Measurement of particle dynamics in a coherent acoustic field

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ABSTRACT.

A variety of forces can be used to separate fine particles from water in industrial processes. A coherent standing acoustic field can apply a pressure difference across a small particle such that the particle is forced to nodes or anti-nodes in the field depending of particle/liquid properties. The motion of an individual particle is the result of the balance of forces on the particle which can include particle-to-particle interactions. This study develops a method to measure individual particle motion using a particle tracking velocimetry (PTV) approach. This method is used to investigate the effects of particle loading in an acoustic field and is used to test the current theory which only addresses single particle motion. The paper focuses on the application and validation of a PTV-based experiential measurement technique used to determine the forces applied on individual particles.

KEYWORDS:

Coherent Acoustic Field, Macro Particle Separation, Particle Clustering, Particle Tracking Velocimetry

INTRODUCTION

When a free single spherical particle is exposed to an acoustic wave, primary and secondary hydrodynamic acoustic forces are developed on the particle. The magnitude of the force is reinforced when the acoustic field is in a standing wave mode, rather than in a plane travelling wave mode. The force can be used effectively to concentrate small particles to some designated location. This process can be used in applications to separate fine particles such as in the manipulation of nanoparticles [1] or in separation of particles in a microchannel [2].

Analytical research has developed over the last century theory to predict the effects of an acoustic force on single particles from different fundamental viewpoints. King [3] derived an expression for the radiation pressure due to an acoustic standing wave field on an incompressible particle suspended in an inviscid fluid by integrating the acoustic pressure on the surface of the particle. Yosioka and Kawasima [4] obtained the primary radiation force on a compressible particle by defining a velocity potential for the displacement velocity of particles. Gorkov [5] took a different approach by defining the potential energy of an ultrasonic wave in the fluid. In all of these approaches, only a single particle is considered. Effects of particle-to-particle interactions, which are obviously important with increasing particle loading, are not addressed.

A study has been undertaken to investigate the potential use of acoustic field for particle separation in macro-scale industrial systems. To investigate the limit to which the single particle theory can be applied an experiment has been designed to determine the forces on individual particles. This paper develops the methodology for determined the forces on particles using velocity data collect with a particle tracking velocimetry (PTV) technique. The effect of total particle concentration on the method for determining velocity measurement is investigated with an emphasis on effects of particle clustering which occurs over time. The paper will also compares experimental results of the motion of many particles due to an acoustic field with the single particle model to assess effects of total particle load.

THEORY AND MODELING

Force on a Single Particle Due to Acoustic Pressure

Assuming that the particles are suspended in an ideal fluid, they are compressible and their diameter is small compared to the wave wavelength of the acoustic field, the force on the particle can be defined as the gradient of the acoustic potential energy (Gorkov [5]). For a standing-wave field where the radiation force is much larger than a progressive wave, the acoustic potential energy (U) can be described by:

$$U = V_0 \left(\frac{\beta_f - \beta_p}{\beta_f} < PE > - \frac{3(\rho_p - \rho_f)}{\rho_f + 2\rho_p} < KE > \right)] \qquad 1$$

where V_0 is the volume of the particle, ρ and β are the density and compressibility, and the subscripts f and p are used for fluid and particle's properties respectively. The time averaged potential $< PE >$ and kinetic $< KE >$ energy densities of the field are defined as:

$$< PE > = \frac{\beta_f < P >^2}{2} \qquad 2$$

$$< KE > = \frac{\rho_f < v >^2}{2} \qquad 3$$

where $< P >$ and $< v >$ are time-averaged pressure and displacement velocity of the wave in a standing field arrangement. These can be given as,

$$p(x, y, z, t) \ = P_0(x, y)\sin{(\kappa z)}\sin{(\omega t)} \qquad 4$$

$$v(x, y, z, t) = v_0(x, y)\cos{(\kappa z)}\cos{(\omega t)}\boldsymbol{k} \qquad 5$$

where P_0 and $v_0 = \frac{P_0}{\rho_f c}$ are the pressure and velocity amplitude of the wave depending on spatial location (x, y, z) with y the axial coordinate in the direction of gravity, c is the fluid sound speed, $\kappa = \frac{2\pi}{\lambda}$ is the wave number of the wave, $\omega = 2\pi f$ is the angular frequency, and $\boldsymbol{k}$ is the unit vector in the axial direction. As the force on a particle due to acoustic pressure is the gradient of the potential energy of the wave, $F = -\nabla U$, the force can be derived as:

$$F = \frac{V_0 P_0^2 \kappa}{4\rho c^2}\eta sin\ (2\kappa z) \qquad 6$$

Here, η is the acoustic contrast factor that characterizes the relationship between the particle and fluid properties that lead to the acoustic force. It is defined as:

$$\eta = \left(\frac{\Lambda + \frac{2}{3}(\Lambda - 1)}{1 + 2\Lambda}\right) - (\frac{1}{3\Lambda\sigma^2})) \qquad 7$$

$$\Lambda = \frac{\rho_0}{\rho} \qquad 8$$

$$\sigma = \frac{c_0}{c} \qquad 9$$

In properties of typical materials that are used in the investigation are presented. The calculated value of the acoustic contrast factor for different (ideal) density ratio and speed of sounds are plotted in Figure 1.

Table 1 Specifications of the materials used in the experiment

Particle	Density(gr./cm^3)	Speed of sound (m/s)
60P18 Hollow Glass	0.6	5600
110P8 Hollow Glass	1.1	5600
Ethanol	0.8	1720
Soda Lime Glass	2.5	8400

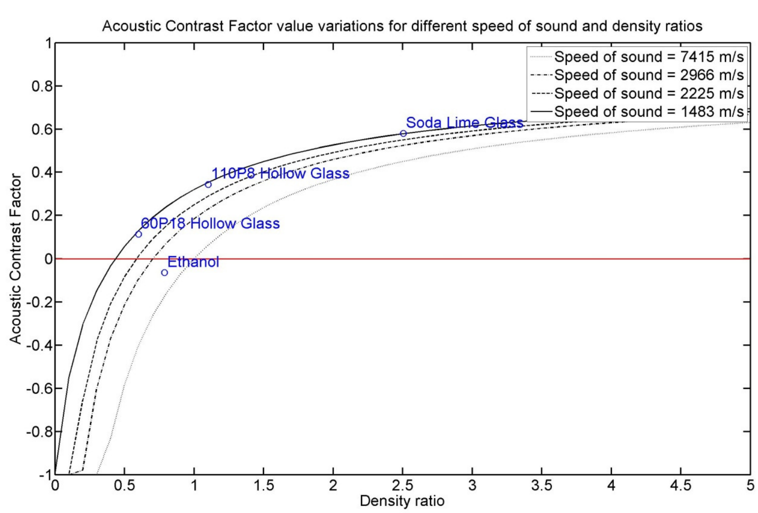
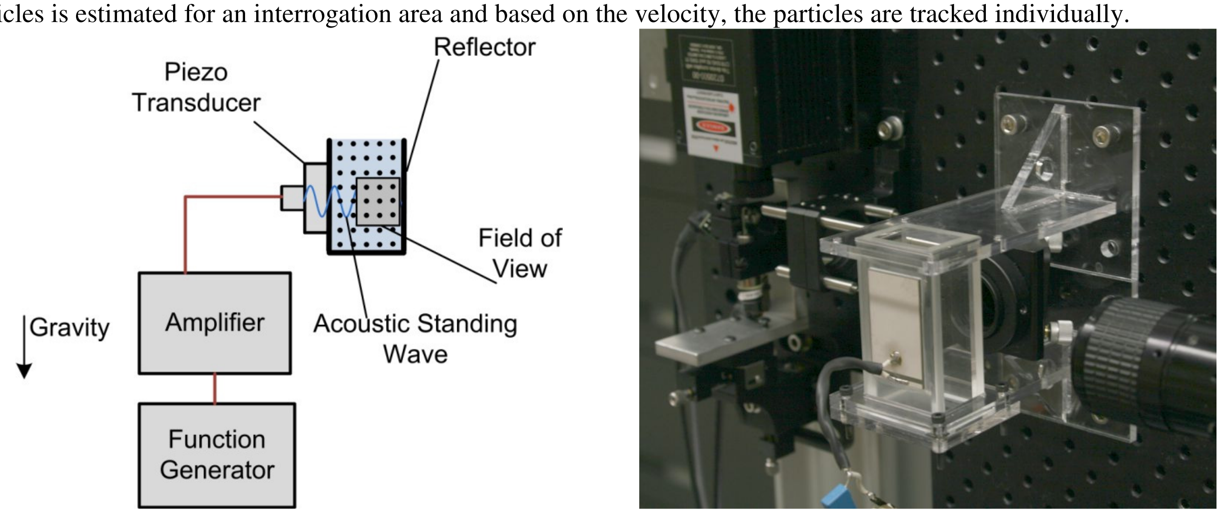


Figure 1 Acoustic Contrast Factor value variations for different speed of sound and density ratios

EXPERIMENTAL SETUP

A schematic of the experimental setup is shown in Figure 2(a) and compared to a digital image in Figure 2(b). The acoustic field is generated in a static fluid (water) contained in a 30×25×65mm glass cell. A 1.5 MHz resonance frequency high power piezo-transducer forms one side of the chamber and is excited with a continuous sine wave. The fluid in the cell is seeded with 11 μm hollow glass spheres with the specific gravity of 0.6. The concentration of particles is measured to be 0.1% by image processing. Images of particles have been collected in a number of configurations. In Figure 2(b) the image shows an imaging configuration with a thin light sheet of a 532 nm laser illuminated the region of interest and a 1024 × 1024 array CCD camera operating at 1000fps coupled with a long working distance microscope is used to image the field. The particle images are processed using a PTV technique to determine velocity of particles. In this technique, the velocity of particles is estimated for an interrogation area and based on the velocity, the particles are tracked individually.



(a) (b)

Figure 2. (a) Schematic of the main components (b) Digital image of the experimental setup

RESULTS AND DISCUSSION

PTV algorithm

Data images of particles forced to the pressure nodes of the acoustic pressure field are captured and processed. In Figure 3, the schematic of the PTV algorithm is shown. The particles are tracked over the whole time period over which densification occurs. Image preprocessing includes de-noising and normalizing the intensity of images which is performed for individual images and is the first step of processing. The particle detection is carried out using commercial software (MATLAB Image Processing Toolbox). Initially using the normalized cut-off threshold of a specific image, the particles are recognized by tracing the exterior of boundaries of objects in the binary image. The recognition algorithm is a Moore-neighbor tracing algorithm modified by Jacob's stopping criteria [6].

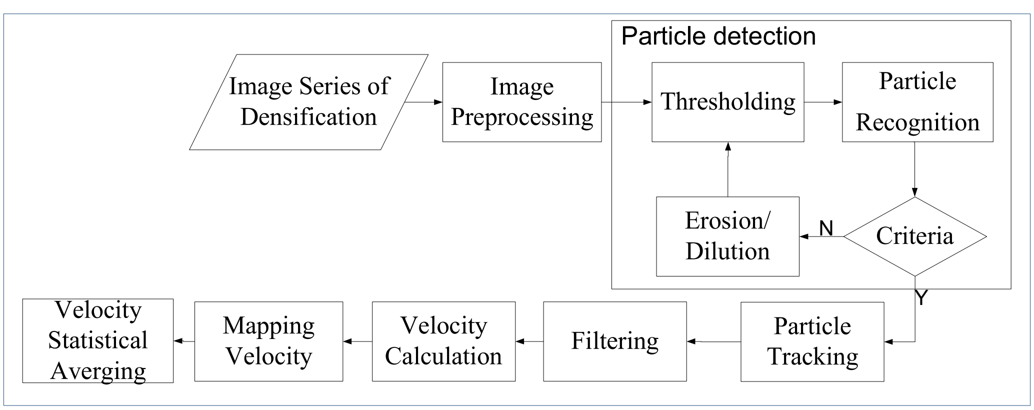
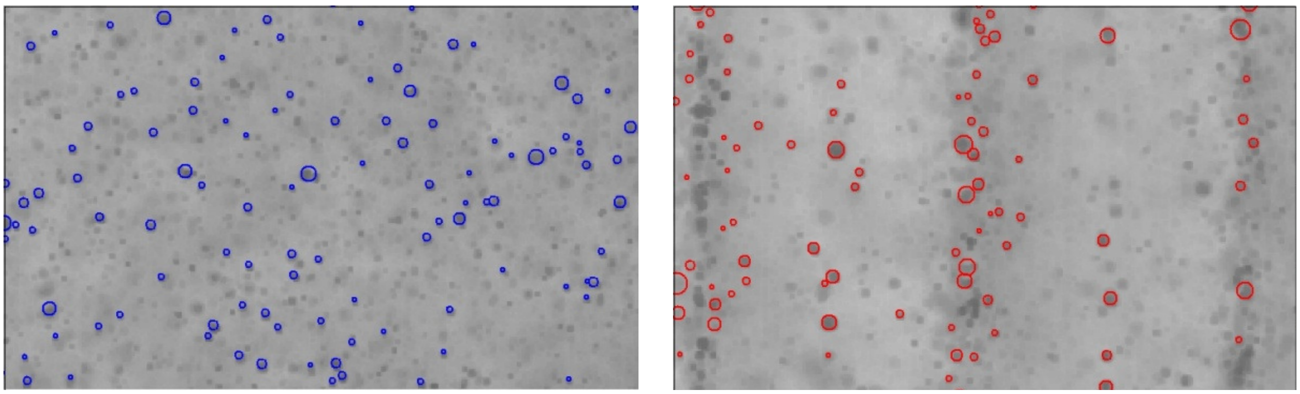


Figure 3. Particle Tracking Velocimetry algorithm

Images post processing

Example raw images shown in Figure 4 of particle fields before and after the acoustic field is applied show the effect of the acoustic field to concentrate particles into vertical bands. In Figure 4(a) a homogeneous particle distribution is shown with in focus particles detected with the PTV algorithm highlighted. In Figure 4(b) the acoustic field, which is oriented vertically, has been applied and the particles have migrated to nodes in the acoustic field. Again, particle detected with the PTV algorithm are highlighted. Both figures highlight a significant problem for determining the velocity and hence the force acting on the particles. For the homogeneous case before the acoustic field is applied, particle detection is relatively easily with the particles being clearly defined. As the particles concentrate under the applied acoustic field, the inter-particle distance reduces and particle images begin to overlap. Individual particle detection therefore becomes more difficult with increasing particle concentration that occurs with time.



(a) (b)

Figure 4. Detection of particles in images in different situations (a)Even distribution of particles before applying the acoustic wave (b)Densification of particles to bands due to the acoustic radiation force

In order to detect the particles based on their intensity, a cut-off coefficient defined compared to the average image intensity was specified. This cut-off coefficient depends on the homogeneity of the particles. The aim here is to choose the cut-off coefficient to have the highest number of the consistently detected particles. To accomplish this, intensity normalized images are studied for different coefficients and the number of detected particles is stored. The normalized standard deviation of detected particles is calculated for each cut-off coefficient value and the result is shown in Figure 5. The aim here is to find the cut-off coefficient at which the minimum standard deviation occurs. This will help in recognizing and tracking the corresponded particles. The minimum value of the standard deviation is located at the cut-off coefficient of 1.2 which is used in later processes.

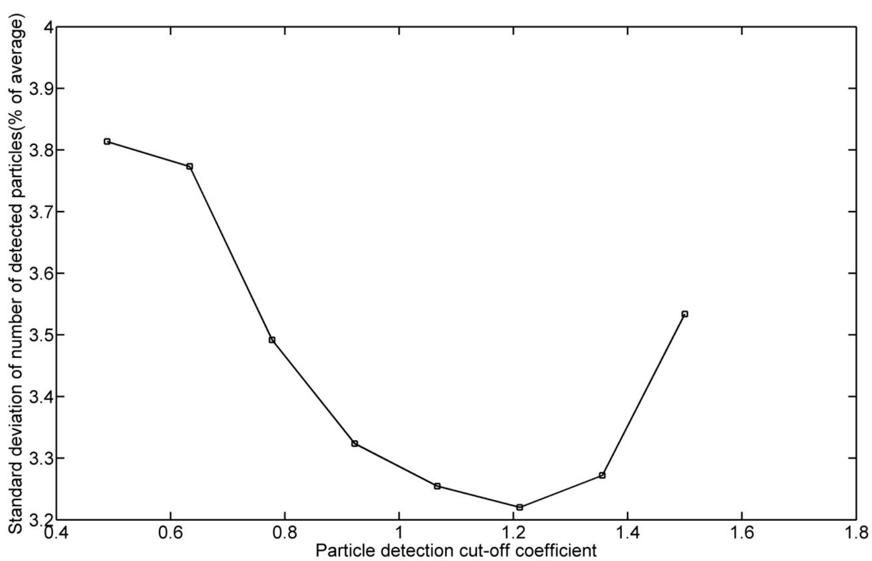


Figure 5. Effect of cut-off coefficient of particle detection on the number of particles found

Image processing can be used to assess the amount of particle concentration that occurs in different regions of the field-of-view. Plots before and after the acoustic field had been applied of particle intensity summed in the vertical direction of the image are shown in Figure 6. This is a semi-quantitative approach that can be used to characterize the level of 'banding' or concentrating of the particle that is occurring. The data plotted in Figure 6 can also be used to locate the position of the nodes and anti-nodes in the acoustic field. The positions of these are highlighted in the figure.

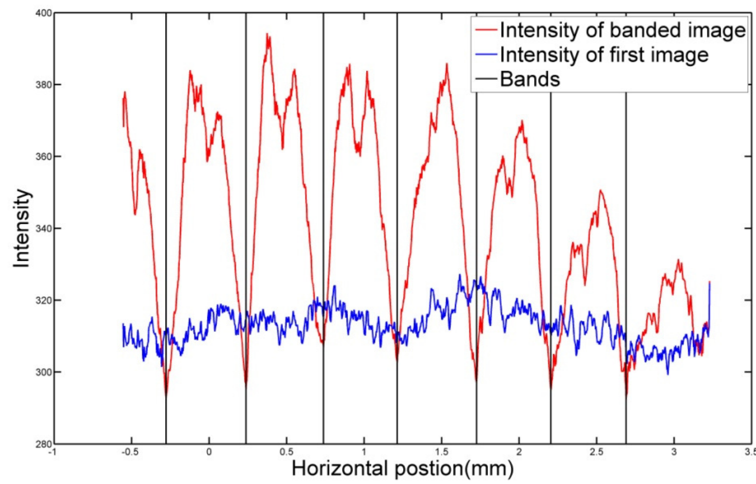


Figure 6. Summed intensity of Figure 3(b) in *x* direction, locations of pressure nodes are found via these intensity plots

Particle tracking

Once particle locations have been determined in the particle recognition section of the PTV algorithm, particle motion can be derived from the time series of images generated by the high speed camera. An example of applying a standard PTV using successive images only is shown in Figure 7. The general motion is as a result of the applied acoustic field and shows particle movement to field nodes. A small vertical component of velocity is present as a result of the effects of gravity. The force on the particle and hence the velocity the particle achieves is a function of position relative to the position of the standing acoustic field and the nodes that it generates.

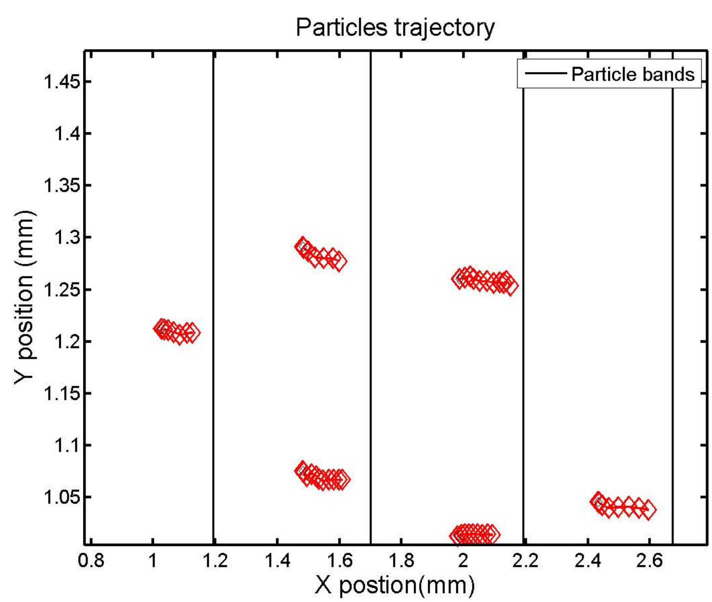


Figure 7. Tracking of particles in densification process

Particle velocity for the entire field can be mapped to the relative positions of nodes in the field. In Figure 8, the experimental measured average horizontal velocity for hollow glass spheres (60P18

Hollow Glass, Potters Industries) in a wavelength is plotted. The particle properties can be found in . Bars have been applied to the experimental data to show the range of measured velocity. This range is mostly due to the distribution of particle size present in the sample. There are other effects such as effect of particle concentration which are going to be studied in further investigations. In the same plot the expected particle velocity determined from single particle theory is also plotted. There are bars also marked as theoretical deviation to show the deviation of the horizontal theoretical velocity due to an imposed particle size distribution. As it is seen, the pattern of the horizontal velocity field matches the one estimated by single particle theory. Also the range of the measured velocity profiles falls within the range calculated based on the standard deviation of the size of particles.

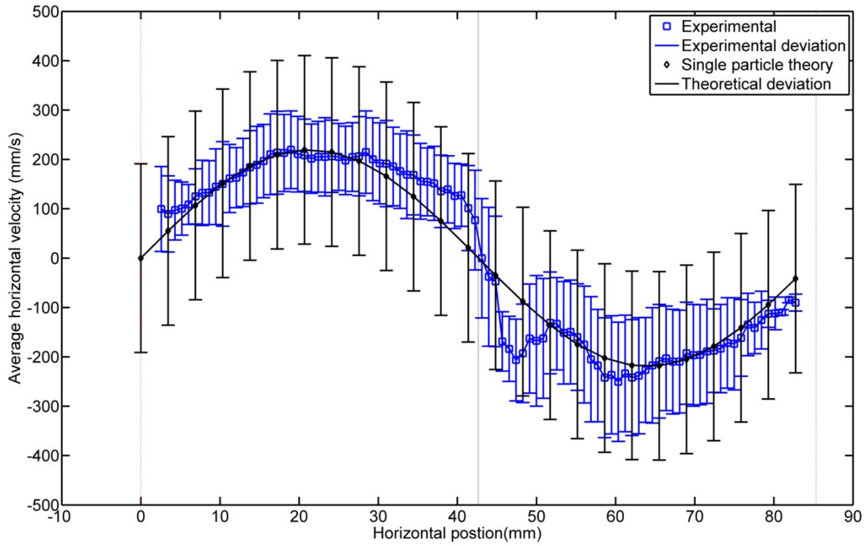


Figure 8. Transverse velocity of particles, experimental vs. single particle theory values

CONCLUSIONS

In this paper, a PTV approach has been used to track the motion of solid particles in water under the influence of an acoustic standing pressure wave. Accounting for the viscous drag force using the Stokes drag equation, buoyancy force and the acoustic radiation pressure, particle motion is simulated and compared with the experimental values obtained by PTV. The results show that the single particle model is capable of predicting the motion of particles at this particle loading. This particle flow field has some unique features, most notable of which is that with time, the homogeneous distribution of particles does not remain and significant clustering of particles occurs. The PTV scheme used tracks particles in these changing conditions and the main parameters are modified to match different conditions. The effect of cut-off coefficient on particle detection is also investigated. The results show that the PTV algorithm is capable of achieving the velocity field at the whole time period of banding of particles.

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