Novel Closed-loop Stiction Detection and Quantification Method via System Identification

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Abstract— In this paper, a novel detection and quantification method of control valve stiction is discussed. A time delay estimation method is presented for processes under an oscillating state. A suitable model structure of valve stiction is chosen prior to conducting valve stiction detection and quantification. Given the stiction model structure, a bounded search space of a stiction model is defined and a constrained optimization problem is performed. The validity of the proposed method is illustrated through industrial examples.

I. INTRODUCTION

In a basic control-loop with a valve, SP, OP, MV, and PV stand for the set point, the controller output, the manipulated variable, and the controlled variable, respectively. Note that SP, OP, and PV are usually recorded on the distributed control system in industry, so are readily available, while the manipulated variables such as flow rate are not always available.

Valve stiction often causes oscillation in control loops. The presence of oscillation increases the variability of the process variables; thus decreases the quality of product and increases energy consumption. It is known that the undesirable behavior of control valves is the biggest single contributor to poor control loop performance and the destabilization of process operations.

Several methods for detection and quantification of valve stiction have been presented in the literature [1]–[8]. However, many of them have some practical limitations one way or other, which have to be considered for real applications in industry. Quantification of control valve stiction is still a challenging issue.

In this paper, a novel stiction detection and quantification strategy based on process model identification is proposed using routine closed-loop operating data. Prior to doing stiction detection and quantification, it is necessary to choose a suitable model structure to describe control valve stiction. Several data-driven valve stiction model structures are available in the literature [3], [7], [9]. In this paper, the one proposed by He et al. [7] will be adopted. Given a valve stiction model structure, a search space of stiction model parameters is determined by using controller output data, which is also called OP. Note that, if a valve stiction model is exactly known, then a time series of manipulated variable (MV) can be calculated from given OP data and the valve

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stiction model. A process model can be estimated by using system identification algorithms with MV and OP data. In this paper, the problem of interest is to find the best unknown stiction model parameters satisfying a mean squared error criterion within a space of valve stiction model parameters. The proposed strategy not only detects but also quantifies valve stiction.

The proposed method has many advantages from the practical implementation point of view. First, it has no requirement to filter original data and can be easily implemented. It can be implemented as an automatic detection tool because it uses only routine operating data. Also it can deal with open-loop data if OP moves in downward and upward directions several times. The effectiveness of the proposed stiction detection and quantification method is demonstrated by industrial examples.

II. TIME DELAY ESTIMATION

The time delay estimator (TDE) is to determine the delay D between two sensor signals: $x_k = s_k + \theta_k$ and $y_k = s_{k-D} + \phi_k$, where s_k is a stationary bandlimited random process and θ_k, ϕ_k are additive Gaussian noise signals. The desire for high resolution delay estimation is particularly evident in equalization and source localization in sonar signal processing. There were many publications on the time delay estimation for this kind of systems. The most common and popular method for TDE is a generalized cross-correlation technique.

A. A Proposed Method

This method requires a process under an oscillating state. This is the typical case in a loop that has oscillations such to an extent that the root cause needs to be determined.. The input and output data collected from the process with oscillation can be used to obtain an estimate of a process time delay.

B. Main Procedure

1) Set $i = 1$ for the first-order plus time delay process and $i = 2$ for the second-order plus time delay process.
2) Collect $u(t)$ and $y(t)$ from the process.
3) Get differences $U_1(t) = u(t+1) - u(t)$ and $Y_1(t) = y(t+1) - y(t)$ for $i = 1$, and $U_2(t) = U_1(t+1) - U_1(t)$ and $Y_2(t) = Y_1(t+1) - Y_1(t)$ for $i = 2$.
4) Calculate cross-correlation functions $R_i(\tau)$ given by
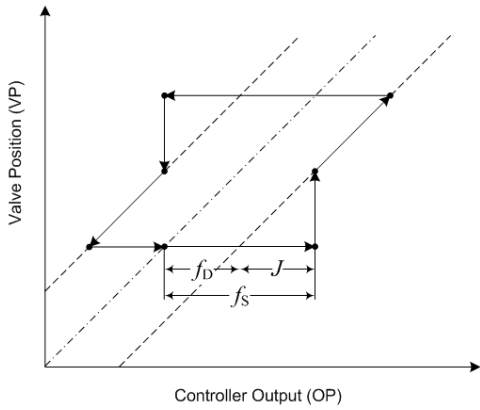
$$R_i(\tau) = \sum_{\tau} U_i(t) Y_i(t+\tau). \qquad (1)$$

Fig. 1. Schematic operation diagram of a sticky valve.

5) Find the largest absolute value $R_i(\tau_0)$ among all $R_i(\tau)$, where τ is a positive integer.
6) Then τ_0 is the process time delay.

In the time delay estimation, the first- or second-order plus time delay system is considered.

III. STICTION MODEL OF CONTROL VALVE

A. General Conception

Fig. 1 shows the schematic operation diagram of a sticky valve, where f_D denotes the kinetic friction band, J the stick band, and f_S the static friction band. Some of definitions of stiction can be found in [9], [10].

B. Physical Model of Valve Sticion

Several physical friction models have been presented in [11]–[13]. One of the commonly used friction models is the Karnopp model. It includes static and moving friction. The disadvantage when applying the friction model to a generic valve is the need to specify a large set of parameters. In order to overcome this disadvantage many researchers developed different kinds of empirical data-driven stiction models. The data-driven models have parameters that can be directly related to plant data and they produce the same behavior as the physical model. The data-driven models need only an input signal and the specification of deadband plus stickband and slip jump. It overcomes the main disadvantages of physical modeling of a control valve.

C. Kano et al. Valve Stiction Model

A valve stiction model was proposed by Kano et al. [3]. The input and output of this valve stiction model are the controller output and the valve position, respectively. The controller output is transformed to the range corresponding to the valve position in advance. The first two branches in the model flow chart check if the upper and the lower bounds of the controller output are satisfied. This valve stiction model has several advantages: (i) It can cope with the stochastic input as well as the deterministic input. (ii) $u_s(t)$, which is the controller output at the moment the valve state changes from moving to resting, can be updated at appropriate timings by introducing the valve state. (iii) It

can change the degree of stiction according to the direction of the valve movement.

D. Choudhury et al. Valve Stiction Model

Choudhury et al. proposed a valve stiction model in [9], where the control signal is translated to the percentage of valve travel with the help of a linear look-up table. The model consists of two parameters, namely, deadband plus stickband S, which is specified in the input axis, and slip jump J, which is specified in the output axis.

E. He et al. Valve Stiction Model

In the valve stiction model of Kano et al., no matter how small $\Delta u(t)$ is, as long as it is greater than zero, the valve will always move and stop, which is not logically correct. The Choudhury et al. stiction model has the same problem. Another issue associated with the Kano et al. and Choudhury et al. models is that the saturation constraints are added to the controller output instead of an actuator (a valve). Based on the typical input-output behavior of a sticky valve, He et al. proposed a new valve stiction model [7], which is simpler and more straightforward in logic. If desired, the saturation constraint can be easily added to $u_v(t)$ after the model calculation.

IV. EXISTING STICTION DETECTION METHODS

A. Open-Loop Methods

Stiction can easily be detected using invasive methods such as the valve travel or bump test. However, to apply such invasive methods across an entire plant site is neither feasible nor cost-effective because of their manpower, cost and time intensive nature. Several methods have been developed to detect valve stiction in the last decade [14]–[16]. However, most methods require either detailed process knowledge or user interaction, which is not desirable for automated monitoring systems.

B. Closed-Loop Methods

Horch [1] presented an automatic detection method based on the cross-correlation function (CCF) between the controller output and the process output, which is applicable to nonintegrating processes. In a continuing work, Horch [10] proposed another method to address the valve stiction in integrating processes by considering the probability distribution of the second derivative of the controlled variable.

Singhal and Salsbury [2] presented a valve stiction detection method based on comparison of areas before and after the peak of an oscillating control error signal, i.e., the difference between the set-point and the process variable being controlled.

Kano et al. [3] presented two valve stiction detection methods: one requires knowing the valve position (VP) and the other is based on the plot of OP and PV with the shape of parallelogram.

He and Pottmann [4] presented a valve stiction detection technique in which the OP is fitted piece-wisely to both triangular and sinusoidal wave using the least square method.

A better fit to the triangle indicates valve stiction, while a better fit to the sinusoid indicates non-stiction. Also in the work, a stiction index (SI) was first defined as the ratio of the mean squared error (MSE) of sinusoidal fitting and the sum of the MSE's of both sinusoidal and triangular fittings. An SI close to zero would indicate non-stiction while an SI close to one would indicate stiction.

In the meantime, Rossi and Scali [5] presented independently a very similar technique to [4]. In [5], the PV signal instead of OP is fitted using three different signal models: relay, triangular, and sinusoidal wave.

He et al. [7], [17] extended the previous work in [4] to cover both self-regulating and integrating processes based on the following observations: In the case of control loop oscillation caused by poor controller tuning or external oscillating disturbance, OP and PV typically follow sinusoidal waves for both self-regulating and integrating processes. In the case of stiction, for self-regulating processes, OP will move like a triangular wave, while for integrating processes such as level control, PV will move like a triangular wave. The basic idea of this detection method is to fit two different functions, i.e., triangle and sinusoid, to the measured oscillating signal, where OP is for self-regulating processes and PV is for integrating processes. A better fit to the triangle indicates valve stiction, while a better fit to the sinusoid indicates non-stiction. The SI metric is used as a criterion to evaluate the existence of valve stiction.

Choudhury et al. [6] presented a method to detect and quantify stiction using routine operating data. The nonlinearity of the loop is tested using bicoherence. If the nonlinearity is detected, stiction is estimated as the maximum width of the cycles of the PV vs. OP plot in the direction of OP. The PV vs. OP plot is fitted to an ellipse and the amount of stiction is estimated to be the maximum length of the ellipse in OP direction, which is called the ellipse-fitting method. The stiction estimated using the method of Choudhury et al. is stated as 'apparent stiction' and it provides indication of the severity of the consequences of the stiction in an oscillatory loop. A simple grid search method for estimating stiction parameters was presented in [18].

Recently, Jelali [8] independently presented a global optimization-based method for quantification of valve stiction in control loops. It calculates an initial approximate guess of S and J, which are the dead-band plus stick band and the slip jump, respectively, using the ellipse-fitting method [6], and searches for the optimum point near the initial guess using genetic algorithms or pattern search.

C. Discussion of the Existing Methods

In Horch [1], one issue is the differentiation of noisy signals. A suitable filter and cut-off frequency have to be carefully chosen in order to filter noise. This can hardly be done automatically since different processes have different system characteristics and different noise levels. It has been observed that even after filtering, the calculation of derivatives amplified moderate amount of noise and blurred

the distinction between the shapes of the two probability distributions [19].

In Singhal and Salsbury [2], there are several practical limitations as mentioned by authors themselves: (i) The method can not be applied to integrating processes. (ii) It cannot distinguish other nonlinearities from stiction. (iii) The error signal must be sampled many times per oscillation period in order to get accurate peak location and areas calculation. (iv) The noise adds variation to the peak and zero-crossing locations which could result in problematic diagnosis.

In Kano et al. [3], the first method can be used only when flow rate or valve position is measured. The second method is not always reliable even when flow rate or valve position is measured as shown in one of their flow control examples.

In He et al. [17], there are two issues: i) When the controller has bad tuning, the fitting method is not effective. ii) For processes with large time delay, the opposite result could be obtained. Controllers with poorly tuned parameters and processes with large time delay often exist in industry, which limits the use of the method.

The method of Rossi and Scali [5] is very similar to that proposed by He et al. [17]. It is only applicable to self-regulating processes.

In Choudhury et al. [6], [18], the ellipse-fitting method has a limitation in the fact that the shape and size of the PV vs. OP plot is sensitive to several factors: The changes of proportional or integral control gain, the process gain, the process time constant, the time delay of the process, phase lags, etc. Hence, 'apparent stiction' that the ellipse-fitting method estimates will differ from real stiction.

In Jelali [8], a good initialization needs for the stiction estimation. The initial point of stiction parameters is obtained by using the ellipse-fitting method and the optimum solution is found near the given initial guess. As noted above, the ellipse-fitting method may not be accurate and hence the optimum solution found near the initial guess may not be the solution that describes the behavior of the control valve the best. Also the strategy for time delay estimation of the process is not well addressed. The genetic algorithms adopted requires a large number of functional evaluations per iteration and storing a considerable amount of information in the computer memory.

V. NOVEL CLOSED-LOOP STICTION DETECTION AND QUANTIFICATION

A. Basic Principle and Important Steps

The basic idea is to convert the stiction detection and quantification problem into a low-order Hammerstein type system identification problem, followed by a global optimization search for the stiction parameters. This idea focuses on finding a non-invasive method to determine if stiction exists in a control valve. It approximates process dynamics by a low-order transfer function model while estimating parameters for the static stiction model to account for the nonlinearity induced by the stiction. We consider that most

industrial processes can be approximated as the first or second order plus time delay process.

It is required to choose a suitable stiction model structure before proceed. The basic steps to follow in the proposed method are:

1) Given a stiction model structure and OP data, effectively bound a search space of unknown stiction model parameters.
2) Choose stiction model parameters from the bounded stiction model space, and a series of manipulated variable (MV′) data is calculated from OP data according to the given valve stiction model.
3) With MV′ and PV data, the process model is identified such that a mean squared error is minimized. Varying stiction model parameters, different process models are obtained.
4) Find the stiction model that describes the characteristics of the control valve behavior the best. Find the minimum model error and get the corresponding process model and stiction model parameters.

The key to success of this procedure lies in the efficient optimal global search of stiction parameters.

B. Stiction Detection Procedure

1) Data Selection: Tests of the proposed method on simulated data showed that it is necessary to select appropriate low and high limits on the sampling time. Our experience also shows that it is necessary to select the sampling time to make sure there are more than 50 data points in an oscillating period.

2) Data Preprocessing: Filtering is not necessary in the proposed method, but detrending the input and output data is important. Detrending is the process of removing the zero order trend (the mean) from the original data and needs not only for input and output data (OP and PV), but also for the generated MV′ data.

3) Stiction Model Structure: We found that the valve stiction model structure of He et al. [7] is not only simple in logic but also closer to real stiction behavior. If we search expected parameters of the mentioned stiction model according to the data, the model with optimal parameters has been shown very close to the real stiction characteristics.

4) Search Space of Stiction Model : A search region of stiction model parameters is defined for constrained optimization. The region of stiction model parameters is determined using OP data and the given stiction model structure.

5) Process Model Identification: Under the assumption that the process is a first- or second-order plus time delay process, the ordinary least square method is suitable for identification. The time delay of the process may either be searched in the optimization or effectively identified by applying the time delay estimation method proposed in Section II.

6) Quantification of Valve Stiction: A cost-effective constrained optimization technique is adopted for comprehensive stiction model parameter search. It finds the stiction model
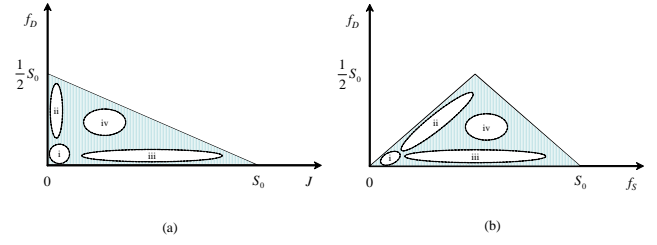

Fig. 2. Spaces of stiction parameters. (a) The space of stiction parameters (f_D, J). (b) The space of stiction parameters (f_D, f_S). (i) Week stiction. (ii) Dead-bend behavior. (iii) Stick and slip behavior. (iv) Dead-bend plus stick and slip behavior.

that describes the characteristics of the control valve behavior the best. The model with a minimum error implies the most possible and realistic stiction model parameters found.

C. Search Space of Stiction Model Parameters

The control valve is a physical link with movement in control loop and the characteristic of its behavior is described by its physical specification. A space of stiction parameters for search can be defined and be specified by using the OP data and the relationship of stiction parameters. Note that $f_D + f_S \leq S_0$, where $f_D \geq 0$, $f_S \geq 0$, and the upper bound S_0 is approximately given by the span of OP. Due to the relation $f_S = f_D + J$, it holds that $2f_D + J \leq S_0$. Fig. 2 (a) illustrates the constrained search space of stiction model parameters (f_D, J). Fig. 2 (b) shows an equivalent search space of stiction parameters in terms of (f_D, f_S). Note that the upper bound S_0 plays an role in constraining the stiction parameter domain. A tight upper bound S_0 can be obtained by estimating the length S on the op-pv plot regardless whether stiction is present or not. As an example, the ellipse-fitting method in [6] can be used.

D. Constrained Optimization Techniques for Parameter Search

There are two principal goals leading to the design of global optimization methods: (i) Global reliability to ensure that the domain is searched sufficiently to provide a reliable estimate of a global solution and (ii) local refinement to produce a fine solution. Most global optimization algorithms have been developed to achieve these two goals by combining a global strategy and a local strategy [20].

1) Multistart Adaptive Random Search: Random search algorithms allow in principle to find a global minimum and the solution does not depend on the starting point. Adaptive random search is known as an efficient random search algorithm with systematic reduction of the size of the search region. Note that combining the basic adaptive random search method with a multistart approach improves the ability to reach the global minimum. The basic procedure of multistart adaptive random search is as follows: (i) Generate a set of random starting points and iterate an adaptive random search algorithm with rough accuracy on each starting point, which obtains a set of approximate local minimum points. (ii) Apply an efficient local search algorithm for the optimum
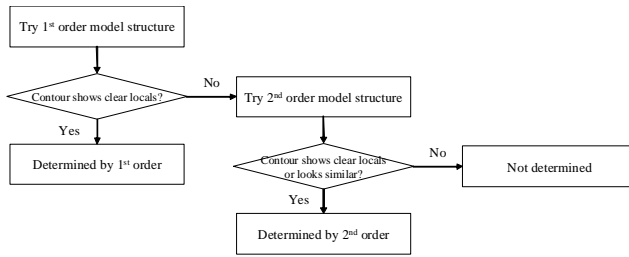
Fig. 3. A simple flow chart for model structure and solution determination.

search in the vicinity of the minimum points found in the step (i). The step (i) attempts to find promising starting points that are more likely to reach the location of a global optimum. The step (ii) is a fine global optimization within a reduced search domain near the approximate local points given in (i). It is suitable to the problems of a large number of local optima and finds a global solution best for complicated models. In realty, it is observed that many local minima are present in the space of stiction model parameters. Various advanced modifications of the above basic algorithm can be found in the literature [21], [22]. Note that in the cases when the model structure of the process is known *a priori*, the multistart adaptive search technique can be applied to the constrained optimization problem to seek a stiction model within the bounded space of stiction model parameters.

2) Contour Map: If the process model structure is unknown, the flow chart in Fig. 3 can help determine a suitable model structure and solution of stiction model parameters. It is noted that the process model structure can affect the local solution structure in the contour map. The basic procedure is as follows: (i) Uniform grid search with 1% to 2% resolution. (ii) Followed by the local search. In the step (ii), the lowest value of the objective function is taken as a starting point for the fine local search.

E. Advantages

i) Simple method and easy implementation. No need to filter the original data. The identification process is to find the most suitable parameters of the valve model and it is effective even under large noises in the output of the process.

ii) Simple process model structure. A dozen of examples, including first order plus time delay process, second order plus time delay process and integrating plus time delay process, are simulated. In these cases, if no stiction exists, the stiction parameters of the valve can be identified as zeros undoubtedly by the proposed method. If stiction exists, the stiction parameters of the valve can be identified with satisfactory performance.

iii) Low computational cost. We directly program a least square algorithm with an analytical structure, and do not use the Matlab system identification toolbox to identify the process. This has shorten the run time greatly. Using the original Matlab function, the run time is more than 5 minutes for a control loop with

1500 data points. Using the proposed method, the average computation time is less than half a minute.

iv) Closed-loop method using routine operating data. This method not only detects stiction but also quantifies it.

iv) It can be proved theoretically that the comprehensive search as we did in the proposed algorithm is necessary for consistency of the estimation. Due to space limit, the proof is not included here.

VI. INDUSTRIAL CASE STUDIES

To demonstrate the validity of the proposed method, industrial control loop data shown in Fig. 4 are considered. Fig. 4 shows time series plots of OP and PV and Fig. 5 shows OP vs. PV plots of the control loop data. It is noted that Loop 1-2 are open-loop data and others are closed-loop data. A computer system with Intel Pentium(R) CPU 3.2GHz and 2GB of RAM was used for computation and the search space of stiction model parameters was bounded by the span of OP. From the case studies, it is seen that Loop 1, 4-5 show mostly stick and slip behavior, Loop 2, 6 show dead-band plus stick and slip behavior, and Loop 3 shows mostly no stiction. It is noted that, for Loop 3-4, 6, the results obtained by the proposed method are in agreement with the result of [6], while for oscillating Loop 2, 5, the proposed results are not in agreement with those of [6]. Stiction quantification results of [8] would also appear to be similar to the results of [6]. Both [6] and [8] rely on the OP vs PV plot method to some extent, and thus show similar results. The proposed method, however, does not depend on an initial guess given by the OP vs. PV plot but is based on a constrained optimization with a bounded space of a stiction model. 'Apparent stiction' obtained from the ellipse-fitting method is different from real stiction and thus may not be reliable as an initial guess. It is also noted that the constrained optimization algorithm adopted in the proposed method is efficient in computation. It takes 19.8 sec. and 33 sec. on average for the 1st order and 2nd order plus time delay model, respectively.

VII. CONCLUDING REMARKS

A novel closed-loop stiction detection and quantification strategy using routine closed-loop operating data is presented based on a model identification approach. A time delay estimation method for processes under an oscillating state is presented. A bounded search region of stiction model parameters to be determined is defined and a constrained optimization problem for valve stiction model estimation is addressed. A cost-effective constrained optimization technique is adopted to find the best valve stiction models representing a more realistic valve behavior in the oscillating loop. Industrial case studies demonstrate the effectiveness of the proposed method.

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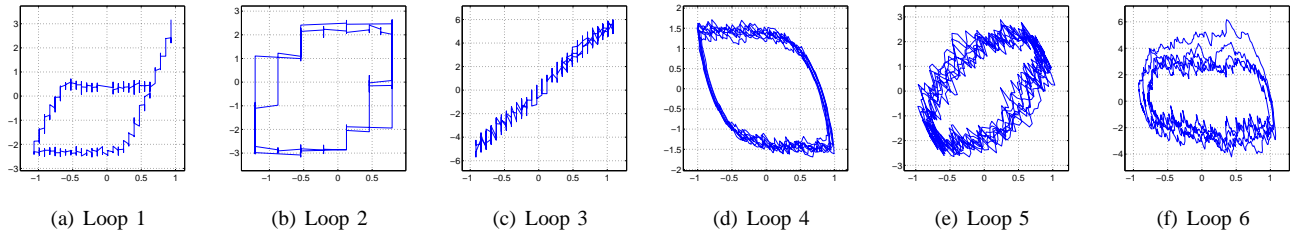
(a) Loop 1 (b) Loop 2 (c) Loop 3 (d) Loop 4 (e) Loop 5 (f) Loop 6

Fig. 5.   OP vs. PV plots of industrial control loop data.

TABLE I

COMPARATIVE STUDY OF INDUSTRIAL CONTROL LOOP DATA

Loop No.	Loop Type	Data Size	Span of OP	f'_S	f'_D	S [6]	CPU time (sec.)	
							1st order	2nd order
1	Pressure control	4,685	2	0.52	0.02	1.38	37.2	49.9
2	Pressure control	900	2	0.82	0.18	2	14.2	21.1
3	Pressure control	2,732	2	0	0	0	17.4	37.9
4	Pressure control	1,000	2	1.70	0	1.65	12.9	15.4
5	Flow control	1,000	2	0.10	0.04	1.23	14.0	19.5
6	Flow control	2,000	2	0.78	0.64	1.84	23.4	54.3

(a) Loop 1 (b) Loop 2
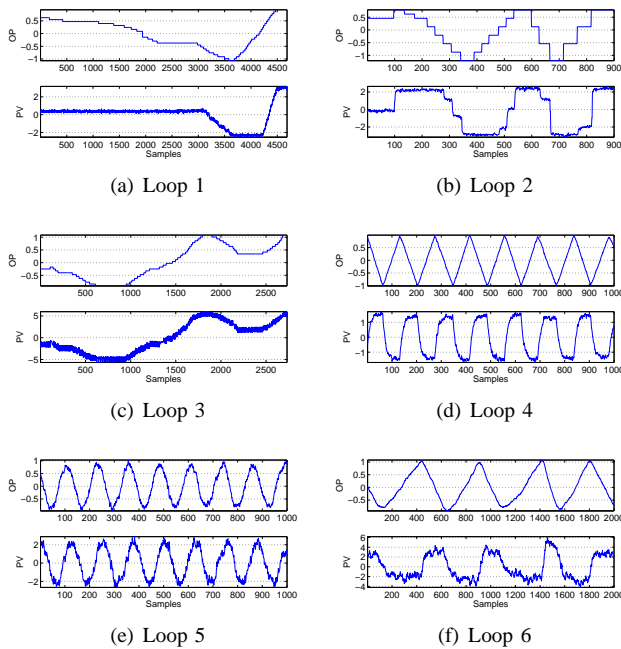
(c) Loop 3 (d) Loop 4

(e) Loop 5 (f) Loop 6

Fig. 4.   Time series plots of industrial control loop data.

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