D. S. Nobes, A. Setayeshgar, M. G. Lipsett, and C. R. Koch. Investigating the particles motion in ultrasonic acoustic wave field using PIV/PTV. In *International Congress on Ultrasonics, Poland*, page 4. AIP, Sept 2011

Investigating the Motion of Particles in an Ultrasonic Acoustic Wave Field Using PIV/PTV

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Abstract. The influence of a multi-wavelength acoustic standing wave field on the motion of micron-sized particles is experimentally investigated using particle image velocimetry/particle tracking velocimetry (PIV/PTV) to examine existing theories describing the radiation force on particles. An ultrasonic acoustic wave is introduced into a column chamber containing a mixture of distilled water and a disperse population of spherical particles. In this system the acoustic field is aligned with gravity to form horizontal bands of particles, which are also influenced by buoyancy and drag forces. Accounting for these forces with the acoustic radiation pressure, the motion of an individual particle is modeled. There is a good agreement between the pattern of the particles motion in experimental results and the predicted single particle motion; however, due to the concentration of particles in the experiment, a difference is observed in the maximum value of the velocity of the particles in the experiment and in the single particle model.

Keywords: Ultrasonic Acoustic Pressure, Macro Particle Separation, Particle Image Velocimetry, Particle Tracking Velocimetry

INTRODUCTION

When a free single spherical particle is exposed to an acoustic wave field, primary and secondary hydrodynamic acoustic forces are developed on the particle. The magnitude of the force is reinforced when the wave is in a standing wave mode, rather than in a plane travelling wave mode. This force can be used effectively to concentrate small particles to some designated locations for various applications, such as manipulation of nanoparticles [1] or separation of particles in a microchannel [2].

Analytical research has been performed to study the acoustic force on single particles from different views. King [3] derived an expression for the radiation pressure due to an acoustic standing wave field on an incompressible particle suspended in an inviscid fluid by integrating the acoustic pressure over the surface of the particle. Yosioka and Kawasima [4] obtained the primary radiation force on a compressible particle by defining a velocity potential for displacement velocity of particles. Gorkov [5] took a different approach by defining the potential energy of ultrasonic wave in the fluid.

Simulation of filtration using the ultrasonic radiation pressure has attracted recent attention by a number of researchers. Trippa *et al* [6] modeled the separation of lipid particles from blood in an ultrasonic separator. Lipkens *et al* [7] investigated the effect of different parameters such as fluid flow on particle motions in ultrasonic standing waves. The present work compares experimental results of the motion of many particles due to an acoustic pressure field with single particle models and investigates the effects of concentration of particles.

THEORY AND MODELLING

Force on a Single Particle Due to Acoustic Pressure

Assuming that particles are suspended in an ideal fluid, the particles are compressible, and their diameter are small compared to the wave wavelength of the acoustic field, the force on the particle is defined as the gradient of the acoustic potential energy [5]. For a standing-wave field where the radiation force is much larger than a progressive wave, the acoustic potential energy can be described by:

$$U = V_0\left[\frac{\beta_f-\beta_p}{\beta_f} < PE > -\frac{3(\rho_p-\rho_f)}{\rho_f+2\rho_p} < KE > \right] \qquad (1)$$

where V_0 is the volume of the particle, ρ and β are density and compressibility, and the subscripts f and p refer to fluid and particle properties respectively. $< PE >$ and $< KE >$ are time averaged potential and kinetic energy densities of the field, respectively, and are defined as:

$$< PE > = \frac{\beta_f < P >^2}{2} \tag{2}$$

$$< KE > = \frac{\rho_f < v >^2}{2} \tag{3}$$

where $< P >$ and $< v >$ are time-averaged pressure and displacement velocity of the wave; and for a standing wave these can be expressed as functions of spatial location and time by

$$p(x, y, z, t) = P_0(x, y)\sin(\kappa z)\sin(\omega t) \tag{4}$$

$$v(x, y, z, t) = v_0(x, y)\cos(\kappa z)\cos(\omega t)\boldsymbol{k} \tag{5}$$

where P_0 and $v_0 = \frac{P_0}{\rho_f c}$ are the pressure and velocity amplitude of the wave as a function of spatial location (x, y, z) with y the axial coordinate in the direction of gravity, c is the fluid sound speed, $\kappa = \frac{2\pi}{\lambda}$ is the wave number of the wave, $\omega = 2\pi f$ is the angular frequency, and $\boldsymbol{k}$ is the unit vector in the axial direction. As the force on a particle due to acoustic pressure is the gradient of the potential energy of the wave, $F = -\nabla U$, this force can be expressed as follows:

$$F = \frac{V_0 P_0^2 \kappa \beta_f}{4}\left(\frac{5\rho_p - 2\rho_f}{\rho_f + 2\rho_p} - \frac{\beta_p}{\beta_f}\right)\sin(2\kappa z) \tag{6}$$

Other Forces Affecting Particle Motion

Considering a single particle in a fluid affected by ultrasonic acoustic pressure, the forces acting on the particle can be summarized as ultrasonic radiation force, buoyancy, and the drag force due to viscosity. The buoyancy force due to the density difference between the fluid and the particle is defined as:

$$F_b = V_0(\rho_p - \rho_f)g \tag{7}$$

which acts downward in the direction of the gravity acceleration g. Considering a small particle and low velocity, the drag force due to viscosity of the fluid can be computed using Stokes drag equation:

$$F_D = 3\pi\mu d u \tag{8}$$

in which μ is the viscosity of the fluid, d is the particle diameter and u is the instantaneous velocity of the particle. The velocity of a single particle affected by these forces is simulated using custom software (MATLAB), solving for an ordinary differential equation describing the particle's motion.

EXPERIMENTAL SETUP

A schematic of the experimental setup is shown in Figure 1.The experiment is carried out in cylindrical chamber filled with mixture of distilled water and 37 micron soda-lime glass sphere particles with the specific gravity of 1.3. The concentration of particles is estimated to be 7% by image processing. A 500 kHz resonance frequency, high-power piezo transducer forms one end of the chamber and is excited with a continuous sine wave produced by a function generator. Above the transducer an acoustic phantom [8] is used to avoid the effects of the acoustic streaming. Images of particles in a thin light sheet of a 532 nm laser are captured using a high resolution camera and long working-distance microscope. Images are processed using a hybrid particle image velocimetry/particle tracking velocimetry (PIV\PTV) (DaVis 8, LaVision GmbH) [9] technique to determine velocity of particles. In this technique, the velocity of particles is estimated for an interrogation area; and particles are tracked individually based on the velocity.

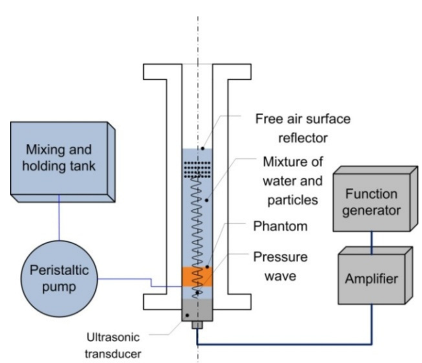
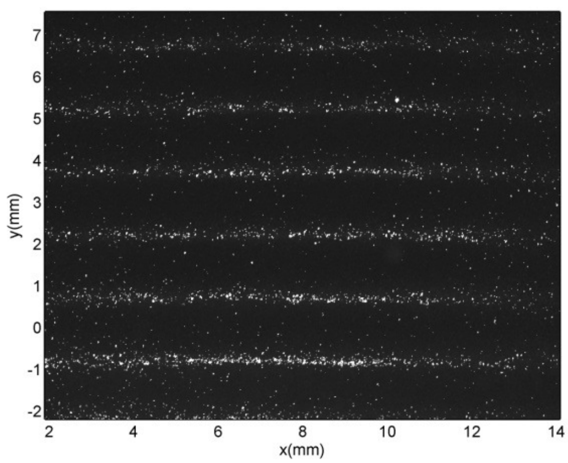


Figure 1. Schematic of the experimental system

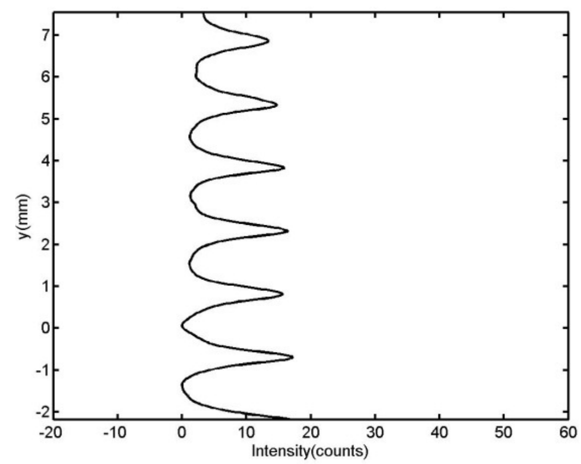
RESULTS AND DISCUSSION

Formation of Bands

Figure 2(a) shows the formation of particles into bands. The gravity force on particles is balanced with the acoustic radiation force at the bands causing local concentration of particles. These bands persist for many seconds, but then the bands begin to collapse with particles streaming downward and then re-establishing a fresh set of bands. This phenomenon may be due to the nature of the acoustic field in a more concentrated region. To quantify the concentration of particles at the bands and the separation efficiency, the intensity of the image in Figure 2(a) is averaged in the *x* direction and plotted versus vertical axis *y*, as shown in Figure 2(b), with the sharp peaks of the intensity corresponding to the centres of the bands. The wavelength of the wave is estimated to be 3.06 mm, which is in good agreement with the wavelength of the excited wave, which is 3.00 mm.



(a)

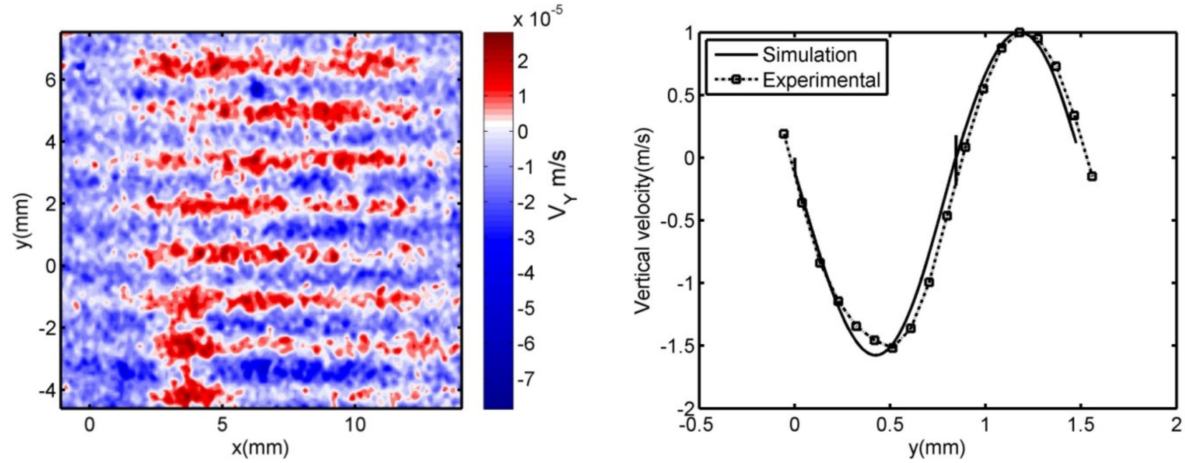


(b)

FIGURE 2. (a)Formation of particles to the bands and (b)Summed intensity of image in (a) in *x* direction

Motion of Particles

The velocity of particles while forming the bands is calculated by utilizing a hybrid PIV/PTV algorithm on the captured images. A contour map of the vertical velocity of the particles forming the bands is shown in Figure 3(a). As it is shown in the colour bar, the magnitude of maximum downward velocity is larger than the maximum of the upward velocity of particles to the bands. This difference is likely due to buoyancy force on the particles. In Figure 3(b), the experimental vertical velocity of the particles is compared with simulation results for particle motion by plotting position on the repeated waveform versus measured velocity. In the simulation the values of parameters match the physical parameters of the experimental setup as described above. As shown in the figure, there are three zero velocity locations in the waveform. The zero-velocity location in the middle of the wave is an unstable zero velocity, in which the particles are forced to the closest bands which are either side of this node. The zero-velocity points at the side of the waveform are stable and particles migrate to these points. In this figure, the result of modeling of the motion of a single particle is also plotted. Experimental and simulation velocity plots are scaled to their maximum value for comparative purposes. The single particle motion simulation predicts particle motion pattern well, except that the maximum velocity for the single particle model is 2.6×10^{-4} m/s, while the maximum velocity of the particles measured in experiments is 1.2×10^{-5} m/s. The main source of this discrepancy is likely the concentration of particles in the experiment. The high concentration does not allow the particles to move as freely as they can individually. This can be compared with the similar effects of hindered settling on the velocity of particles. The experimental settling velocity is computed to be 7.2×10^{-5} m/s which is less than the settling velocity value (6.4×10^{-4} m/s) predicted by the Stokes drag theory, again due the concentration of particles in the experiment. There is a backflow of fluid due to sedimentation of other particles to the bottom of the container which reduces the mean velocity of the sedimentation process relative to the Stokes results for a single sphere [10].



(a) (b)

FIGURE 3.(a)Vertical velocity contour of particles while formation of bands, (b)Experimental and simulation velocity comparison of banding of particles

CONCLUSIONS

In this paper, motion of particles under the influence of an acoustic standing pressure wave has been investigated using PIV/PTV. Accounting for the viscous drag force using the Stokes drag equation, buoyancy force, and the acoustic radiation pressure, particle motion is simulated and compared with the experimental values. The results show that the model is capable of predicting the pattern of the motion of the particle in a wavelength; however, there is a difference in the maximum value of the velocity of the particles in the experimental and simulation model which is likely due to the concentration of the particles in the experiment.

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