The Lumped Capacitance Method

Patricio F. Mendez
University of Alberta
ChE 314 Heat Transfer

Contents

1 The Lumped Capacitance Method

In this section we will learn about transient heat transfer in a particular condition, which happens to be common in practice: the case in which all the points inside the control volume are at approximately the same temperature. In this case, we can *assume* that all points inside the control volume (e.g those at the center and those at the surface) are exactly the same temperature, and we can estimate the heat stored using a single value of temperature.

The lumped capacitance method is mathematically very convenient, because in the absence of thermal gradients inside the control volume, the only independent variable is time t, and the equations resulting from this analysis are ordinary differential equations much easier to solve than partial differential equations.

The term "lumped" implies that all mass within the control volume is considered as the same. The term capacitance comes from an analogy between heat transfer and electrical circuits which we will learn later in this class.

The methodology that follows will always be the same

1. Determine the control volume and establish an energy rate balance
2. Assign expressions to q_{in}, q_{out}, q_{gen}, and q_{st} using our knowledge of heat transfer mechanisms and heat stored
3. After replacing the heat transfer mechanisms and heat stored in the energy balance, we obtain a differential equation
4. Solve the differential equation

What differs for each case is the expressions involve, which can result in very easy or very difficult equations to solve. The learning in this unit is to apply our knowledge to a very broad variety of practical problems, and provide answers that are both insightful and qualitative.

Fortunately, many aspects of the the lumped capacitance method are quite intuitive. Using and developing this intuition will be invaluable for this course, and essential in our life as engineers.

2 Lumped Capacitance with Convection

2.1 Example: Convection cooling of a copper sphere

Consider a copper sphere of 6 cm diameter. The sphere is initially at 90°C, and it is put in stagnant air in a large room at 20°C. Consider a natural convection coefficient $h = 10\frac{\mathrm{W}}{\mathrm{m^2K}}$. The density of copper is $\rho = 9000\frac{\mathrm{kg}}{\mathrm{m^3}}$, and its specific heat $C = 385\frac{\mathrm{J}}{\mathrm{kgK}}$.

2.1.1 Final temperature

What will be the temperature of the sphere after a long period of time? Before doing any formal analysis, it is important to develop an intuitive sense of the problem. Taking this class is not needed to answer this question!

After a long period of time the sphere will be in thermal equilibrium with the air of the room. We only know that the room is "large," so we can assume (very conveniently), that the temperature of the room is not affected by the heat of the sphere. Therefore, the final temperature of the sphere will be 20°C.

2.1.2 Initial rate of heat transfer

What is the initial rate of the heat leaving the sphere? For this case, for now, let's consider only convection

To answer this question we need to set a control volume around the sphere as shown in Figure 1. This question asks about q_{out}, which we are assuming is by convection:

$$\begin{aligned}
q_{\mathrm{out}} =& q_{\mathrm{conv}} && (1)\\
=& A_{\mathrm{s}} q''_{\mathrm{conv}} && (2)\\
=& A_{\mathrm{s}} h(T_{\mathrm{s}} - T_\infty) && (3)\\
=& A_{\mathrm{s}} h(T - T_\infty) && (4)\\
=& 4\pi R^2 h(T - T_\infty) && (5)\\
=& 4\pi 0.03^2 \cancel{\mathrm{m}^2} 10 \frac{\mathrm{W}}{\cancel{\mathrm{m}^2}\cancel{\mathrm{K}}}(90\,^\circ\cancel{\mathrm{C}} - 20\,^\circ\cancel{\mathrm{C}}) && (6)\\
=& 7.9\mathrm{W} && (7)
\end{aligned}$$

where T is the uniform temperature for all points inside the control volume, then $T_s = T$.

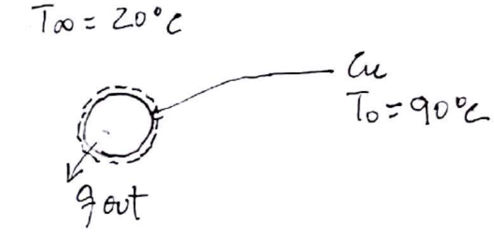


Figure 1: Schematic of a copper sphere with diameter of 6 cm cooled by stagnant air

2.1.3 Initial temperature rate

What is the initial cooling rate of the sphere in this case? To answer this question we must perform an energy balance for the control volume of Figure 1:

$$\cancel{q_{\text{in}}} - q_{\text{out}} + \cancel{q_{\text{gen}}} = q_{\text{st}} \tag{8}$$

Since the sphere is hotter than the room we can assume that no heat comes into the sphere ($q_{\text{in}} = 0$), and we can also assume that there is no heat generation within the sphere ($q_{\text{gen}} = 0$). All these hypotheses could be changed later, the heat balance approach is a very general and very powerful methodology. The heat stored will be:

$$q_{\text{st}} = \frac{d(mi)}{dt} \tag{9}$$

$$= m\frac{di}{dt} \tag{10}$$

$$= V\rho c\frac{dT}{dt} \tag{11}$$

where the control volume is a closed system, and we are assuming no phase change (the ball is never hotter than 90°C, much below copper's melting temperature of 1085°C). Combining Equations 8, 4, and 11 we obtain:

$$-A_\mathrm{s}h(T-T_\infty) = V\rho c\frac{dT}{dt} \tag{12}$$

$$\frac{dT}{dt} = -\frac{A_\mathrm{s}h}{V\rho c}(T-T_\infty) \tag{13}$$

The initial cooling rate of the sphere corresponds to dT/dt at the initial time when T =90°C, resulting in dT/dt =-1.21°C/min. The negative sign indicates the system is cooling, while a positive sign would have indicated a system heating up. This analysis can be extended without difficulty to the case in which there are phase changes.

Is the cooling rate constant with time? How does the value of cooling rate calculated vary with time? Just like before, we do not need to have taken this class to answer this question. We know it starts cooling at the beginning (we just calculated it), and we know that at some point it will reach the air temperature and stop cooling. So, no, the cooling rate is not constant; eventually the cooling rate will become zero.

2.1.4 Temperature evolution

Temperature evolution means a graph of temperature as a function of time (T vs. t). In contrast, a temperature profile means temperature as a function of location (e.g. T vs. x).

No spurious ups and downs are expected in this cooling process, so intuitively we can draw Figure 2. In this curve it is important to highlight the following features

- The curve starts at T =90°C at time t =0.
- At long times ($t=\infty$) the curve is asymptotic to the fluid temperature: $T=T_\infty$)
- At the start the cooling rate is finite (we calculated it above!). Finite means it is not zero, and not infinity. This finite rate results in a slope at the starting point, and it is indicated in the graph.

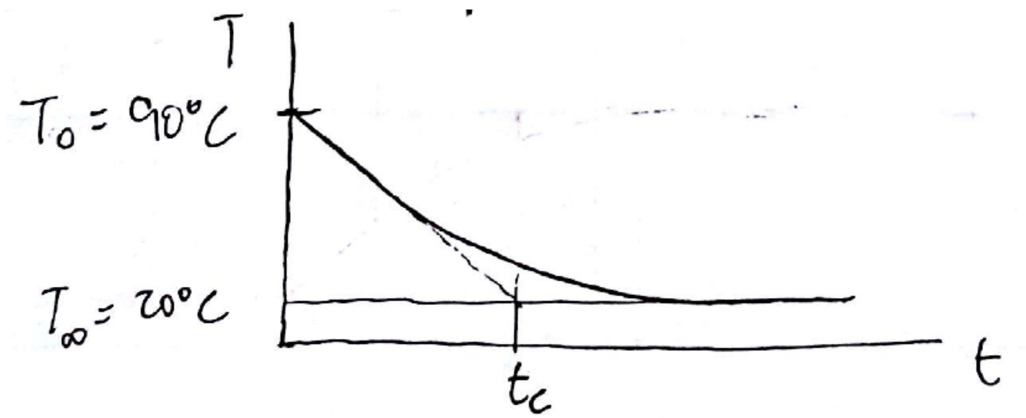


Figure 2: Schematic of the temperature evolution. t_c is the time constant (characteristic time of cooling).

2.1.5 Concept of characteristic time

A common question in practice is "how long does the sphere take to cool down?" This question implies that there is a "characteristic time" t_c (or "time constant") that characterizes the cooling and is the answer to this question. This question makes intuitive sense, and has practical implications; however, a straight answer is elusive; inspection of Figure 2 shows that there is no sharp point giving an unequivocal answer. Indeed, there are many possible answers that would "make sense, " for example considering a "half life," which is the time it would take for the sphere to reduce the temperature difference with the fluid by half. Other criterion would be when the temperature difference is reduced by 90%, 95%, or 99%. All these criteria have been used in engineering.

In this class we will use a criterion for characteristic time that makes as much sense and is as arbitrary as those mentioned above. Because it has the advantage of resulting in simpler mathematics, it is the concept we will adopt.

The characteristic time (or other characteristic values such as characteristic length) that we will use in this class consists in extrapolating the tangent at the starting point until it crosses the asymptote of final state. In Figure 2 this corresponds to the intersection of the tangent at the beginning with the line $T = T_\infty$ =20°C, which is marked as t_c.

Using the concept of characteristic time defined above, we can write

$$\left.\frac{dT}{dt}\right|_{t=0} = \frac{T_\infty - T_0}{t_\mathrm{c}} \tag{14}$$

$$\begin{aligned} t_c &= \frac{T_\infty - T_0}{\dot{T}_0} \qquad (15) \\ &= \frac{20\,^\circ\mathrm{C} - 90\,^\circ\mathrm{C}}{-1.21\frac{^\circ\mathrm{C}}{\mathrm{min}}} \\ t_c &= 57.9 \mathrm{min} \end{aligned}$$

Note that in this case, the difference in temperatures is consistent with the difference in times (last minus first), and signs matter. Also, the notation $\dot{T}$, and dT/dt are *exactly* equivalent, and apply only to time derivatives. The former is the only vestige of the notation used by Newton when he developed calculus, while the latter is the more widely adopted notation of Leibnitz when he developed calculus simultaneously with Newton. The notation was not the only difference between these to major mathematicians...

2.1.6 Governing equation

The energy balance for the control volume is a differential equation for predicting temperature evolution (Equation 13), which can be written as

$$\frac{dT}{dt} + \frac{A_\mathrm{s}h}{V\rho c}T = \frac{A_\mathrm{s}h}{V\rho c}T_\infty \tag{16}$$

This is an ordinary differential equation (ODE), of first order and linear. It is not homogeneous, as it contains a constant term.

The independent variable is time t, and the dependent variable is temperature $T(t)$. To solve this equation we will need only one initial condition:

$$T(0) = T_0 = 90\,^\circ\mathrm{C} \tag{17}$$

Parameters are those magnitudes that can change from problem to problem, but are constant for a given problem. In this case, time and temperature can have different values for the particular problem considered, but things such as the density, are considered constant. The parameters in this equation are seven: A_s, h, ρ, c, V, T_∞, T_0.

This problem would be very difficult to explore experimentally. A factorial design of n points per variable would result in n^7 experiments! A change of variables will be convenient:

2.1.7 Solution of the differential equation

Change of variable and parameters: A convenient first step in solving the differential equation is to change variables or parameters to simplify the solution:

$$T - T_\infty = \theta \tag{18}$$
$$\frac{A_s h}{\rho C V} = K \tag{19}$$

thus:

$$\frac{dT}{dt} = \frac{d\theta}{dt} \tag{20}$$

Substituting into Equations 16 and 17 we obtain:

$$\frac{d\theta}{dt} + K\theta = 0 \tag{21}$$
$$\theta(0) = \theta_0 \tag{22}$$

where $\theta_0 = T_0 - T_\infty$

Equation 21 is now homogeneous, which facilitates its solution. In this example, after a long period of time, the copper sphere is in steady state with the external environment, so $d\theta/dt = 0$; therefore $\theta \to 0$ at long times.

Expression of the characteristic time: The constant K has a physical meaning. We can explore it by determining the initial cooling rate of the solution. From Equation 21, we obtain

$$\left.\frac{d\theta}{dt}\right|_{t=0} = -K\theta_0 \tag{23}$$
$$\left.\frac{dT}{dt}\right|_{t=0} = -K(T_\infty - T_0) \tag{24}$$

Solving for K, and comparing with Equation 15:

$$K = \frac{dT/dt|_{t=0}}{T_0 - T_\infty} = \frac{1}{t_\mathrm{c}} \tag{25}$$

The physical meaning of the constant K is that it is the inverse of the characteristic time t_c, thus:

$$t_\mathrm{c} = \frac{V\rho c}{A_\mathrm{s} h} = \tau \tag{26}$$

where the notation τ is commonly used to name a time constant.

Integration of the differential equation: An exponential functional form is proposed based on experience, and the parameters involved need to be determined.

$$\theta(t) = C_1 \exp(rt) \tag{27}$$

$$\frac{d\tau}{dt} = C_1 r \exp(rt) \tag{28}$$

Replacing into Equation 21:

$$\cancel{C_1} r \cancel{\exp(rt)} + K \cancel{C_1} \cancel{\exp(rt)} = 0 \tag{29}$$

$$r = -K = -\frac{1}{\tau} \tag{30}$$

From the boundary conditions:

$$\theta(0) = C_1 = \theta_0 \tag{31}$$

resulting in

$$\theta(t) = \theta_0 \exp\left(-\frac{t}{\tau}\right) \tag{32}$$

or, using the original variables and parameters:

$$T(t) = T_\infty + (T_0 - T_\infty)\exp\left(-\frac{t}{\tau}\right) \tag{33}$$

$$T(t) = T_\infty + (T_0 - T_\infty)\exp\left(-\frac{A_\mathrm{s} h}{V\rho c} t\right) \tag{34}$$

This solution is valid for both heating and cooling case. It shows that for $t = 0$, $T = T_0$, for $t \to \infty$, $T \to T_\infty$, and that there is a characteristic time t_c (or time constant τ) that gives an idea of the time it takes for the system (in this case a copper sphere cooling down) to reach its final temperature.

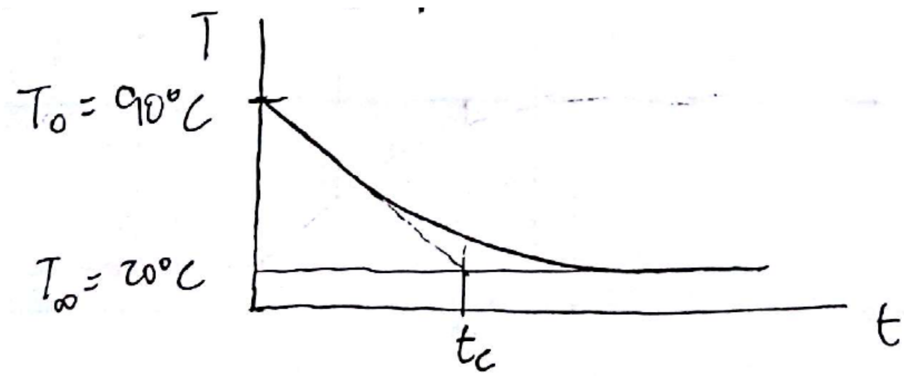


Figure 3: Temperature evolution for this example

3 Lumped Capacitance with Radiation

If the sphere considered above colled down by radiation, there would be many similarities and differences. The energy balance would still be like in Equation 8, except q_out would now be by radiation:

$$q_\mathrm{out} = q_\mathrm{rad} = A_\mathrm{s}\varepsilon\sigma(T^4 - T_\mathrm{sur}^4) \tag{35}$$

When using radiation in the energy balance we obtain:

$$-A_\mathrm{s}\varepsilon\sigma(T^4 - T_\mathrm{sur}^4) = V\rho c\frac{dT}{dt} \tag{36}$$

resulting in the following differential equation:

$$\frac{dT}{dt} + \frac{A_\mathrm{s}\varepsilon\sigma}{V\rho c}T^4 = \frac{A_\mathrm{s}\varepsilon\sigma}{V\rho c}T_\mathrm{sur}^4 \tag{37}$$

Similarly to Equation 16, this is an ordinary differential equation (ODE), of first order and not homogeneous, as it contains a constant term. The independent variable is time t, and the dependent variable is temperature $T(t)$. To solve this equation we will need only one initial condition:

$$T(0) = T_0 \tag{38}$$

In contrast with Equation 16, this equation is non-linear, as the dependent variable T is raised to the 4th power. The parameters in this equation are seven: A_s, ε, σ, ρ, c, V, T_sur, T_0.

The solution of this equation is complex; however, it has important similarities with the solution for lumped capacitance with convection (Equation 33). Both equations start at $T = T_0$, and both reach a steady state of $T = T_\infty$. In addition, both have a characteristic time to reach steady state, with important similarities too.

3.1 Characteristic time in lumped capacitance with radiation

In Section 2.1.5 we defined a characteristic time as that corresponding to the time to reach steady state at the initial cooling (or heating) rate. For the case of radiation, the initial cooling rate can be obtained from Equation 36:

$$\left.\frac{dT}{dt}\right|_0 = \dot{T}_0 = -\frac{A_\mathrm{s}\varepsilon\sigma(T_0^4 - T_\mathrm{sur}^4)}{V\rho c} \tag{39}$$

Replacing into Equation 15, we obtain

$$t_\mathrm{c} = \tau = \frac{V\rho c(T_0 - T_\mathrm{sur})}{A_\mathrm{s}\varepsilon\sigma(T_0^4 - T_\mathrm{sur}^4)} \tag{40}$$

If we were to factor the difference of squares in the denominator, we would see that the characteristic time can be expressed in terms of the equivalent convection coefficient for radiation $h_{\mathrm{rad},0}$ calculated based on T_0 and T_sur.

$$t_\mathrm{c} = \tau = \frac{V\rho c}{A_\mathrm{s}h_{\mathrm{rad},0}} \tag{41}$$

which is exactly the same as we have seen in Equation 26 for lumped capacitance for convection! Indeed, if we were to calculate lumped capacitance with radiation as if it was convection using $h_{\mathrm{rad},0}$ the results would be very similar, as long as the ratio between T_0 and T_sur is not too far from 1.

4 Lumped Capacitance with Latent Heat

4.1 Example: Snow Making

Consider a snow-making machine. It projects droplets with diameter of 0.5 mm into air at -20°C. If the initial temperature of the droplet is 6°C, how long does it take to become ice? Thermal properties of water or ice at 0°C can be found at Table A6 in the textbook: density $\rho = 1000\frac{\text{kg}}{\text{m}^3}$, specific heat $c = 4217\frac{\text{J}}{\text{kgK}}$ and the latent heat of ice melting into water is $i_{\text{sf}} = 333.7\times10^3\frac{\text{J}}{\text{kg}}$. Assume the droplet temperature is uniform through the volume, and the convection coefficient is $h = 200\frac{\text{W}}{\text{m}^2\text{K}}$.

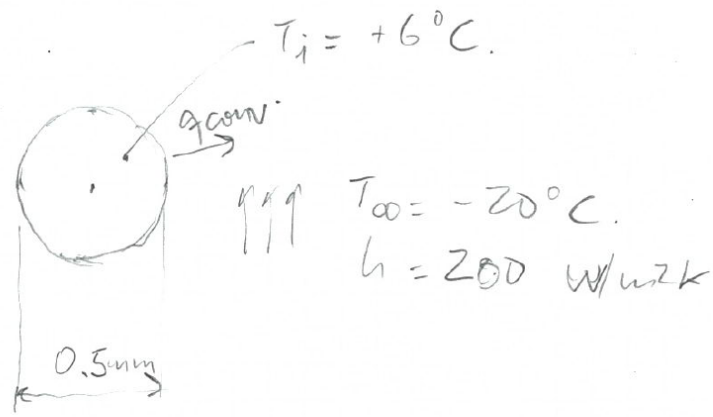


Figure 4: Schematic of a droplet of 0.5-mm diameter at initial temperate of 6°C being cooled by the environment of -20°C through convection

There are two stages from the liquid state to solid state:

- Stage I: cooling of liquid droplet ($0 < t < t_1$, $T > T_{\text{sf}} = 0\,°\text{C}$)
- Stage II: solidification of droplet ($t_1 < t < t_2$, $T = T_{\text{sf}}0\,°\text{C}$)

where t_1 is the time to first reach freezing temperature, and t_2 the time at which the droplet froze completely. Calling t_{sf} the time spent in Stage II

$$t_2 = t_1 + t_{\text{sf}} \tag{42}$$

4.1.1 Stage I: Cooling of liquid droplet

From Equation 33, and the definition of the time constant (Equation 26) we can solve for time t:

$$t = -\tau \ln \frac{T - T_\infty}{T_0 - T_\infty} \tag{43}$$

$$\tau = \frac{\rho c V}{A_s h} = \frac{\rho c \frac{4\pi R^3}{3}}{4\pi R^2 h} = \frac{1}{3}\frac{\rho c R}{h} \tag{44}$$

$T = 0°\text{C}$, $T_\infty = -20°\text{C}$, and $T_0 = 6°\text{C}$, $\Rightarrow \ln \frac{T-T_\infty}{T_0-T_\infty} = -0.26$

$$\tau = \frac{1}{3}\frac{\rho c R}{h} = \frac{1}{3}\frac{1000\frac{\text{kg}}{\text{m}^3} \times 4217\frac{\text{J}}{\text{kgK}} \times 0.25 \times 10^{-3}\text{m}}{200\frac{\text{W}}{\text{m}^2\text{K}}} = 1.76\text{s}$$

$$t_1 = -\tau \ln \frac{T_{\text{sf}} - T_\infty}{T_0 - T_\infty} = 0.46\text{s}$$

4.1.2 Stage II: Solidification of droplet

In stage II we cannot use Equation 33 because the droplet temperature is constant during freezing. In this case we need to go back to basics and apply a batch energy balance between time t_1 and time t_2:

$$\cancel{Q_{\text{in}}} - Q_{\text{out}} + \cancel{Q_{\text{gen}}} = Q_{\text{st}} \tag{45}$$

$$Q_{\text{st}} = -V\rho i_{\text{sf}} \tag{46}$$

$$Q_{\text{out}} = A_s h(T_{\text{sf}} - T_\infty)t_{\text{sf}} \tag{47}$$

resulting in

$$-\rho V i_{\text{sf}} = -A_s h(T_{\text{sf}} - T_\infty)t_{\text{sf}} \tag{48}$$

$$t_{\text{sf}} = \frac{V\rho}{A_s h}\frac{i_{\text{sf}}}{(T_{\text{sf}} - T_\infty)} \tag{49}$$

for a sphere: $V = \frac{4\pi R^3}{3} A_s = 4\pi R^2$, thus:

$$\begin{aligned} t_{\text{sf}} &= \frac{1}{3}\frac{\rho i_{\text{sf}} R}{h(T - T_\infty)} \\ &= \frac{1}{3}\frac{1000\frac{\text{kg}}{\text{m}^3} \times 333.7 \times 10^3\frac{\text{J}}{\text{kg}} \times 0.25 \times 10^{-3}\text{m}}{200\frac{\text{W}}{\text{m}^2\text{K}}(10\,°\text{C} + 20\,°\text{C})} = 6.95\text{s} \end{aligned}$$

The total time for freezing the droplet is then $t_2 = t_1 + t_{\text{sf}} = 7.41\text{s}$, most of which spent in freezing at 0°C.

5 Lumped Capacitance with Multiple Elements

Lumped capacitance is also useful when the control volume encloses different elements with different masses and enthalpies. For example consider the cooling of a car engine assuming lumped capacitance. As long as we can assume that the temperature is uniform and the same for each element of the engine: block, pistons, crankshaft, etc, we can apply the equations we have developed considering the heat stored between time t_1 and time t_2 as:

$$Q_{\text{st}} = \sum_j m_j i_j \bigg|_2 - \sum_j m_j i_j \bigg|_1 \tag{50}$$

or for heat rates:

$$q_{\text{st}} = \frac{d \sum_j m_j i_j}{dt} \tag{51}$$

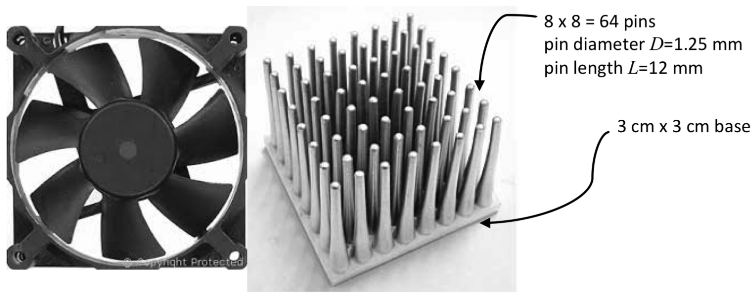
In both cases, the enthalpies correspond to the same uniform temperature of the system inside the control volume. The equations above are valid independently of the presence of phase transformations.

6 Lumped Capacitance with Heat Generation

In many aspects, lumped capacitance with heat generation is the same as we have seen so far, we state a control volume and energy balance, we identify heat transfer mechanisms, and we end up with an ordinary differential equation. In all previous cases, the final temperature of the system was that of the surrounding environment (T_∞ or T_{sur}). When there is heat generation this changes, and the final temperature is a balance between the heat generated and the ability to dissipate that heat.

6.1 Example: Transient heating of a CPU

An old Pentium microprocessor is cooled using the heat sink indicated below, which sits right downstream from a fan. The processor clock speed is limited by the cooling ability of the heat sink. Assume that all cooling happens by convection in the heat sink, and that the processor can be treated as a lumped capacitance. In this problem, the CPU was at 30°Cwhen it was turned on with a clock speed of 200 MHz, the heat sink area is A_s =2.9 10^{-3} m^2, the air blown by the fan is at 20°C, the heat transfer coefficient is h =100 W/m^{2}K, the mass of the whole system is 10 g, and the specific heat is c =1100 J/kg K.



Clock Speed	Power
75 MHz	8.0 W
90 MHz	9.0 W
100 MHz	10.1 W
120 MHz	11.9 W
133 MHz	11.2 W
150 MHz	11.6 W
166 MHz	14.5 W
200 MHz	15.5 W

Figure 5: Fan and heat sink for cooling a CPU. The table on the right relates the CPU clock speed to the power consumed

The energy rate balance in this case is

$$\cancel{q_\text{in}} - q_\text{out} + q_\text{gen} = q_\text{st} \tag{52}$$

The heat transfer mechanisms involved are

$$q_\text{out} = q_\text{conv} = A_\text{s} h (T - T_\infty) \tag{53}$$

$$q_\text{st} = mc\frac{dT}{dt} \tag{54}$$

where m is the mass of the chip. Also, we know q_gen =15.5 W from Figure 5. Putting the mechanisms and the energy balance together results in

$$-A_\text{s} h (T - T_\infty) + q_\text{gen} = mc\frac{dT}{dt} \tag{55}$$

which is a linear ordinary differential equation of first order, non-homogeneous. In this case we need only one initial condition, which is the initial temperature T_0 =30°C. We can rewrite Equation 55, and the problem is completely captured as

$$\begin{cases} \dfrac{dT}{dt} + KT = KT_\infty + \dfrac{q_\text{gen}}{mc} \\ T(0) = T_0 \end{cases} \tag{56}$$

where

$$K = \frac{A_s h}{mc} \tag{57}$$

6.1.1 Steady-state temperature with heat generation

The system of equations 56 is very similar to Equation 16, except that the constant term that makes the equation non-homogeneous now also includes a heat generation term. In steady state, (when $dT/dt = 0$), the final temperature of this system is not any more the temperature of the fluid! We know this from experience: the CPU gets hot when turned on.

Assuming $dT/dt = 0$ inEquation 56 we can solve for temperature in steady state T_{ss}:

$$T_{ss} = T_\infty + \frac{q_{gen}}{Kmc} \tag{58}$$

$$T_{ss} = T_\infty + \frac{q_{gen}}{A_s h} \tag{59}$$

which for the values of this problem corresponds to 20°C+53.45°C=73.45°C. If during maximum power use, the fan stopped, and the CPU could only be cooled by natural convection ($h \approx 10$ W/m^{2}K, the superheat due to heat generation would be 534.5°C (for a steady-state temperature of 554.5°C). The chip would obviously be destroyed well before reaching steady-state.

6.1.2 Solution of the differential equation

A similar change of variable is convenient to solve this problem:

$$\theta(t) = T(t) - T_{ss} \tag{60}$$

This change of variable is essentially the same as in Equation 18, in which the steady-state temperature is subtracted. The difference is that in Equation 18 the steady state temperature is T_∞, while now the steady-state temperature is given by Equation 59.

The solution with the change of variable is the same as before:

$$\theta = \theta_0 \exp\left(-\frac{t}{\tau}\right) \tag{61}$$

with

$$\tau = \frac{1}{K} = \frac{mc}{A_\mathrm{s}h} \tag{62}$$

resulting in

$$T(t) = T_\mathrm{ss} + (T_0 - T_\mathrm{ss})\exp\left(-\frac{t}{\tau}\right) \tag{63}$$

The time constant in this problem is 37.93 s, indicating the order of magnitude of time to reach steady state. If the fan did not turn on when the chip started to use power, the value of h would be ten times smaller, and the time constant would be ten times larger: 379.3 s.

6.1.3 Time to failure

If the fan stops during peak power use, then the chip temperature will rise until reaching failure. For this old chip, let's consider failure temperature T_fail =100°C; modern gaming chips with smaller features should never exceed 80°C.

To calculate the time to reach failure, we need to solve Equation 63 for $t = t_\mathrm{fail}$ when $T = T_\mathrm{fail}$:

$$t_\mathrm{fail} = \tau \ln \frac{T_0 - T_\mathrm{ss}}{T_\mathrm{fail} - T_\mathrm{ss}} \tag{64}$$

In this case, if we assume that the chip was in steady-state at peak power, the starting temperature for the temperature rise in this calculation would be T_0 =73.45°C, and the time constant associated with it would be 379.3 s, corresponding to cooling only through natural convection. The steady-state temperature in this case is only theoretical, since the chip will fail and stop drawing power well before reaching it: T_ss =554.5°C. Replacing these values into Equation 64, we obtain t_fail =21.53 s. We see that although the time constant to reach steady state is long, in this case, the time to failure is much shorter. This is because the chip, when cooled by the fan, is very close to the failure temperature already, and it needs to climb only 16°C, not hundreds, to fail.

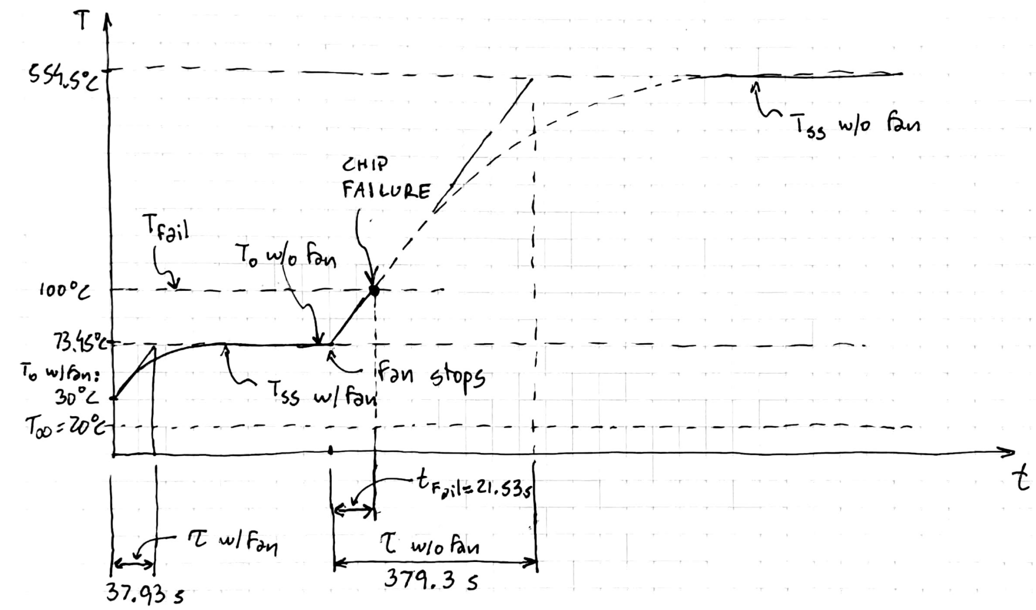


Figure 6: Temperature evolution for normal operation and failure when fan stops for an old Pentium CPU

Acknowledgement

This set of notes is based on class notes from previous years by Prof. Mendez, and was first typed and illustrated by PhD student Ying Wang as part of her Fraser and Shirley Russell Teaching Fellowship.