This is joint work with Sergey Neshveyev and Makoto Yamashita at Oslo.

11. James Mingo, *Queen's University*

    Infinitesimal operators and infinitesimal independence

    A self-adjoint matrix and a principal minor are very close in a number of ways, in particular an old theorem of Cauchy says that the eigenvalues of the major intertwine those of the minor. As the size of the matrices increases, the difference becomes infinitesimal. I will discuss some interesting connections between the interlacing result and free independence. This is joint work with Pei-Lun Tseng (NYU Abu Dhabi).

12. Matthias Neufang, *Carleton University and University of Lille*

    Banach algebra structures on the space of trace class operators over locally compact quantum groups

    I shall give a survey of our results on Banach algebra structures on the space of trace class operators $T(L_2(G))$ over a locally compact quantum group G. Indeed, the latter carries two dual products which, in the group case, are operator analogues of the convolution product in the group algebra and the pointwise product in the Fourier algebra, respectively. We study in particular the corresponding module structures on $B(L_2(G))$. As an application, we characterize quantum group amenability in terms of injectivity in the category of $T(L_2(G))$-modules. We also present the very recent result that the space of trace class operators of trace 0 carries naturally a Jordan-type product, stemming from the two dual products, which is associative: this gives rise to a new Banach algebra capturing quantum group duality. We show, for instance, that this algebra has a bounded approximate identity if and only if G and its dual $\hat{G}$ are both co-amenable.

The talk is mainly based on joint work with J. Crann, Z. Hu and Z.-J. Ruan.

13. Zhuang Niu, *University of Wyoming*

    A classification of Villadsen algebras

    The class of Villadsen algebras is a class of simple unital approximately homogeneous (AH) C*-algebras which do not absorb the Jiang-Su algebra. Let us consider the Villadsen algebras with seed spaces finite products of a given finite-dimensional contractible metrizable space. Then they actually can be classified by the K0-group together with the radius of comparison. This is a joint work with George Elliott and Chunguang Li.

14. Nico Spronk, *University of Waterloo*

    Unitarizable groups

    The class of unitarizable groups arises very naturally when studying representation theory of locally compact, even discrete groups. They are a class of groups which generate natural von Neumann algebras and hence Fourier-Stieltjes algebras. I wish to talk about various aspects of the theory of these including many open questions.

15. Vladimir Troitsky, *University of Alberta*

    Orthomorphisms and disjointness preserving operators

    Orthomorphisms and disjointness preserving operators are two classes of operators that appear naturally in Banach lattices and, in particular, Banach function spaces. In the talk, we will provide an overview of properties of these operators. We will also discuss their representations as multiplication operators and weighted composition operators.

Contributed Talks

1. Benjamin Anderson-Sackaney, *University of Saskatchewan*

   C*-Simplicity and Related Notions for Quantum Groups

   A group is C^*-simple if its reduced C^*-algebra is simple. Two of the most historically important notions used for proving a group is C^*-simple are Powers' averaging property and topologically free boundary actions. These techniques led to remarkable dynamical characterizations of C^*-simplicity and, as well, the unique trace property, a property that is intimately related to C^*-simplicity. More recently these techniques have been extended successfully to the study of the unique trace property for quantum groups, however, a suitable analogue of a topologically free boundary action remains elusive. In fact, besides the establishment of many quantum examples via a quantum version of Powers' averaging property, not progress has been made on C^*-simplicity for quantum groups. In this talk we will introduce the notion of a stably strong C^*-faithful action of a discrete quantum group, which we note that in the case of a group acting on a topological boundary, is equivalent to topological freeness. We are able to use this notion to recover C^*-simplicity of unimodular free unitary quantum groups. We will also discuss boundary maps and their involvement with Powers' averaging property and C^*-simplicity.

   This talk is based on work-in-progress that is joint with Roland Vergnioux.

2. Aaron Davey, *MacEwan University*

   A note on positive bilinear maps

   This talk concerns positive maps between -algebras, particularly when those positive maps are multilinear. We construct examples of positive bilinear maps that are not 2-positive, and therefore are not completely positive bilinear maps. Paulsen and Smith showed that completely bounded bilinear maps are in one-to-one correspondence with completely bounded linear maps. We show that a similar correspondence does not hold for positive bilinear and linear maps. In particular, we observe that a similar correspondence does not hold if we replace complete boundedness with completely positivity.

3. Dylan Gawlak, *Queen's University*

   On the Orthogonal Weingarten Calculus and Free Probability

Let $\{O_i\}_{i=1}^n$ be a collection of independent Haar distributed orthogonal matrices and let $X = O_1 + O_1^t + \cdots + O_n + O_n^t$. As the size of the matrices tends to infinity, independent Haar distributed orthogonal matrices become real second order free, a condition stronger than regular free independence. This condition allows us to apply the techniques of free probability to the fluctuation moments of real random matrix models. Using the techniques of free probability and the orthogonal Weingarten calculus, we shall analyze the asymptotic distribution of X as well as the asymptotic distribution of the fluctuations. We shall also discuss the asymptotic infinitesimal distribution of Haar distributed orthogonal matrices and it's interpretation in term of annular noncrossing diagrams.

4. Saikia Hridoyananda, *University of Manitoba*

A non-commutative boundary for the dilation order

Arveson's hyperrigidity conjecture focuses on the unique extension property (UEP) of representations of C*-algebras with respect to a generating operator system. The states that are maximal in the dilation order fully encapsulate the cyclic representations of a C*-algebra with the UEP. The set of all maximal states form a norm-closed set which remains stable under absolute continuity. In this talk, we will discuss an equivalent characterization of the dilation maximal states in terms of a boundary projection. Subsequently, we will state a reformulation of Arveson's hyperrigidity conjecture in terms of the noncommutative topological properties of this boundary projection. This is a joint work with Raphal Cloutre.

5. Adam Humeniuk, *Mount Royal University*

C*-covers of operator algebras and the semi-Dirichlet property

A C*-cover of a (non-selfadjoint) operator algebra A is a C*-algebra generated by a completely isometric copy of A. Famously, there is a unique minimal C*-cover, called the C*-envelope. The C*-covers of an operator algebra are ordered and form a complete lattice. I will argue that studying the whole lattice, beyond just the envelope, is a useful and interesting thing to do.

Also interesting is to study nice classes of C*-covers. In joint work with Elias Katsoulis and Christopher Ramsey, we showed that the restricted class of C*-covers with the semi-Dirichlet property is a complete sublattice. As a consequence, for any semi-Dirichlet operator algebra, there is a maximal semi-Dirichlet C*-algebra, which is a new universal object that generalizes familiar examples, such as Cuntz-Pimsner-Toeplitz algebras of C*-correspondences.

6. Boyu Li, *New Mexico State University*

   Universality of simple cycle reservoirs using dilation theory

   Reservoir computation models form a subclass of recurrent neural networks with fixed non-trainable input and dynamic coupling weights. Reservoir models have been successfully applied in a variety of tasks and were shown to be universal approximators of time-invariant fading memory dynamic filters under various settings. Simple cycle reservoirs (SCR) have been suggested as severely restricted reservoir architecture, but well suited for hardware implementations without performance degradation in many practical tasks.

   We use a technique from dilation theory to show that SCRs are capable of universal approximation of any linear reservoir system with continuous readout and hence any time-invariant fading memory filter over uniformly bounded input streams.

   This is a joint work with Robert Simon Fong and Peter Tiňo

7. Vasily Melnikov, *University of Alberta*

   A quantized Grothendieck closedness theorem

   Abstract: Suppose that (M, τ) is a tracial von Neumann algebra. In the spirit of a classical result of Grothendieck, we investigate under what conditions on M is a closed linear subspace $E \subset M$ finite-dimensional whenever E is closed in some non-commutative Haagerup L_p-space ($p \neq \infty$). As a striking application of our results, the inclusion map $ML_1(M, \tau)$ factors through $C(K)$ or $L_1(\mu)$ if, and only if, M is isomorphic as a Banach space to an Abelian von Neumann algebra.

8. Mehdi Moradi , *University of Ottawa*

   Finite-dimensional approximation properties of traces

   One of the main ideas in the classification of injective factors is the notion of hypertrace introduced by Connes in 1976 and successfully exploited by Haagerup, Popa, Kirchberg, and most recently by Tikuisis-White-Winter and Schafhauser to solve many long-standing problems.

After introducing the C*-analog of hypertraces, namely, amenable traces and their cousins quasidiagonal and locally finite-dimensional traces, defined for the first time by N. Brown in 2006, I will answer basic questions already mentioned by Brown. I will finish my talk by stating a theorem that I hope sheds some light on the interrelation of logically distinct types of traces and other well-known problems. This talk is based on joint work with Massoud Amini.

9. Ana Savu, *University of Alberta*

   Quantum Markov chain

   Quantum Markov chains are dynamics that generalize the classical Markov chain to quantum setting. Like in the classical setting, the evolution of the quantum chain in the future depends only on the current state and not on history. However, the transition to the next instance is achieved by acting linearly on the wave function and not on the distribution of the system. The probability distribution of the quantum chain is obtained by evaluating the trace on a set of operators and completely positive maps replace transition probabilities. As an example rolling a die in the quantum realm is presented.

10. Paul Skoufranis, *York University*

    Bi-Free Independence and Asymptotics of Random Quantum Channels

    It has been 10 years since the first paper on bi-free independence was published by Voiculescu. Since then, the theory of bi-free probability has been well developed. In this talk, a basic introduction to the main concepts of bi-free probability will be provided. Subsequently, upcoming work connecting bi-free probability to the asymptotics of random quantum channels will be discussed.

11. Brendan Steed, *University of Victoria*

    A Generalized Heisenberg Cycle

    Using the annihilation and creation operators, $\frac{d}{dx} \pm x$, together with their relation to the harmonic oscillator, $-\frac{d^2}{dx^2} + x^2$, the authors Butler, Emerson, and Schulz constructed a spectral, called the *Heisenberg cycle*, over the crossed-product C^*-algebra $C_u(R)R_d$ generated by the discretely topologized real numbers acting by translation on the C^*-algebra of uniformly continuous bounded functions on R. In this talk, we present a generalization of the Heisenberg cycle; A spectral triple over the crossed-product $C_u(R^n)R^n_d$ generated by letting the discretely topologized underlying group structure of R^n act by translation on the C^*-algebra of uniformly continuous functions over R^n. The operator for this generalized Heisenberg cycle comes from replacing $\frac{d}{dx} \pm x$ with the action of $d + d\rho$ on suitable differential forms in $\Lambda^* R^n$, where d is the exterior derivative and $d\rho$ multiplication by the 1-form $\sum_{i=1}^n x_i dx_i$. We conclude the talk with an analysis of the ζ-functions associated to the Heisenberg cycle, and a discussion of some notable subalgebras in $C_u(R^n)R^n_d$ along which the Heisenberg cycle may be pulled-back to produce other notable spectral triples.

12. Tomasz Szczepanski, *University of Alberta*

    On the chain of commuting operators and Lomonosov's Invariant Subspace Theorem

    In 1973 V. Lomonosov proved that if T is an operator on a Banach space X such that it commutes with a non-scalar operator S and S commutes with a non-zero compact operator K, then T has an invariant subspace. We call T a Lomonosov operator and abbreviate the chain of commuting operators described by this theorem to T-S-K.

    For a long time it wasnt clear if there are operators that are not of Lomonosov type. The answer to this question came from the construction of the first counterexample of an operator without invariant subspaces (P. Enflo (1976)). After that it was still an open question if every operator T with an invariant subspace can be connected via chain T-S-K. Finally, in 1980 D. Hadwin, E. Nordgren, H. Radjavi, and P. Rosenthal gave an example of a non-Lomonosov operator that does have an invariant subspace.

    In 2000 V. Troitsky showed that the chain of commuting operators in Lomonosov's Theorem cannot be extended to four operators, namely T-S_1-S_2-K doesn't imply that T has an invariant subspace. Based on this observation in this talk we show that the non-Lomonosov operator from the paper of D. Hadwin et al. can be connected via chain T-S_1-S_2-K. This poses a question, if every operator can be connected via chain T-S_1-S_2-K. We show that this is true for a certain class of operators including isomorphisms on ℓ_p spaces. The question of the existence of operators that require a chain of more than 4 operators connecting them with a non-zero compact operator remains open.

13. Ian Thompson, *University of Manitoba*

Rigidity and uniqueness of approximations for operator subsystems

We investigate various forms under which an operator subsystem forms a test set in its generating C*-algebra. This philosophy underlies Arveson's notion of hyperrigidity, which was partially inspired by a function theoretic statement of Šaškin. In our work, we develop noncommutative formulations of Šaškin's theorem. This results in a method of encoding a global rigidity constraint in terms of a single representation. We also discuss the impact that our work has in relation to a recent counterexample to Arveson's hyperrigidity conjecture. This is joint work with Raphaël Clouâtre.

14. Venkata Karthik Timmavajjula, *University of New Brunswick, Fredericton*

    Noncommutative Riemannian Geometry on S_3

    We extend calculations of Majid and Raineri to compute the moduli space of generalized Hodge star operators on S_3 as a finite set. Here, S_3 is equipped with the bi-invariant calculus induced by Ad-invariant generating set consisting of transpositions. If time permits, well look at the corresponding vacuum Maxwells equations.

15. Dan Ursu, *University of Münster*

    Structure of intermediate subalgebras of crossed products of free actions

    Let G be a discrete group acting on a compact Hausdorff space X, and consider the reduced crossed product $C(X)_\lambda G$. Assume that the action is free, and moreover that the group G has an additional weak assumption known as the Approximation Property (AP). In joint work with Matthew Kennedy, we show that any intermediate subalgebra B satisfying $C(X) \subseteq B \subseteq C(X)_\lambda G$ is canonically the reduced crossed product arising out of a partial dynamical subsystem of the action of G on X. We also show that the converse is true, namely that if the action is not free, then there always exists an intermediate subalgebra that is not of this form.

16. Matthew Wiersma, *University of Winnipeg*

    Traces on locally compact groups

    We conduct a systematic study of tracial states on -algebras of locally compact groups in relation to the structure of the underlying group. We introduce the trace kernel, and examine its relation to the von Neumann kernel and to small invariant neighbourhood (SIN) quotients. Restricting our attention to amenable traces, we show for property (T) groups that amenable trace kernels coincide with von Neumann kernels. We also show for totally disconnected groups that amenable trace separation implies the factorization property. As an application of our results, we address problems involving simple AF embeddability and quasi-diagonality for -algebras of locally compact groups.