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HCCI Combustion Timing Control with Variable Valve Timing

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Abstract— **Homogeneous Charge Compression Ignition (HCCI) is a promising concept for combustion engines to reduce both emissions and fuel consumption. In HCCI engines, a homogeneous air-fuel mixture auto-ignites due to compression, which is unlike traditional spark ignition or diesel engines where ignition is started with either a spark or fuel injection. HCCI combustion control is a challenging issue because there is no direct initiator of combustion in these engines. Variable Valve Timing (VVT) is one effective way to control the combustion timing in HCCI engines. VVT changes the amount of trapped residual gas and the effective compression ratio both of which have a strong effect on combustion timing. In order to control HCCI combustion, a physics based control oriented model is developed that includes the effect of trapped residual gas on combustion timing. The control oriented model is obtained by model order reduction of complex chemical kinetic reaction mechanisms. This method allows different fuels to be incorporated using a standard methodology and fills the gap between complex models with highly detailed chemical kinetics and simple black box models that have been used in model based control. The control oriented model is used to develop ignition timing PI control using simulation. The PI control modulates the trapped residual gas using variable valve timing as the actuator. The results indicate that the controller can track step changes in HCCI combustion timing.**

I. INTRODUCTION

High thermal efficiency and low nitrogen oxide and particulate matter emissions are the main advantages of HCCI combustion. However, the start of combustion in HCCI engines is difficult to control since there is no direct initiator of combustion. HCCI combustion is controlled by the chemical kinetics of the trapped charge. To achieve ignition timing control of HCCI combustion, the mixture composition, temperature and pressure at the Inlet Valve Closing (IVC) can be controlled. VVT enables quick changes in the amount of trapped hot residual gases inside the cylinder and is a simple and precise actuation method to set condition at IVC for the desired auto-ignition timing.

A variety of different actuators have been used for HCCI combustion timing control such as: compression ratio [2]; intake air temperature [3]; external Exhaust Gas Recirculation (EGR) [4]; valve timing [5]; and dual-fuel [6]. Each of these actuators have their own challenges when implemented in real engines. Variable compression ratio systems are complex and changing the compression ratio in real time (cycle-by-cycle) is a challenging issue. Intake air heating is impractical because of the energy required to heat the intake air and the long heater response time compared to an engine cycle. Controlling the combustion timing by varying the auto-ignition properties of the fuel requires at least two fuels which adds to the complexity of the system. VVT reduces residual gas heat loss and achieves fast cycle-by-cycle control response compared to external EGR but adds complexity to the system.

Control algorithms for HCCI combustion timing control using VVT are being actively developed. A PID controller is designed to control negative valve overlap and the intake valve closing on a single cylinder HCCI engine [7]. LQR control is implemented on a single cylinder variable valve actuated engine [8]. A layered closed loop control algorithm is developed for an HCCI gasoline engine equipped with VVT [9]. An MPC algorithm is applied to control combustion timing with both VVT and dual-fuel as actuators [10] and using a system identification model while a physics based model is used in [5].

The goal of smooth, fast control of HCCI combustion timing using variation of trapped residual gas still remains to be achieved. In this work, PI control of HCCI combustion timing using VVT actuation is developed based on a control oriented model. Although only PI control is developed here, the reduced order model can be used for more complex model based control. The model is reduced order model developed from a complex physics based model that since it is physics based can easily be adapted to other engines and fuels [11]. The control algorithm is tested in simulation and the combustion timing is tracked accurately for commanded step changes.

II. CONTROL ORIENTED MODEL

A Control Oriented Model (COM) of a single cylinder HCCI engine is developed using model reduction of a Detailed Physical Model (DPM) [11]. The HCCI four stroke cycle is modeled as a sequence of continuous processes: intake, compression, combustion, expansion and exhaust. In the COM, the system of interest is the instantaneous contents of a cylinder. This system is open to the transfer of mass, the enthalpy and energy in the form of work and heat. The cylinder is modeled as a time variant volume and the cylinder contents are represented as one continuous medium. It is assumed that there is no spatial variation in properties within a cylinder at any instant of time. The COM consists of two main parts: the IVC sub-model which calculates the in-cylinder gas temperature at IVC and the residual gas fraction and cylinder sub-model that gives θ_{50} and other cycle information. This model has been developed for n-heptane fuel by taking complex chemical kinetics [1] and using a systematic model reduction procedure. In the future,

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the COM can be easily modified for other fuels by using model reduction of the chemical kinetics corresponding to the appropriate fuel.

A. IVC sub-model

At the start of the cycle, fresh reactants are inducted into the cylinder. Intake air and fuel are assumed to instantaneously mix with the trapped residual gas in the cylinder at IVC. To determine the thermodynamic state of the mixture at IVC, the contents of the in-cylinder mixture must first be determined. The contents of the cylinder at the current cycle k depends on fresh reactants inducted during cycle k and residual gases from the last cycle, $k-1$. For lean combustion of n-heptane with internal EGR and no external EGR, the inducted charge becomes:

$$\begin{aligned}&\phi_k C_7H_{16} + c_1(\alpha_{i,k}(c_2-\phi_{k-1})+c_2)O_2 + \\ &c_3(c_2+\alpha_{i,k})N_2 + c_4\alpha_{i,k}\phi_{k-1}CO_2 + c_5\alpha_{i,k}\phi_{k-1}H_2O\end{aligned} \tag{1}$$

where α_i, ϕ and k represents the residual gas fraction, fuel equivalence ratio and cycle number respectively and c_i are constants listed in Table I. The values of α_i are obtained from the formation of the energy balance for the cylinder gas over the intake process from IVO to IVC. The fuel equivalence ratio and residual gas fraction are calculated [12], [14] as:

$$\phi_k = \frac{n_{f,k}}{n_{fs,k}} \tag{2}$$

$$\alpha_{i,k} = \frac{A \ \ P_{EVO,k-1}}{A \ \ P_{EVO,k-1} + B \ \ T_{EVO,k-1}} \tag{3}$$

and $n_{f,k}$ is the number of fuel moles, $n_{fs,k}$ is the number of fuel moles for stoichiometric combustion, $P_{EVO,k-1}$ is the pressure at exhaust valves open, and $T_{EVO,k-1}$ is the temperature at exhaust valves open. The factors A and B are:

$$\begin{aligned}A &= V_{EVC,k-1}\Omega M_f T_{int,k} M_{air}\\ B &= M_{res}P_{int,k}(V_{IVC,k}-V_{IVO,k})\Omega M_{fuel} + \\ &\quad c_6 R M_{res}(m_{f,k}-m_{f,k-1})T_{int,k}M_{air}\end{aligned}$$

where Ω is the engine speed, $V_{IVC,k}$ is the in-cylinder volume at IVC, $V_{IVO,k}$ is the in-cylinder volume at inlet valves open, and $T_{int,k}$ is the intake charge temperature.

Temperature at IVC, T_{IVC}, is the temperature of products and reactants after full mixing and is calculated as:

$$T_{IVC,k} = \frac{C_{2,k}\alpha_{i,k}T_{EVO,k-1} + C_{1,k}T_{int,k}}{C_{1,k} + C_{2,k}\alpha_{i,k}} \tag{4}$$

where

$$\begin{aligned}C_{1,k} &= \phi_k C_{p,C_7H_{16}} + c_1 C_{p,O_2} + c_3 C_{p,N_2}\\ C_{2,k} &= c_4\phi_{k-1}C_{p,CO_2} + c_5\phi_{k-1}C_{p,H_2O}\\ &\quad + c_3 C_{p,N_2} + c_1(c_2-\phi_{k-1})C_{p,O2}\end{aligned}$$

The intake process is assumed to take place at atmospheric pressure as the variable valve timing engine usually operated at wide-open throttle with the valve timing controlling the amount of air.

$$P_{IVC,k} = P_{int,k} \tag{5}$$

B. Cylinder sub-model

Once the intake valve closes, the cylinder charge is constant (assuming no blow-by) and the compression of the inducted charge made up of fuel, air and internal EGR is assumed to be isentropic. This results in the following equations:

$$T_{SOC,k} = (\frac{V_{IVC,k}}{V_{SOC,k}})^{\gamma_c-1}T_{IVC,k} \tag{6}$$

$$P_{SOC,k} = (\frac{V_{IVC,k}}{V_{SOC,k}})^{\gamma_c}P_{IVC,k} \tag{7}$$

where γ_c is determined from DPM [11]. The crank angle of start of combustion is calculated from simplified Arrhenius equation [12] as:

$$\begin{aligned}\theta_{SOC,k} = \theta_{IVC} + &\frac{K_{th}\Omega}{A}e^{\left(\frac{E_a}{RT_{IVC,k}(\frac{V_{IVC,k}}{V_{TDC}})^{\gamma-c_1}}\right)}\\ &\frac{(\alpha_{i,k}(c_7\phi_{k-1}+c_8)+c_7\phi_k+c_8)}{c_1^b(\phi_k)^a(\alpha_{i,k}(c_2-\phi_{k-1})+c_2)^b}\\ &(\frac{RT_{IVC,k}V_{TDC}}{P_{IVC,k}V_{IVC,k}})^{a+b} + \theta_{offset}\end{aligned} \tag{8}$$

where the values of a, b, A, θ_{offset}, E_a and K_{th} are listed in Table I.

Based on the correlation developed in [12] and [13] $\Delta\theta$ is calculated as:

$$\Delta\theta_k = c_9 \ \ c_{10}^{\phi_k} \ \ c_{11}^{T_{IVC,k}(\frac{V_{IVC,k}}{V_{SOC,k}})^{\gamma-1}} \ \ c_{12}^{\theta_{SOC,k}} \tag{9}$$

Finally, θ_{50}, the crank angle of 50% mass fraction burned, is calculated as:

$$\theta_{50,k} = \theta_{SOC,k} + 0.5\Delta\theta_k \tag{10}$$

In order to determine the thermodynamic state of the system after combustion, the first law of thermodynamics is applied to the system. In-cylinder gas temperature after combustion is calculated as:

$$T_{AC,k} = \frac{D_k + (D_{1,k}-N_{1,k}R)T_{2,k} - D_{1,k}c_{14} + D_{2,k}c_{14}}{D_{2,k} - RN_{2,k}} \tag{11}$$

where

$$\begin{aligned}D_k &= c_{13}LHV_{C_7H_{16}}\phi_k\\ D_{1,k} &= \phi_k C_{p,C_7H_{16}} + c_1(\alpha_{i,k}(c_2-\phi_{k-1})+c_2)C_{p,O2} + \\ &\quad c_3(c_2+\alpha_{i,k})C_{p,N_2} + c_4\alpha_{i,k}\phi_{k-1}C_{p,O2} + c_5\alpha_{i,k}\phi_{k-1}C_{p,H_2O}\\ D_{2,k} &= c_5(\phi_k+\alpha_{i,k}\phi_{k-1})C_{p,H_2O} + c_4(\phi_k+\alpha_{i,k}\phi_{k-1})C_{p,CO_2} + \\ &\quad c_3(c_2+\alpha_{i,k})C_{p,N_2} + c_1(c_2+\alpha_{i,k}-\alpha_{i,k}\phi_{k-1}-\phi_k)C_{p,O_2}\\ N_{1,k} &= \phi_k + c_1(\alpha_{i,k}(c_2-\phi_{k-1})+c_2) + c_3(c_2+\alpha_{i,k}) + \\ &\quad c_4\alpha_{i,k}\phi_{k-1} + c_5\alpha_{i,k}\phi_{k-1}\\ N_{2,k} &= c_5(\phi_k+\alpha_{i,k}\phi_{k-1}) + c_4(\phi_k+\alpha_{i,k}\phi_{k-1}) + c_3(c_2+\alpha_{i,k}) + \\ &\quad c_1(c_2+\alpha_{i,k}-\alpha_{i,k}\phi_{k-1}-\phi_k)\end{aligned}$$

By applying the ideal gas law before and after combustion, the in-cylinder pressure after combustion is calculated as:

$$P_{AC} = \frac{N_{2,k}}{N_{1,k}}P_{SOC,k}\frac{T_{AC,k}}{T_{SOC,k}} \tag{12}$$

In HCCI engines, large amounts of energy are released during a short period of time. For this reason, a pressure rise rate threshold is usually monitored to keep the combustion noise under certain level. The rate of pressure rise is calculated from

$$PRR_k = \frac{P_{AC,k} - P_{SOC,k}}{\Delta\theta_k}. \tag{13}$$

and can be included as a model output in order to properly control the engine.

The expansion process, which takes place until the opening of the exhaust valve, is assumed to be isentropic. The temperature and pressure of the in-cylinder gas at EVO are calculated as

$$T_{EVO,k} = \left(\frac{V_{AC,k}}{V_{EVO,k}}\right)^{\gamma_e - 1} T_{AC,k} \tag{14}$$

$$P_{EVO,k} = \left(\frac{V_{AC,k}}{V_{EVO,k}}\right)^{\gamma_e} P_{AC,k} \tag{15}$$

Values of all model parameters and constants are listed in Table I. These model parameters are determined from DPM: γ_c, K_{th}, c_{13} and θ_{offset}. While these parameters: γ and γ_e are determined from experimental data. The parameters: a, b, A and E_a are taken directly from the literature [12], [17].

TABLE I

MODEL PARAMETERS AND CONSTANTS

a	0.25	c_3	41.4
b	1.5	c_4	7
E_a	15098	c_5	8
R	0.8314	c_6	2
K_{th}	2.31×10^{-6}	c_7	4
γ	1.37	c_8	52.4
γ_e	1.34	c_9	2.07×10^{-18}
γ_c	1.32	c_{10}	3.55
A	5.1×10^{11}	c_{11}	0.993
θ_{offset}	85.2	c_{12}	1.16
c_1	11	c_{13}	0.87
c_2	1	c_{14}	298

C. State, Input and Output Variables

The COM in standard form is:

$$x_{k+1} = f(x_k, u_k, w_k) \tag{16}$$
$$y_k = g(x_k, u_k, w_k). \tag{17}$$

where x is the state of the system, u is the control input and w is a disturbance. The input, the state, and the output for this system are:

$$u = \begin{bmatrix}\theta_{IVO} & \theta_{EVC}\end{bmatrix} \tag{18}$$
$$x = \begin{bmatrix}\alpha_i & T_{IVC} & \theta_{SOC} & \Omega & \phi\end{bmatrix} \tag{19}$$
$$y = \theta_{50} \tag{20}$$

III. COM MODEL VALIDATION

Experimental validation of the COM is performed on a single cylinder research engine with the specifications listed in Table II. Both steady-state and transient operation are tested and the model values of θ_{50} and fuel equivalence ratio are compared to the experimental data. For Transient engine operation validation, a step change in EVC and IVO timing is performed and the results plotted in Figure 1. This figure shows a step change to both EVC and IVO using the VVT system and the resulting θ_{50} of the COM model and engine are plotted showing that the model captures the transient dynamics. The COM reaches the final values earlier compared to the experimental data because of the simplifications in IVC sub-model and because the COM neglects the fuel wall wetting dynamics. Comparison of the COM's accuracy in predicting θ_{50} shows the average and root mean square error of 1.74 CAD and 1.65 CAD respectively.

The COM is further validated at several steady-state engine operating points. For all cases shown, the external EGR is zero. The tests are conducted for a fuel equivalence ratio range between 0.2 and 0.4 and an engine speed range of 800-900 RPM as shown in Figure 2. Figure 2(a) shows θ_{50} variation as a function of engine speed with a fixed injected fuel energy of 0.39 kJ. This figure shows that a decrease in the engine speed causes combustion timing, θ_{50}, to advance, since decreasing the engine speed increases the amount of time for the auto-ignition reactions to occur and the HCCI combustion is a time based process dominated by the time scales of the reactions [18]. Figure 2(b) shows θ_{50} for different equivalence ratios at a constant engine speed of 825 RPM. The reactivity of the fuel tends to increase from lean to rich conditions, advancing θ_{50}, which is consistent with the literature [19].

Next, the COM is validated more for variations in IVC and EVC timing while holding all other parameters constant. Figure 3(a) shows θ_{50} for different EVC timing and shows that, combustion timing advances when EVC timing is retarded. In-cylinder gas temperature is reduced due to expansion and more fresh charge is inducted into the cylinder due to low in-cylinder pressure at IVO. Mixture composition has key role on HCCI combustion phasing control in this case and combustion advances because more fuel is inducted to the cylinder. Figure 3(b) shows θ_{50} as IVC timing is varied. This figure shows that combustion timing, θ_{50}, advances when IVC timing is advanced. The in-cylinder residual gas mass fraction increases when IVC timing is advanced causing mixture temperature at IVC to increase and combustion timing to advance.

A comparison of the COM and the 100 cycles of averaged experimental cylinder pressure traces at Ω=825 RPM and ϕ=0.3 with 120 degree symmetric negative valve overlap is shown in Figure 4. This figure shows that the model predicts combustion timing, θ_{50}, within 0.58 degrees for this case.

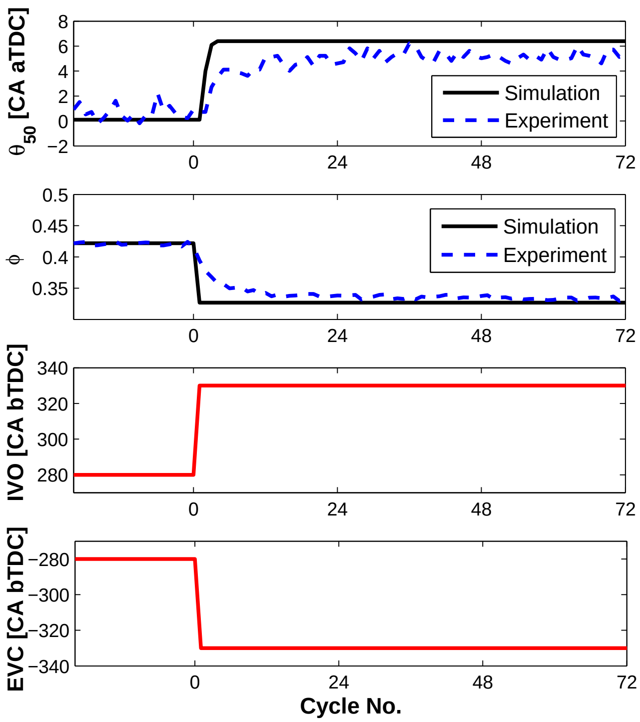

Fig. 1. Experimental validation of the developed model for a step change in EVC and IVO timing symmetric to Top Dead Center (ON=0; Ω=823±12 RPM)

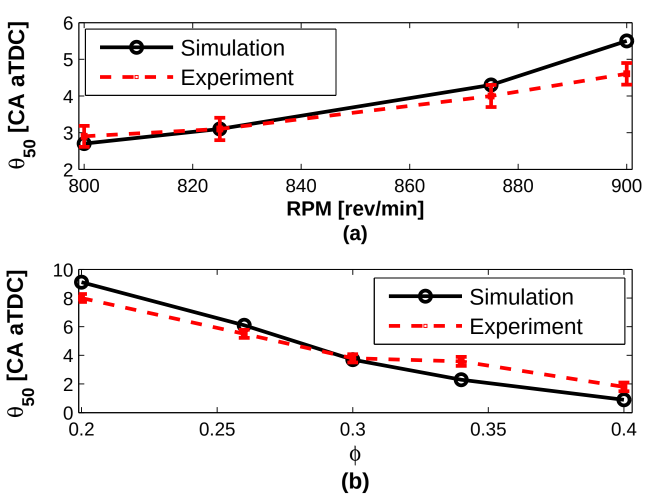

Fig. 2. Comparison of predicted and measured θ_{50} (a) Injected Fuel Energy = 0.39 kJ, speed varies (b) Ω=825 RPM, load varies

Further model validation for other operating conditions is described in [11].

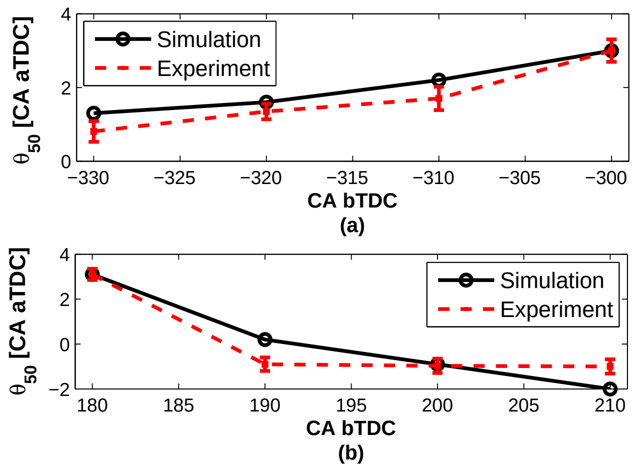

Fig. 3. Comparison of predicted and measured θ_{50} (a) EVC timing changes (b) IVC timing changes

TABLE II

SINGLE CYLINDER RESEARCH ENGINE SPECIFICATIONS [15]

Bore	97 mm
Stroke	88.9 mm
Compression ratio	13.9:1
Connecting rod length	159 mm
Rated Speed	823±12 RPM
IVO [bTDC]	280
IVC [bTDC]	180
EVO [bTDC]	-180
EVC [bTDC]	-280

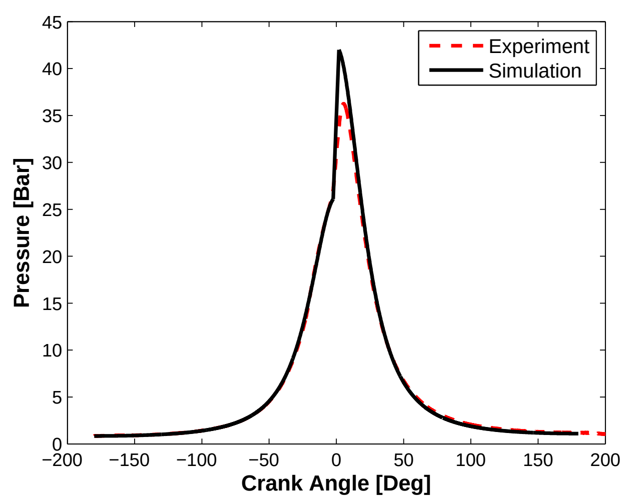

Fig. 4. Experiment Pressure Trace vs. Model (Ω=825 RPM, ϕ=0.3, ON=0)

IV. PI CONTROLLER DESIGN

A PI controller is developed that uses θ_{50} as a measured input and adjusts the intake and exhaust valves timing (IVO, EVC) to get varied amount of the internal residual gas in the cylinder to adjust the combustion timing. Symmetric Negative Valve Overlap (NVO) is used as a variable valve timing strategy. This strategy involves early closing of the exhaust valves followed by late intake valve opening symmetric to top dead center. This technique helps improving the combustion stability and extend the HCCI operation range [20], [21]. Using simulation the COM is used to design a PI controller for θ_{50} control. The controller is a standard PI control that is synchronous with the engine cycle (k)

$$u_k = u_{k-1} + (k_p + 0.5k_iT)e_k + 0.5k_iTe_{k-1} \tag{21}$$

$$EVC_k = EVC_{k-1} + u_k \tag{22}$$

$$IVO_k = IVO_{k-1} - u_k \tag{23}$$

where $e = \theta_{50}(Ref) - \theta_{50}(Meas)$ is the error and T, k_p and k_i are the sample time, proportional and integral gains respectively ($k_p = 2$ and $k_i = 3$). The designed controller is tested in simulation on the complex model [11] and the tracking performance of the designed controller is shown in Figure 5. The PI controller can track the desired θ_{50} trajectory with a rise time of 3 to 4 engine cycles and the maximum overshoot of 0.8 CAD. No steady state error is observed which is attributed to the integral term. The effect of measurement noise on tracking performance of the PI controller is studied by adding a Gaussian disturbed noise with standard deviation of 1.2 CAD to the measurement of θ_{50} after cycle 55 (see Figure 5(a)). The noise level was determined based on the available experimental data. The PI controller maintained tracking despite the measurement noise in the feedback signal.

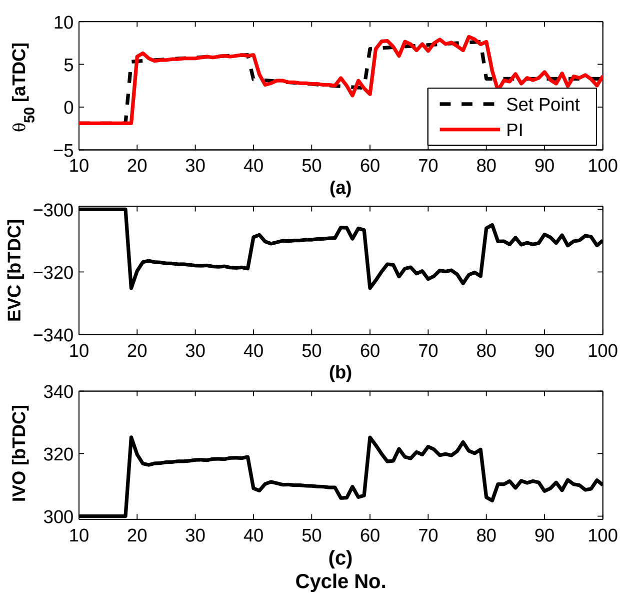

Fig. 5. Simulation: Tracking Performance of PI controller (a) Engine Plant Output (b and c) controller inputs

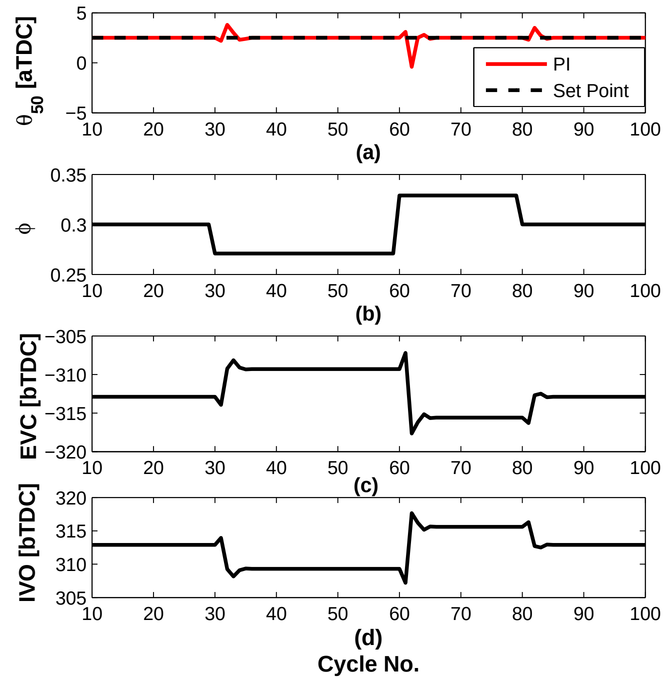

Fig. 6. Disturbance rejection: fuel equivalence ratio (engine load) (a) θ_{50} (b) Disturbance (c and d) Controller Inputs

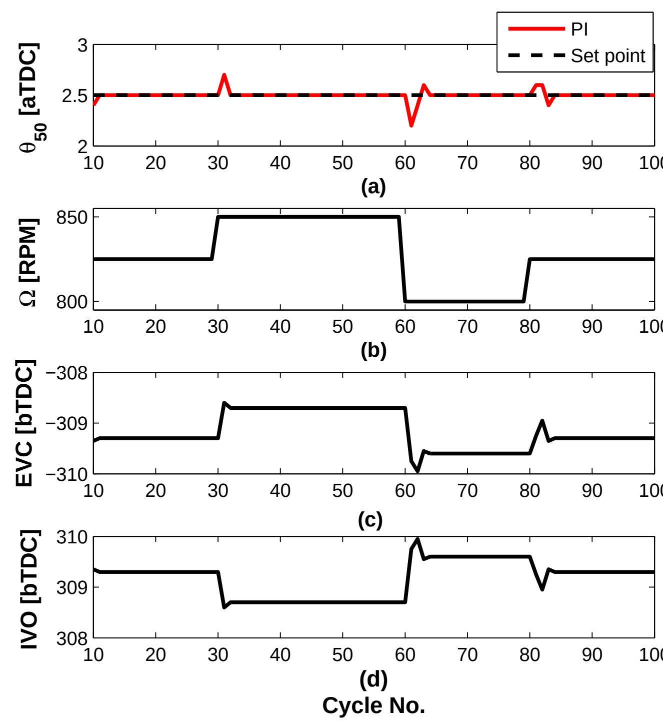

Fig. 7. Disturbance rejection: Engine speed (a) θ_{50} (b) Disturbance (c and d) Controller Inputs

The controller is also tested with the disturbances of varying engine loads and varying engine speed. In Figures 6 and 7, the disturbance rejection properties of both controllers are compared for positive and negative disturbance step changes. The results show that the PI controller has a reasonable disturbance rejection characteristic and the integral action causes the steady state error to go zero.

V. CONCLUSIONS

A reduced order control oriented model for HCCI combustion timing control that includes the effect of trapped residual gas is developed and validated on transient and steady-state experimental data. The model is accurate and computationally efficient needing 8 ms to simulate an HCCI cycle (on a 2.66 GHz Intel processor). Since the control oriented model is physics based and combustion model is obtained by model order reduction from more complex chemical kinetics the parametrization of the model for different fuels and engines should be straightforward. Using the control oriented model, a PI controller is developed to control combustion timing by modulating the IVO and EVC timing symmetric to the top dead center. Performance of the controller is checked against the detailed HCCI model. The results show that the model based PI controller can track the desired combustion timing trajectories with a maximum rise time of 4 engine cycles and the controller performs well in maintaining a desirable engine operation during load and engine speed disturbances. This indicates that the PI control strategy is feasible for real-time HCCI combustion control.

DEFINITIONS/ABBREVIATIONS

DPM	Detailed Physical Model
COM	Control Oriented Model
EVC	Exhaust Valve Closing
EVO	Exhaust Valve Opening
IVO	Inlet Valve Opening
IVC	Inlet Valve Closing
NVO	Negative Valve Overlap
ON	Octane Number
aTDC	after Top Dead Center
bTDC	before Top Dead Center
int	Intake
θ_{50}	50% mass fraction burned
res	Residual
AC	After Combustion
RPM	Revolutions Per Minute
PRR	Pressure Rise rate
LHV $[J/kg]$	Low Heating Value
SOC	Start of Combustion
R $[JK^{-1}mol^{-1}]$	Ideal Gas Constant
Ω [rev/min]	Engine Speed
m [kg]	Mass
V $[m^3]$	Volume
T [K]	Temperature
P [Pa]	Pressure
C_p $[kJ/kg^oC]$	Specific Heat at Constant Pressure
M $[kg/kmol]$	Molecular Weight
E_a $[K]$	Arrhenius rate parameter
n_{fs}	Number of Fuel Moles for Stoichiometric Combustion
n_f	Number of Fuel Moles

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