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THEORETICAL AND EXPERIMENTAL STUDY OF HYDROCYCLONE PERFORMANCE AND EQUIVALENT SETTLING AREA

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ABSTRACT

Predicting the performance of a solid-liquid separation process can help in comparing different separators for selection and design. This can be applied to hydrocyclone technology which is used widely in industry due to being an inexpensive device that is easy to operate and maintain and which has no moving parts. Environmental concerns and technological issues in separation processes are motivating the design of higher efficiency systems with less capital and operating costs. There is a need therefore for, methods to compare different separation technologies.

In spite of extensive research into hydrocyclone performance, a mathematical model that can predict the performance of a hydrocyclone for comparison with other centrifugal separators is rare in the literature. The main objective of this research is to apply theoretical and experimental approaches to study hydrocyclone performance in order to propose an applicable separation performance model that represents the whole hydrocyclone operating range. A mathematical model is developed to explore the performance of the separator and to predict the hydrocyclone's equivalent area as compared to a continuous gravity settling tank. A performance chart that can be used for selection and design of hydrocyclones is the result of the model.

INTRODUCTION

Centrifugal sedimenting separators can be categorized into moving wall and fixed wall types. Hydrocyclones are centrifugal separators with a fixed wall and no moving parts which have found several applications in industry due to their simple operation and large centrifugal acceleration. Comparing other types of centrifuge separators such as basket [1], scroll decanter [2] and disc-stack machine [3], hydrocyclones can provide a high centrifugal acceleration [1]. They also require less space, lower capital investment, and have lower maintenance and operating costs. However, quantifying the separation performance is still needed to allow comparison of all centrifugal devices, including hydrocyclones.

Centrifuge separators are usually designed through a scaling procedure using scaling factors [4]. The equivalent area factor is a scaling parameter proposed by applying assumptions that include Stokes' law for particle motion, plug flow at the inlet and an evenly distributed particle sizes in the centrifuge device [5]. This factor is usually augmented by empirical correlations to overcome simplifying assumptions. Equivalent area factor, which is also known as the theoretical capacity factor [6], is the area of a gravity settling tank that has the same separation performance as the centrifuge separator at the same flow rate. It is an indication of separation device performance compared to a continuous gravity settling tank and can be used for comparing different sedimenting based separators.

Equivalent area factor is introduced [7] by considering a cut size particle sedimenting in a centrifuge separator. This particle is located in a position that has the same probability of being separated or not. This factor is calculated for bottle centrifuges, disc stacks and decanters [5] as a function of centrifuge design parameters (dimensions and speed of rotation) and is proposed to be used for scaling up and comparison between centrifuges.

Developing charts and guidelines for selecting and designing of centrifuge separators is studied in the literature [8]–[11]. Performance and energy charts for comparing hydrocyclones and disc centrifuges over a limited range is shown in [6]. This is followed by Lavanchy et al. [12] who proposed a performance chart for comparing different types of centrifugal separators along with hydrocyclones. Although the equivalent area concept is employed, information on the development of the equivalent area factor for hydrocyclones is not given. This understanding is needed to allow modeling of the equivalent area factor (capacity factor) for hydrocyclones that can be used for developing a performance chart.

Rietema [13] used a similar framework but focused on predicting the cut size of particles in hydrocyclones rather than developing an equivalent area factor. Then the cut size particle that is separated in the residence time in a hydrocyclone should be precisely fed into the center of the inlet pipe of a hydrocyclone. The obtained relation is augmented using the results from experiment to improve the assumptions and then a design method is proposed to design hydrocyclones which is called Rietema's optimum design [13].

Despite the large amount of research on hydrocyclone performance, a mathematical model that can be employed for predicting the equivalent area factor of a hydrocyclone is still lacking. The current work develops a model to predict the equivalent area factor for hydrocyclones for the case of a dilute feed where particle-particle interactions are negligible. This model relates the equivalent area to typical operating parameters such as pressure drop or flow rate and design parameters of a hydrocyclone. This model is used to predict the performance region of the device that is comparable with other centrifuge separators. The model is also compared to two well-known hydrocyclone designs suggested by Rietema [13] and Bradley [14].

MODELING

For a uniform distribution of particles in operation, Ambler [5] correlated volume flow rate and gravity settling velocity for centrifuge separators. A relation is derived for equivalent area factor Σ (with SI units of m^2) - also termed the theoretical capacity factor- such that:

$$Q = 2v_g\Sigma \tag{1}$$

where Q is flow rate and v_g is settling velocity under gravitational acceleration (not under centrifugal acceleration) for 50% cut size particle, where 50% of particles (by mass) which are larger (smaller) than this size pass through each of the outlets of the separator. The settling velocity under gravitational acceleration, v_g is calculated using Stokes' law [15] as:

$$v_g = \frac{\Delta\rho d^2}{18\mu} g \tag{2}$$

where d is the particle 50% cut size diameter, $\Delta\rho$ is density difference between phases, μ is dynamic viscosity of the fluid and g is gravitational acceleration.

To develop a model for an equivalent area factor in hydrocyclones the same method is used here as for other centrifuge separators. This approach applies residence time theory first developed by Rietema [13], to hydrocyclone theory. This model is mainly used for predicting the cut size rather than equivalent area factor. In residence time theory a particle at a 50% cut size can either go to the overflow or underflow of a hydrocyclone if it enters the hydrocyclone from the center of inlet pipe. Radial velocity v_r of a particle that moves in a hydrocyclone due to centrifugal acceleration is related to the tangential velocity component by [16]:

$$v_r = \frac{\Delta\rho d^2}{18\mu} \frac{v_\theta^2}{r} \tag{3}$$

where v_θ is tangential component of the particle velocity and r is the radius of the rotation orbit of the particle.

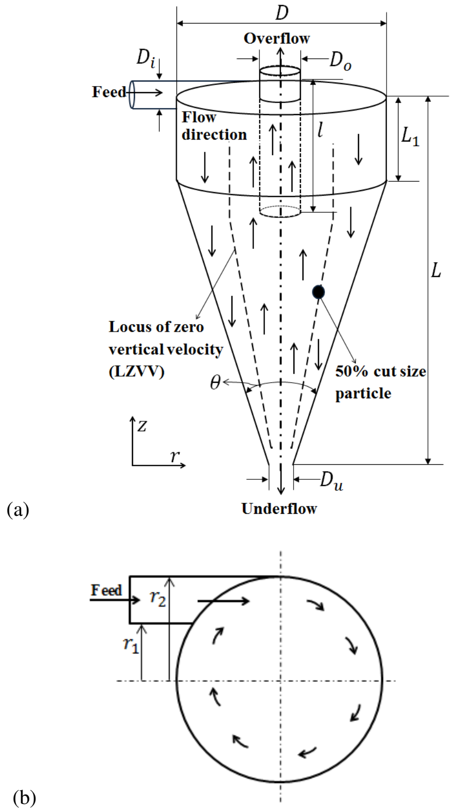

Figure 1. (a) A schematic of a hydrocyclone showing basic components and operation principles, (b) cross section of cylindrical portion and inlet pipe with its walls distances from hydrocyclone centerline

Figure 1 shows the components of a hydrocyclone and a cross section of inlet pipe and the upper cylindrical portion (L_1) of a hydrocyclone. Radial velocity of the particle is related to its vertical velocity component v_z using the chain rule. Assuming that flow near the wall follows the hydrocyclone cone shape [16], leads to:

$$v_r = \frac{dr}{dt} = \frac{dr}{dz}\frac{dz}{dt} = \frac{D}{2L} v_z \tag{4}$$

where dr and dz are line elements in the radial and vertical directions, dt is the time element, D & L are hydrocyclone diameter and total height respectively- as shown in Figure 1(a). Replacing the particle radial velocity from Eq. (3) into Eq. (4) leads to:

$$\frac{\Delta\rho d^2}{18\mu}\frac{v_\theta^2}{r}=\frac{D}{2L}v_z \tag{5}$$

The vertical velocity component can be estimated from the inlet flow rate in the upper hydrocyclone cylindrical section region as [17]:

$$v_z=\frac{4Q}{\pi D^2} \tag{6}$$

With this in Eq. (5) using Eq. (2) and rearranging, the flow rate is obtained as:

$$Q=2v_g\frac{\pi DL}{4g}\frac{v_\theta^2}{r} \tag{7}$$

Comparing the terms in Eq. (1) and Eq. (7) and noticing that the flow rate in an equivalent area is associated with overflow flow rate (the portion of feed flow that leaves the hydrocyclone through overflow pipe as shown in Figure 1(a)) , Σ can be defined as:

$$\Sigma=\frac{\pi DL}{4g}\frac{v_\theta^2}{r}(1-R_f) \tag{8}$$

where R_f [16] is the "flow ratio" in the hydrocyclone and equals the ratio of underflow (the fraction of feed flow that leaves the hydrocyclone through underflow pipe as shown in Figure 1(a)) to inlet flow rate.

The tangential velocity v_θ is obtained based on an operating parameter such as the inlet flow rate. This velocity component is related to the radius r (distance between hydrocyclone centerline and particle) such that [14]:

$$v_\theta=\frac{C}{r^n} \tag{9}$$

where n is an empirical exponent normally between 0.4 and 0.9 [14]. C is a constant that can be obtained from a mass balance using:

$$Q=\int v_\theta dA=H\int_{r_1}^{r_2}\frac{C}{r^n}dr \tag{10}$$

where H is the inlet pipe depth (assuming a rectangular shape for inlet pipe), r_1 and r_2 are distances from the center of the hydrocyclone to the inlet pipe walls (Figure 1(b)) and perpendicular to H such that $r_2=D/2$ and $r_1=r_2-w$ where w is inlet pipe width. Solving the integral results in the constant C:

$$C=\frac{Q(1-n)}{H(r_2^{1-n}-r_1^{1-n})} \tag{11}$$

Then v_θ is obtained by substituting Eq. (11) in Eq. (9).

$$v_\theta=\frac{Q(1-n)}{H(r_2^{1-n}-r_1^{1-n})r^n} \tag{12}$$

This can be replaced in Eq. (8) and since the cut size particle starts its journey in the hydrocyclone from the center of the inlet pipe, $r=(D-r_2+r_1)/2$ and the equivalent area factor is calculated as a function of flow rate as:

$$\Sigma=\frac{\pi DLQ^2(1-n)^2(1-R_f)}{4gH^2(r_2^{1-n}-r_1^{1-n})^2\left(\frac{D-r_2+r_1}{2}\right)^{2n+1}} \tag{13}$$

The main difference between a hydrocyclone and a fixed wall centrifuge separators is that in a hydrocyclone the centrifugal acceleration is a function of flow rate and not just the dimensions and design parameters. This is apparent in Eq. (13).

It is also important to relate the equivalent area factor to the hydrocyclone pressure drop. The pressure drop ΔP in a hydrocyclone is obtained from the tangential velocity component and by integrating in the radial direction between the overflow radius $(D_o/2)$ and hydrocyclone radius $(D/2)$ such that:

$$\Delta P=\int_{D_o/2}^{D/2}\frac{\rho v_\theta^2}{r}dr=\int_{D_o/2}^{D/2}\frac{\rho C^2}{r^{2n+1}}dr \tag{14}$$

where ρ is the fluid density. This leads to:

$$\Delta P=\frac{2^{2n}\rho C^2}{2nD^{2n}}\left[\left(\frac{D}{D_o}\right)^{2n}-1\right] \tag{15}$$

Combining Eq. (15) and Eq. (11) gives the relation between the pressure drop and flow rate:

$$\Delta P=\frac{2^{2n-1}\rho}{n}\left(\frac{Q(1-n)}{D^nH(r_2^{1-n}-r_1^{1-n})}\right)^2\left[\left(\frac{D}{D_o}\right)^{2n}-1\right] \tag{16}$$

Taking the flow rate as a function of pressure drop from Eq. (16) with Eq. (13) results in a relation for the equivalent area factor in terms of pressure drop. Therefore:

$$\Sigma=\frac{\pi LnD^{2n+1}(1-R_f)\Delta P}{\rho g(D-r_2+r_1)^{2n+1}\left[\left(\frac{D}{D_o}\right)^{2n}-1\right]} \tag{17}$$

and using $r_2-r_1=w$ this simplifies to:

$$\Sigma=\frac{\pi LnD^{2n+1}(1-R_f)\Delta P}{\rho g(D-w)^{2n+1}\left[\left(\frac{D}{D_o}\right)^{2n}-1\right]} \tag{18}$$

Considering that H does not appear in Eq. (18), a more general relationship is possible. For circular inlet pipes, by assuming $w=D_i$ where D_i is the inlet pipe diameter, the equivalent area is:

$$\Sigma=\beta\frac{L(1-R_f)\Delta P}{\rho g} \tag{19}$$

for a circular inlet hydrocyclone, where

$$\beta = \frac{\pi n}{(D/D_o)^{2n} - 1}\left(\frac{1}{1 - D_i/D}\right)^{2n+1} \qquad (20)$$

This indicates that for geometrically similar hydrocyclones β depends on the value of n, the ratio of inlet (D_i) and overflow (D_o) diameters to hydrocyclone diameter (D). Hydrocyclone diameter might affect the value of β indirectly through affecting the value of n. In Eq. (19) the interaction of particles is assumed negligible when predicting the performance of hydrocyclones. This requires a low concentration of solid particles in the feed flow. In addition, Stokes' law for calculating radial velocity of particles is also assumed.

EXPERIMENT

Experimental apparatus:

For evaluating the developed model of equivalent area factor an experimental setup has been designed and built. Figure 2 shows the process flow diagram of the setup. Flow is pumped into the hydrocyclone using a centrifugal pump that provides different inlet flow rates and hence pressures. Pressures are recorded at three locations; before the inlet and after both outlets. Pressure drop is obtained from the pressure difference between inlet and underflow as the overflow pressure is close to atmosphere and remains constant at all conditions.

A second pump is attached to the underflow pipe and allows independent control of the underflow flow rate and hence provides different pressures to the underflow without encountering clogging in the underflow pipe. This is advantageous compared to using a valve for manipulating the underflow flow rate as a valve in the underflow pipe can easily block [16]. Each pump is controlled using a separate VFD (variable frequency drive).

Flow at the inlet and underflow is measured by coriolis flow meters (Promass83I Endress+Hauser Ltd; Optimass7300 KROHNE Messtechnik GmbH). Control of the experimental hardware such as pumps and mixer and for communicating to the sensors in different measuring devices custom software (LabWindows™/CVI, National Instruments Inc.) has been developed.

Hydrocyclone:

A commercial hydrocyclone (U2-GMAX FLSmidth Krebs Hydrocyclones) is used in the experiment as the device under test. The main design parameters of the hydrocyclone are shown in Table 1 assuming a similar geometry as in Figure 1. In the experiments, with only water in the system, inlet flow rate and the flow ratio (R_f) vary from ~1 to ~3 m^3/hr and from ~0.1 to ~0.7 respectively.

Table 1: U2-GMAX Krebs hydrocyclone dimensions in mm

Parameter	D	D_i	D_o	L_1	L	l
Value (mm)	50	22	12	62	890	95

RESULTS AND DISCUSSION

Evaluation of β factor:

The equivalent area in Eq. (19) is a function of design parameters that include hydrocyclone length and diameters that are appear in β factor in Eq. (20). To evaluate this factor, its value is calculated for two well-known hydrocyclone design configurations: a Rietema and Bradley as well as the U2-GMAX Krebs and for n ranging from 0.4 to 0.9. Table 2 shows the geometric proportions and their value for these three hydrocyclones. The results of calculating β are shown in Figure 3.

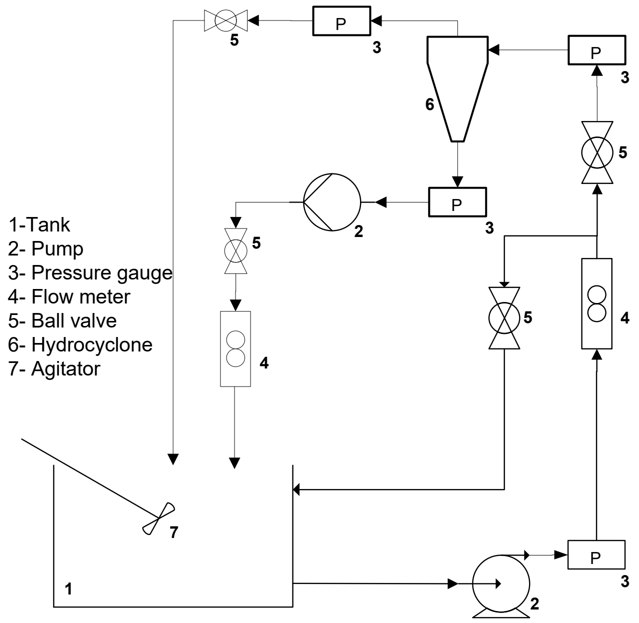

Figure 2: Process flow diagram for the test setup

Table 2: Geometric properties for hydrocyclone design for Bradley and Rietema hydrocyclone designs [18] and U2-GMAX Krebs hydrocyclone as shown in Figure 1

Geometric proportion	Bradley	Rietema	Krebs-U2-GMAX
D_i/D	1/7	0.28	0.44
D_o/D	1/5	0.34	0.24
L_1/D	1/2	0.28	1.24
L/D	-	5	17.8
l/D	1/3	-	1.9
θ	9°	20°	-

As it can be seen from Figure 3 the Rietema and Krebs hydrocyclone provide higher values for β than the Bradley hydrocyclone. The values for U2-GMAX Krebs and Rietema hydrocyclone are almost the same despite different aspect ratios. The ratio of $\beta_{Rietema}/\beta_{Bradley}$ changes from ~2.5 to ~5 for possible values of n, as shown in Figure 4. Hence, it can be predicted that a Rietema (or Krebs) hydrocyclone will provide a higher equivalent area than the Bradley hydrocyclone. This will increase further as $\beta_{Rietema}/\beta_{Bradley}$ ratio increases with increasing values of n.

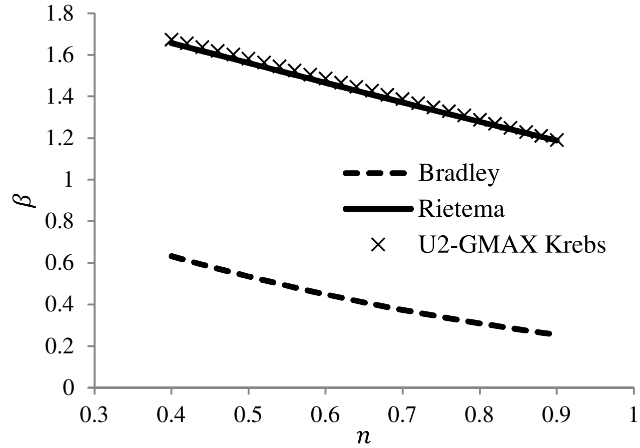

Figure 3. Value of β for different n for Rietema, Bradley and U2-GMAX Krebs design hydrocyclones

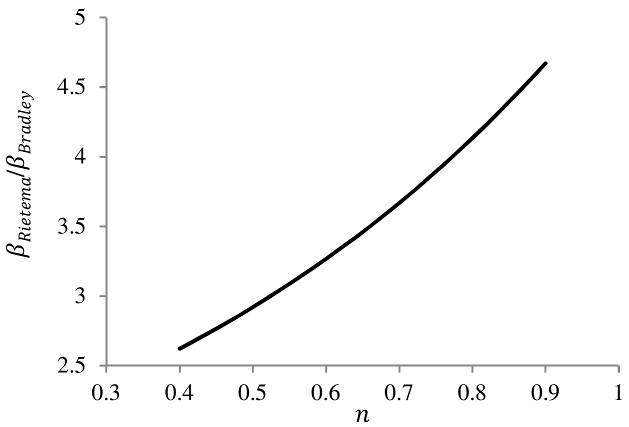

Figure 4. Ratio of β factor for Rietema hydrocyclone and Bradley hydrocyclone

Prediction of equivalent area factor:

According to Eq. (19) the equivalent area factor for hydrocyclones can be evaluated for an applied pressure drop across a hydrocyclone. Pressure drop in hydrocyclones is usually in the range ~30 kPa to ~600 kPa [16] and normally lower pressure drops are associated with hydrocyclones with larger diameters. To find the equivalent area factor, a minimum and a maximum pressure drop over this range is applied and a range of hydrocyclone diameters from 0.01 m to 1 m is used. The values for the minimum and maximum pressure drop applied for different hydrocyclone diameters are shown in Figure 5.

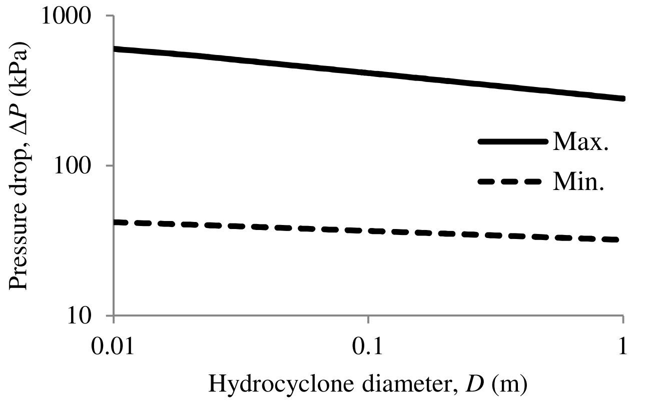

Figure 5. Minimum and maximum pressure drop across hydrocyclones with different diameter for evaluating equivalent area factor

For these values of minimum and maximum pressure drops and assuming $R_f \ll 1$, Σ is calculated from Eq. (19) assuming water as the transport liquid with the density of 1000 kg/m^3. The results are shown in Figure 6 and Figure 7 for the Rietema and Bradley hydrocyclones respectively. It is apparent from these two figures the Rietema hydrocyclone has a higher equivalent area for a given hydrocyclone at a similar pressure drop and this is in agreement with the earlier prediction in the previous section. This means that a Rietema hydrocyclone can support higher flow rates than a Bradley hydrocyclone for the same pressure drop and the same cut size. In other words, if both a Rietema design and Bradley design hydrocyclones are operated under similar conditions (i.e. flow rate and pressure drop), based on the values from the model a Rietema design hydrocyclone will have higher separation efficiency.

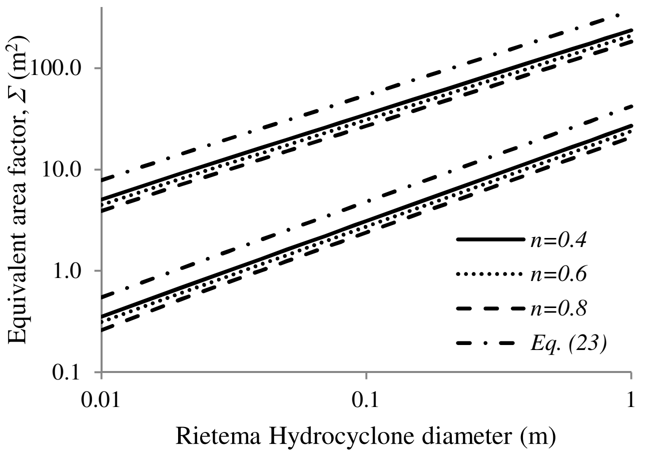

Figure 6. Values of minimum and maximum equivalent area factor for different hydrocyclone diameters in Rietema design and for Rietema's optimum design as in Eq. (19)

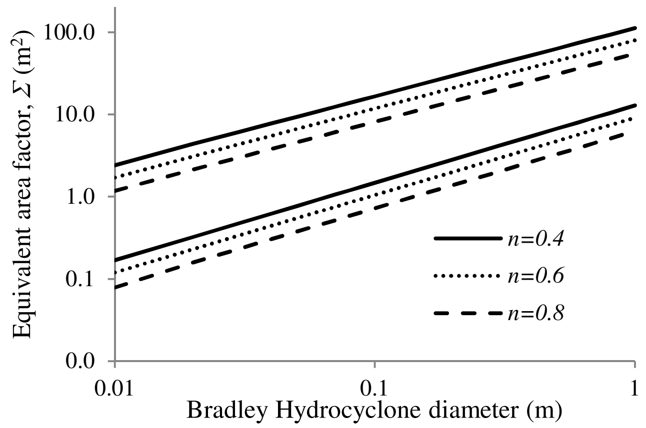

Figure 7. Values of minimum and maximum equivalent area factor for different hydrocyclone diameters in Bradley design

It can be seen from Figure 6 and Figure 7 that the value of the minimum equivalent area factor falls below 1 m^2 for some hydrocyclones for both Rietema and Bradley hydrocyclones. This indicates that a lower performance than a continuous gravity settling tank with the settling area of 1 m^2 is possible. This highlights that improper design of a hydrocyclone separation system may lead to lower efficiency than a gravity settling tank. This should be avoided in practice and can be avoided by increasing the pressure at the hydrocyclone inlet or by changing the hydrocyclone diameter.

Figure 6 and Figure 7 also show the dependency of equivalent area factor to the parameter n (the power coefficient in tangential velocity component). While equivalent area factor in a Rietema hydrocyclone does not change significantly with changes in n, this factor is more affected by the value of n for a Bradley hydrocyclone. This means that predicting the precise value of equivalent area for Bradley hydrocyclone requires detailed information about the tangential velocity profile. This can be reasonably ignored however for a Rietema hydrocyclone design for the values tested here and according to Figure 6.

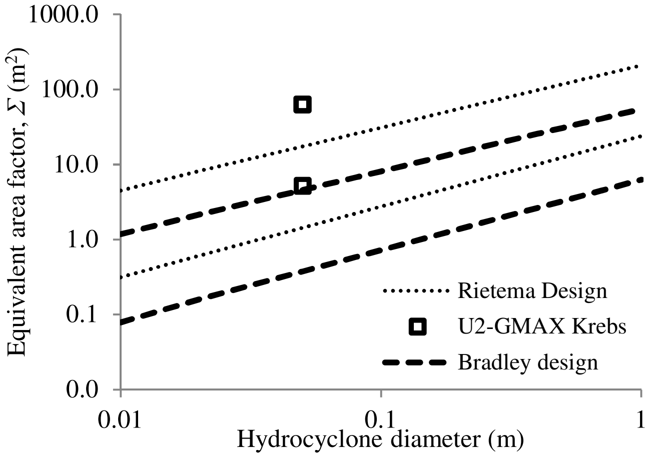

Figure 8: Comparison of equivalent area factor for Rietema ($n = 0.6$) and Bradley ($n = 0.8$) design and a 5 cm Krebs hydrocyclone

Figure 8 compares the minimum and maximum equivalent area for the Rietema, Bradley and U2-GMAX design hydrocyclone. While Rietema and the given Krebs hydrocyclone have the same value for β (Figure 3), a 5 cm Krebs hydrocyclone gives higher equivalent area than a 5 cm Rietema designed hydrocyclone. This is mainly due to the length of the hydrocyclone that appears in the model (Eq. (19)).

The developed model is also evaluated using data from manufacturer data sheet. For 1650 kg/m^3 density difference between solid and liquid phases and assuming liquid viscosity of water as 0.001 Pa.s, 50% cut size and flow rate are obtained from manufacturer data sheet [19] for five reported pressure drops. With this information, values of equivalent area factor are calculated using Eq. (1) and Eq. (2). These values are compared with the equivalent area factor obtained from the model assuming flow ratio as 0.05. Figure 9 shows the comparison of the results from the model and the manufacture data sheet. Good agreement is observed between the developed model and the data indicating the model can predict the hydrocyclone performance.

The discrepancy between the model and the data points is mainly attributed to the fact that the equivalent area factor does not include inherent inefficiencies that appear in all type of equipment such as turbulence, geometry etc. [6]. It could also be partially due to the simplifications and assumptions used in deriving the equivalent area factor. For instance, while this hydrocyclone geometry is similar to the geometry used in the model, the cone shape does slightly deviate from a straight line.

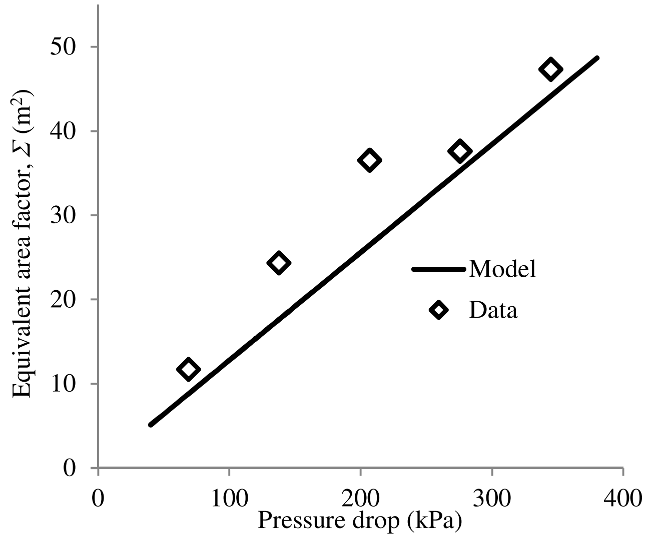

Figure 9: Comparison of developed model for equivalent area factor ($R_f = 0.05$) for a 5 cm Krebs hydrocyclone with the data resulted from manufacturer data sheet [19]

Adjustable coefficient to match experiment:

Rietema's [13] original work proposed a residence time theory for hydrocyclones and used it to derive a relation for predicting the particle cut size. Although no attempt to calculate an equivalent area factor was made, it is possible to obtain a relation for equivalent area factor from the experimentally adjusted relation proposed by Rietema for hydrocyclones. Rietema suggests the following equation for cut size in a hydrocyclone (following our notation):

$$d^2\Delta\rho L\Delta P = 3.5\mu\rho Q \qquad (21)$$

which contains an experimentally determined parameter of 3.5. This relation is thought to be related to a hydrocyclone with aspect ratios known as Rietema's optimum design [13]. Rearranging Eq. (21) to be analogous to Eq. (1) and using Eq. (2) gives a Σ relationship of a hydrocyclone as:

$$Q = 2v_g\left(\frac{18L\Delta P}{7\rho g}\right) \qquad (22)$$

and

$$\Sigma = 2.57\frac{L(1-R_f)\Delta P}{\rho g} \qquad (23)$$

The equivalent area factor resulted from Rietema research in Eq. (23) is comparable to Eq. (19) by comparing the value of β as shown in Figure 3 with the constant coefficient 2.57 in Eq. (23). A discrepancies exist for values of β from Eq. (19) and the constant of 2.57 for all applicable values of n. An adjusting coefficient can be defined to compensate the discrepancy between the experimentally supported relation of Rietema and relation from the current modeling. As a result the equivalent area factor in general is corrected such that:

$$\Sigma = A_c\beta\frac{L(1-R_f)\Delta P}{\rho g} \qquad (24)$$

where A_c is determined for every hydrocyclone design experimentally. Calculating the values of A_c for Rietema's optimum design and averaging leads to $A_c = 1.82$ for $n = 0.66$. The equivalent area that directly results from Eq. (23) is shown in Figure 6 for the same minimum and maximum pressure drop as in Figure 4 and for $R_f \ll 1$. This is otherwise associated to the equivalent area from Eq. (24) considering $n = 0.66$ and $A_c = 1.82$. As it can be seen from Fig. 5, Eq. (23) predicts a higher equivalent area than (19) and this is mainly attributed to the simplifying assumptions in developing the model which is adjusted by applying the above mentioned adjusting coefficient.

For $n = 0.66$, Eq. (16) gives the following relation for pressure drop for Rietema's optimum design:

$$\Delta P = 0.7\rho\left(\frac{Q}{D^{0.66}H(r_2^{0.34}-r_1^{0.34})}\right)^2 \qquad (25)$$

This relation can be applied for a Rietema hydrocyclone as Rietema did not proposed a pressure drop equation to support his model for predicting the cut size.

Experimental results:

For the U2-GMAX Krebs hydrocyclone, experiments are performed for several underflow conditions. The experimental results are used to evaluate equivalent area factor for the hydrocyclone. This includes studying the flow rates, flow ratio and pressure drops. To keep the understanding simple for the new design method no solid particles are used –only water is used as a liquid.

Figure 10 shows the flow ratio changes as a function of pressure drop at three different feed pump speeds set at 40, 50 and 60 Hz on the VFD. The feed pump speeds are used in reporting the results instead of inlet flow rates as the inlet flow rate changes slightly at each pump speed due to variations in underflow pressure. Increasing the pressure drop in the hydrocyclone leads to an increase in underflow flow rate. The flow ratio increases at each constant feed flow rate (associated with a constant feed pump speed). This is expected due to higher flow in the underflow pipe. More pressure drop is related to higher underflow pump speed and hence higher underflow flow rate where the inlet flow rate is constant.

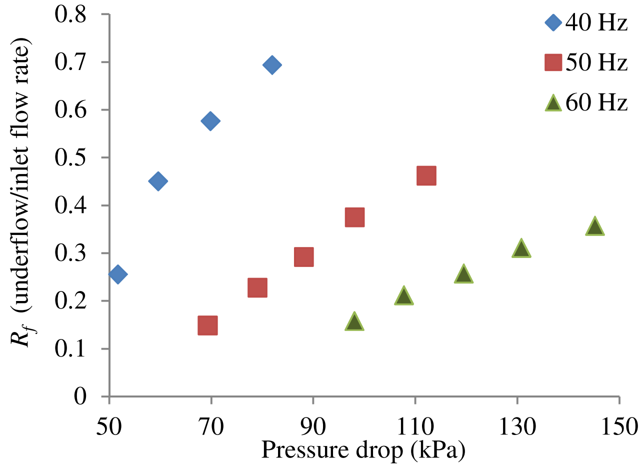

Figure 10: Effect of pressure drop on flow ratio at three feed pump speeds

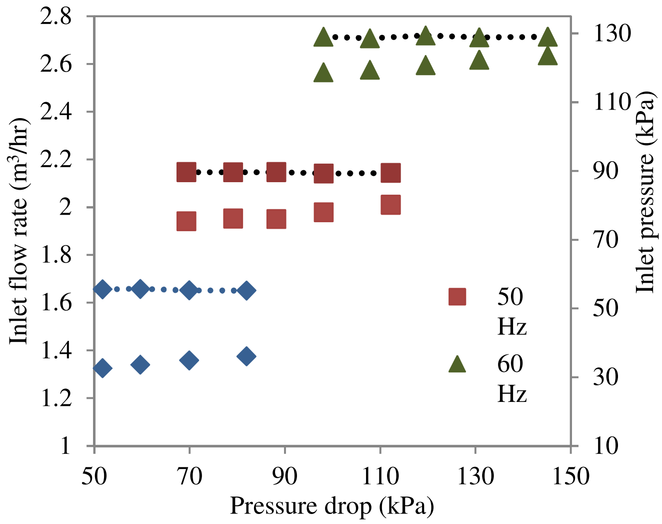

Figure 11: Effect of pressure drop on inlet flow rate and inlet pressure at three feed pump speeds. Dotted line and markers are representing pressure drop; markers alone are representing flow rate

The effect of system pressure drop on the inlet flow rate and inlet pressure is shown in Figure 11. There the inlet flow rate increases slightly with increasing pressure drop while the inlet pressure remains constant. At every feed pump speed this variation is solely related to changes in the underflow operating conditions (pressure/flow rate) as this is the only parameter that is changing.

According to [16], the trends observed in Figure 10 and Figure 11 are similar to the effects of changing the underflow diameter. Figure 12 indicates the trends of inlet and underflow flow rate as a function of the ratio of underflow to overflow pipes diameter [16]. Comparing these trends with Figure 10 and Figure 11 it is clear that using the pump in the underflow is effective in simulating changes in the underflow diameter.

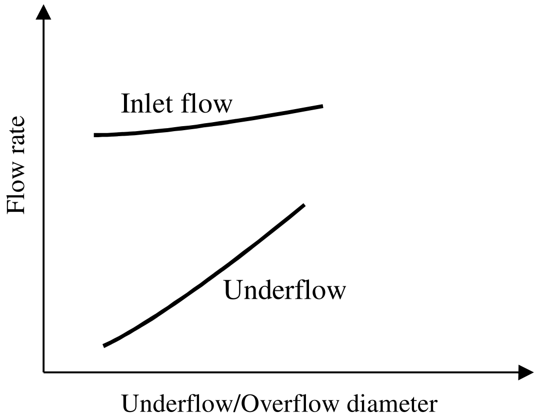

Figure 12: General terends in the underflow and inlet flow rate with underflow pipe diameter in hydrocylones [16]

The underflow pressure changes versus total pressure drop in the hydrocyclone is shown in Figure 13. This figure shows the capability of having both back pressure and suction in the underflow pipe by connecting the pump in the underflow. Decreasing the underflow pressure (more suction) leads to more pressure drop as expected.

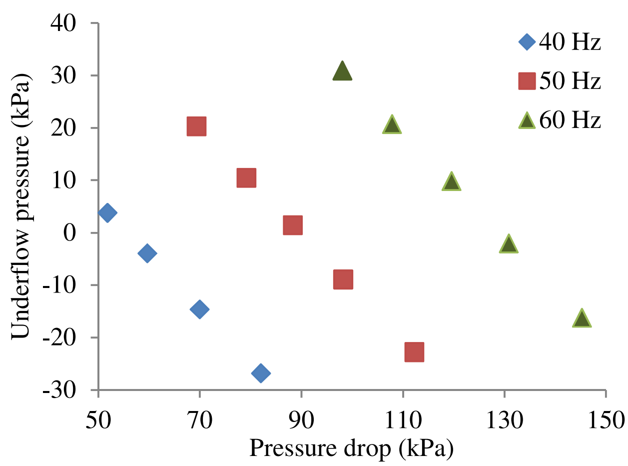

Figure 13: Effect of changes in underflow pressure on pressure drop at three feed pump speeds

Finally, the values of flow ratio and pressure drop from the experiment are applied to equation Eq. (19) combined with aspect ratios for U2-GMAX hydrocyclone to estimate the equivalent area factor and the results are shown in Figure 14. In this figure the equivalent area varies from ~5 to ~14 m^2 for various pressure drops that are in the range shown in Figure 8 for the Krebs hydrocyclone. For this Krebs hydrocyclone the same equivalent area at different pressure drops and flow ratios can result as shown in Figure 14. This indicates that different operating conditions may lead to similar separation efficiency which may be useful in practical applications of hydrocyclones.

These results with only water are promising and indicate that the test rig with an underflow pump is of utility for further studies in predicting hydrocyclone separation performance and equivalent area factor. More experimental investigation of equivalent area but adding solid particles to the water is planned.

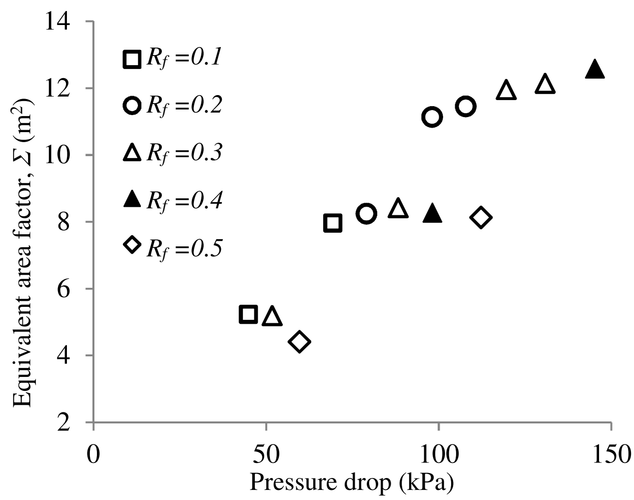

Figure 14: Evaluation of equivalent area for U2-GMAX Krebs hydrocyclone combining the flow ratio and pressure drop values from the experiment with the model

CONCLUSIONS

Comparing the performance of a hydrocyclone with a similar device such as a centrifuge separator is important in selecting and designing a separation system. Since equivalent area factor is using for comparing centrifugal separators an equivalent area factor for hydrocyclones is developed.

A mathematical model for predicting the equivalent area factor for hydrocyclones is described. The model is compared with hydrocyclone designs of Rietema and Bradley. It is observed that equivalent area factor in hydrocyclones varies from less than 1 m^2 to ~400 m^2 and depends on the size and the operating condition of the hydrocyclone. It is found that for similar pressure drop and hydrocyclone diameter, Rietema hydrocyclone can provide higher equivalent area than the Bradley hydrocyclone.

A 5 cm Krebs hydrocyclone is experimentally tested using only water. Changes of the underflow diameter are approximated by using a pump connected to the underflow pipe. It is shown that the Krebs hydrocyclone can provide higher equivalent area factor than the Bradley and Rietema design hydrocyclones of the same diameters.

Prediction of separation efficiency using solid particles in the system is planned.

NOMENCLATURE

A_c	Adjusting coefficient
C	Constant in tangential velocity component
D	Hydrocyclone diameter
d	Particle 50% cut size diameter
D_i	Inlet pipe diameter
D_o	Overflow pipe diameter
dA	Area element
dr	Line elements in the radial direction
dt	Time element
dz	Line elements in the vertical direction
g	Gravitational acceleration
H	Inlet pipe depth
L	Hydrocyclone total height
l	Hydrocyclone vortex finder length
L_1	Hydrocyclone cylindrical section length
n	Exponent in tangential velocity component
Q	Flow rate
r	Radius of the rotation orbit
r_1 & r_2	Distances from the center of the hydrocyclone to the inlet pipe walls
R_f	Flow ratio (underflow/inlet flow rate)
v_g	Settling velocity under gravitational acceleration
v_r	Radial velocity component
v_z	Vertical velocity component
v_θ	Tangential velocity component
w	Inlet pipe width
β	A factor in modeling
ΔP	Pressure drop
$\Delta\rho$	Density difference between phases
θ	Hydrocyclone cone angle
μ	Fluid dynamic viscosity
ρ	Fluid density
Σ	Equivalent area factor (capacity factor)

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