**Plumes in rotating fluid and their transformation into tornados**

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We examine the evolution of buoyant axisymmetric plumes as they are influenced by background rotation in a uniform density ambient fluid. The source Rossby number is sufficiently large that rotation does not directly affect the plume at early times. However, on a time scale on the order of half a rotation period, the plume becomes deflected from the vertical axis. For some experiments and simulations, the deflection persists and the flow precesses about the vertical axis. In other cases, shortly after being deflected, the plume laminarizes near the source to form a near-vertical columnar vortex, which we refer to as a “tornado”. Tornado formation is intermittent, appearing in some experiments and not in others even if the source and background rotation parameters are identical. Simulations reveal that this is a consequence of competing dynamics that occur on comparable time-scales. As a consequence of entrainment, vertical vorticity builds up within the plume reducing the Rossby number and suppressing vertical motion at distances progressively closer to the source. Meanwhile, the azimuthal flow around the vicinity of the source increases, which acts to suppress turbulence in the near-source flow. Although the former process ultimately acts to deflect the plume off-axis, in some instances the swirl around the source succeeds in laminarizing the flow, resulting in tornado formation. Experiments and simulations show that tornado formation is more likely if the plume at the source is “lazy”, meaning it has a deficit of momentum with respect to buoyancy compared with that for a pure plume.

1. Introduction

A plume consists of light fluid that rises in an ambient fluid having relatively larger density or, equivalently, of a dense fluid that descends in a lighter ambient fluid, these cases being physically equivalent in a Boussinesq fluid. In most environmental and industrial circumstances, plumes are turbulent and so entrain ambient fluid as they rise or descend. This serves both to reduce the density contrast between the plume and ambient fluid and also to lower the vertical speed of the plume with distance from its source. Morton \textit{et al.} (1956) derived an elegant model predicting the vertical change in volume and momentum flux (and consequently the change in radius, vertical velocity and
buoyancy) of a statistically steady turbulent axisymmetric plume in a stationary, uniform density ambient fluid. Their model assumed that the radial speed of ambient fluid being drawn into the plume at a particular height was proportional to the vertical speed of the plume itself at that height. As such, the ambient fluid played a passive role by supplying fluid, but otherwise had no dynamic influence on the plume evolution. Observations of plumes emanating from hydrothermal vents in the oceanic abyss (Lupton et al. 1985) as well as those associated with deep convective wintertime mixing in high-latitude seas (MEDOC Group 1970; Clarke & Gascard 1983; Schott & Leaman 1991) motivated laboratory experiments (Maxworthy & Narimousa 1994; Whitehead et al. 1996; Fernando et al. 1998) and simulations (Jones & Marshall 1993; Pal & Chalamalla 2020) that examined the influence of background rotation upon convection from a localized or distributed source. In studies of convection from a localized source, it was predicted that if the ambient fluid was sufficiently deep, then the Rossby number associated with the fluid in the plume would become order unity at a distance $H_f \equiv (B_0/f^3)^{1/4}$ from an effective point source, in which $B_0$ is the buoyancy flux and $f$ is the Coriolis parameter (Jones & Marshall 1993; Fernando et al. 1998). Beyond this point, the plume exhibited noticeable anticyclonic rotation as it ceased to expand radially. In addition to suppressing ambient fluid entrainment, rotation suppressed three-dimensional turbulent motions, effectively laminarizing the plume beyond the distance $H_f$ (Speer & Marshall 1995). The resulting column of dense rotating fluid was prone to baroclinic instability, resulting in the breakup of the column into eddies.

In the case of deep-ocean convection, the ratio of the width to the depth of the convecting region is large. And so one may not expect the dynamic influence of the ambient fluid to be significant. However, the 2010 Deepwater Horizon accident in the Gulf of Mexico has inspired renewed interest in the dynamics of rotating plumes. Over the course of 87 days, oil was continuously discharged at the ocean floor rising as a plume from an effective point source. The pathway of oil toward the surface was influenced by the multiphase composition of the effluent, the ambient stratification and likely by the Earth’s rotation (Fabregat Tomàs et al. 2015, 2016, 2017; Frank et al. 2017). Through numerical simulations examining a moderate Rossby number plume impinging upon a stratified layer, Fabregat Tomàs et al. (2016) noted that rotation acted to set up an adverse vertical pressure gradient within the plume that caused the fluid near the source to be deflected from the vertical and consequently to precess anticyclonically. This deflection and anticyclonic precession was also observed in laboratory experiments of plumes in a rotating uniform density ambient fluid (Frank et al. 2017). They found the mean deflection of the plume from the vertical to be $30^\circ \pm 11^\circ$ and the mean precession frequency to be approximately $0.2f$ ($= 0.4\Omega$, in which $\Omega$ is the background angular rotation frequency).

The observation of the deflection and precession of a rotating plume suggests the ambient flow near the plume may play a more dynamic role in the plume evolution than simply being a source of entrained fluid. In part, the radial flow of ambient fluid toward the entraining plume would be deflected by Coriolis forces so as to set up a cyclonic circulation around the plume, which could act to suppress entrainment (Helfrich & Battisti 1991; Fernando et al. 1998). In a recent numerical examination of laminar rotating plumes emanating from the base of cylindrical domain, Martins et al. (2020) noted inward spiraling motion toward the source in the bottom boundary layer of the domain. Such spiraling motion is anticipated at all depths surrounding a turbulently entraining plume. Furthermore, because a plume differentially entrains fluid with depth, the radial motion of the ambient is expected to have vertical shear. However, the relatively slow far-field ambient motion is strongly influenced by rotation, which has the effect of
Figure 1. Side-view snapshots of rotating plume experiments in which the dyed plume a) develops into a tornado (four left-most panels) and b) begins to precess without forming a tornado (three stacked images on the right). The experiment shown in a) has the parameters given by C1 in table 2. Each panel in a) shows an area around the plume that is 30 cm wide and 80 cm deep with the nozzle at the top of each frame. The experiment shown in b) has the same parameters as C1 except that the source reduced gravity, $g'_{00} = 47 \text{ cm/s}^2$, is $4 \text{ cm/s}^2$ smaller. Only the flow to a depth 15 cm below the source is shown in these cases. In (a) the horizontal dark feature near the top of the images shows where the surface (seen from below) intersects the rear wall of the tank.

suppressing vertical shear. Therefore the ambient flow is horizontally divergent, and this necessarily should lead to vertical motion in the vicinity surrounding the plume. These motions and their consequent impact upon the plume evolution near the source are examined in detail here. In particular, we show that, under some circumstances, the ambient cyclonic circulation that builds up around the plume can act effectively to reduce the local Rossby number so as to laminarize the plume first near the source and then extending far from the source to form a coherent vortex, which we refer to as a “tornado”. For example, figure 1 shows snapshots taken from two experiments of rotating plumes, one in which a tornado forms (figure 1a) and one with similar parameters in which the plume ultimately precesses with no tornado formation (figure 1b). This phenomenon occurred most often in experiments and simulations of “lazy” plumes meaning that the momentum flux relative to the buoyancy flux at the source was smaller than that of a pure plume (Hunt & Kaye 2005).

The paper is organized as follows. Some basic theoretical concepts for plumes and rotational effects are reviewed in §2. In §3, the set-up and analysis of experiments are described with some quantitative results presented therein, specifically the characterization of which source parameters could result in tornado formation. The details of the numerical simulations and the analysis of their results are given in §4. In light of the experiment and simulation analyses, general conditions leading to possible tornado formation are discussed in §5. Conclusions are provided in §6.

2. Theoretical Preliminaries

Here we review theories essential for the interpretation and analysis of the experiments and simulations. First we review the theory for statistically steady pure and lazy plumes in a stationary ambient. Thereafter we consider the influence of rotation upon plumes and
the surrounding ambient fluid. Finally, we review the theory for the stability of “trailing vortices” which, like the tornado, are columnar vortices having axial as well as azimuthal velocity.

2.1. Plume theory

In the absence of rotation, the properties of a statistically steady pure plume can be estimated from the “MTT” model of Morton et al. (1956). For convenience, we suppose the plume consists of buoyant fluid rising from a localized source.

In a uniform density ambient fluid, the reduced gravity, $g'$, and vertical velocity, $w$, of the plume are here assumed to have a Gaussian structure such that

$$w(z,r) = W_0(z) \exp(-r^2/b^2), \quad g'(z,r) = g_0'(z) \exp(-r^2/b^2),$$

(2.1)

in which $b = b(z)$ is a measure of the plume width that changes with vertical distance, $z$, from the source, as shown in figure 2a. The centreline velocity, $W_0$, and reduced gravity, $g_0'$, as well as $b$ satisfy the coupled equations (Morton et al. 1956)

$$\frac{d}{dz}(b^2W_0) = 2\alpha b W_0, \quad \frac{d}{dz}(b^2W_0^2) = 2b^2 g_0', \quad B_0 \equiv \frac{1}{2} \pi b^2 W_0 g_0' = \text{constant},$$

(2.2)

respectively representing conservation of mass, momentum and buoyancy. In the last equation the buoyancy flux, $B_0$, is unchanging with distance from the source because the ambient fluid has uniform density. The entrainment constant, $\alpha$, can vary depending on whether the flow is a jet ($g_0' = 0$) or plume, with a typical value for the latter being $\alpha \simeq 0.1$. If the plume originates from a point source of buoyancy at $Z = 0$, a self-similar solution of (2.2) can be found:

$$b = \frac{6\alpha}{5} Z, \quad W_0 = \left(\frac{25}{12\pi} \frac{1}{\alpha^2}\right)^{1/3} B_0^{1/3} Z^{-1/3}, \quad g_0' = \frac{2}{3} \left(\frac{25}{12\pi} \frac{1}{\alpha^2}\right)^{2/3} B_0^{2/3} Z^{-5/3},$$

(2.3)

in which the distance $Z$ from the point source is illustrated schematically in figure 2a. The corresponding volume and momentum fluxes obey the respective power laws

$$Q(Z) = \pi b^2 W_0 = 3 \left(\frac{25}{12\pi} \frac{\alpha^2}{25}\right)^{2/3} B_0^{1/3} Z^{5/3}, \quad M(Z) = \frac{1}{2} \pi b^2 W_0^2 = \frac{3}{2} \left[\frac{25}{12\pi} \frac{\alpha^2}{25}\right]^{1/3} B_0^{2/3} Z^{4/3}.$$

(2.4)

A plume originating from a nozzle of finite radius, $b_0$, with volume flux $Q_0$ has mean vertical velocity at the source of $w_0 = Q_0/(\pi b_0^2)$. The corresponding source Reynolds number, $\text{Re}_0 = w_0 b_0/\nu$, is assumed to be sufficiently large ($\text{Re}_0 \gtrsim 100$) that the flow is turbulent. The source buoyancy relative to its source momentum is assessed by the
source Richardson number, $Ri_0$, defined by (Hunt & Kaye 2005)

$$Ri_0 \equiv \frac{5}{4\alpha} \frac{g'_0 b_0}{w_0^2}.$$  \hspace{1cm} (2.5)

Here $g'_0$ is the reduced gravity at the source located at $z = 0$. The flow from a finite-sized source is equivalent to that of a pure plume if $Ri_0 = 1$, in which case the nozzle opening is located at a distance $z_v = 5b_0/(6\alpha)$ above the virtual point source, so that $z = Z + z_v$ (see figure 2a).

For a non-pure plume, consideration of the height of the virtual origin, $z_0$, and the effective source vertical velocity, $w_{0*}$, is crucial for the examination of possible laminarization of a rotating plume, particularly if it is lazy as characterized by $Ri_0 > 1$. In this circumstance there is a deficit of momentum compared to buoyancy relative to their ratio in a pure plume (Caulfield 1991; Hunt & Kaye 2001, 2005). For a lazy plume, the vertical velocity initially increases upon leaving the nozzle as the plume adjusts its momentum flux relative to its buoyancy flux through reducing or even suppressing entrainment until the local, $z$-dependent plume Richardson number

$$Ri_p(z) \equiv \frac{5}{4\alpha} \frac{g'_0 b}{W_0^2}$$  \hspace{1cm} (2.6)

approaches that of a pure plume: $Ri_p \to 1$. The maximum vertical velocity, $W_{00}$, is reached at a distance from the source where $Ri_p = 5/4$, so that

$$W_{00} = \frac{4^{2/5}}{5^{1/2}} w_0 \frac{Ri_0^{1/2}}{(Ri_0 - 1)^{1/10}} \simeq 0.78w_0 \frac{Ri_0^{1/2}}{(Ri_0 - 1)^{1/10}}.$$  \hspace{1cm} (2.7)

The location, $z_0$, of the virtual origin of the far-field pure plume is situated above from the source if $Ri_0$ is sufficiently large; its location is close to where $b(z) = W_0$. Likewise, the volume and momentum fluxes can be measured as functions of $Z \equiv z - z_0$ and fit to power laws far above $z = z_0$ (see figure 2b). Then, using (2.4), the effective source vertical velocity is

$$w_{0*} = 2M(Z = z_v)/Q(Z = z_v),$$  \hspace{1cm} (2.8)

in which $z_v = 5b_0/(6\alpha)$ is distance from the virtual origin (at $z = z_0$) where the equivalent pure plume has the same radius as the source (see figure 2b). In practice, the prediction (2.7) is found to be close to the measured estimate using (2.8). On this basis, we suppose $W_{00} \simeq w_{0*}$ and use (2.8) to characterise the effective source vertical velocity.

In the case of extremely lazy plumes, $Ri_0 \gg 1$, Hunt & Kaye (2005) derived approximate formulae for the change with $z$ of fluxes near the source. In particular, they showed that the vertical velocity increases with distance $z$ from source according to

$$w \simeq w_0(1 + 4g'_0 z/w_0^2)^{1/2},$$  \hspace{1cm} (2.9)

a result that could be derived from the MTT equations (2.2) by setting the entrainment coefficient, $\alpha$, to zero, in which case $g'_0$ is constant. Hence, even for a moderately lazy plume, entrainment is expected to be reduced between the source and $z_0$, if not suppressed altogether if $Ri_0 \gg 1$. 
2.2. Effects of rotation

Assuming axisymmetry, the equations for continuity, and radial and azimuthal momentum conservation for an inviscid fluid are given, respectively, by

$$\frac{1}{r} \frac{\partial}{\partial r} \left( ru_r \right) + \frac{\partial w}{\partial z} = 0,$$

(2.10)

$$\frac{\partial u_r}{\partial t} = -u_r \frac{\partial u_r}{\partial r} - w \frac{\partial u_r}{\partial z} + fu_\theta + \frac{1}{r} u_\theta^2 - \frac{\partial P}{\partial r} \simeq fu_\theta + \frac{1}{r} u_\theta^2 - \frac{\partial P}{\partial r},$$

(2.11)

$$\frac{\partial u_\theta}{\partial t} = -w \frac{\partial u_\theta}{\partial z} - (f + \zeta) u_r \simeq -fu_r,$$

(2.12)

in which $f = 2\Omega$ is the Coriolis parameter, defined in terms of the background angular velocity $\Omega$. Here we have defined $P \equiv p/\rho_a$ to be the dynamic pressure normalized by the ambient fluid density, and $\zeta = [\partial_r (ru_\theta)]/r$ to be the vertical component of vorticity. The rightmost approximations in (2.11) and (2.12) assume that outside the plume the advection terms and vertical vorticity are negligible, as is confirmed by analysis of numerical simulations.

The relative importance of the Coriolis force acting on an eddy within the plume having size on the order $b$ and speed on the order of the centreline vertical velocity, $W_0$, is typically assessed by a $z$-dependent Rossby number defined as (Speer & Marshall 1995; Fernando et al. 1998)

$$Ro(z) \equiv \frac{W_0(z)}{fb(z)}.$$

(2.13)

Above the virtual origin (possibly lying above the source if the plume is sufficiently lazy), $Ro(z)$ decreases with distance from the source as $W_0$ decreases and $b$ increases with increasing $z$. For a pure plume, substitution of (2.3) into (2.13) yields

$$Ro(Z) = \frac{5}{6\alpha_f} \left( \frac{25}{12\pi \alpha^2} \right)^{1/3} \frac{B_0^{1/3}}{Z^{4/3}} \simeq 33.7 \frac{B_0^{1/3}}{f Z^{4/3}},$$

(2.14)

in which $Z$ is the distance from the virtual origin (see figure 2a).

Experiments by Fernando et al. (1998) showed that the radius of a plume ceased to increase with $z$ after reaching a critical distance from the virtual origin

$$H_f \simeq (5.5 \pm 0.5) \left( \frac{B_0}{f^3} \right)^{1/4}.$$

(2.15)

Substituting this into (2.14) gives the corresponding Rossby number at this distance to be approximately $Ro_c \simeq 3.4$. Thus, at least during its initial evolution, rotation has negligible influence upon the plume at distances $z$ where $Ro(z) \gg Ro_c$. As the plume evolves, however, rotation non-negligibly influences the flow within the plume due to entrainment of the surrounding ambient fluid. If we suppose that the approximation in (2.12) holds close to edge of the plume then, at least during its early time evolution, the characteristic azimuthal velocity $U_\theta$ within the plume should increase linearly with time according to

$$U_\theta/W_0 \sim \alpha_f ft,$$

(2.16)

in which $\alpha_f$ is an entrainment coefficient. For a lazy plume, $\alpha_f$ should increase with $z$ from a reduced value where $Ri_{p} \gg 1$ near the source to the usual $\alpha \simeq 0.1$ where the flow acts more like a pure plume at distances $z$ where $Ro(z) \gg Ro_c$. Equation (2.16) suggests that on a time-scale on the order $1/(\alpha_f ft)$, the characteristic azimuthal velocity will become comparable to the centreline vertical velocity, $W_0$. Furthermore, there should be
a corresponding linear increase in time of the characteristic vertical vorticity, \( \zeta_0 \sim 2U_\theta/b \), within the plume. Thus we define a plume Rossby number that depends on time as well as \( z \):

\[
\text{Ro}_p(z,t) = \frac{|U|}{(f + \zeta_0)b\theta},
\]

(2.17)

in which \( U = (U_\theta, W_0) \) and \( b_\theta \) is the radius at which the azimuthal velocity equals \( U_\theta \). In our analysis of experiments and simulations we define \( U_\theta \) to be the maximum azimuthally averaged azimuthal velocity. Even if the source Rossby number, \( \text{Ro}_0 = w_0/(fb_0) \), is large, vorticity is expected to dominate the plume Rossby number near the source after a time on the order of \( b_0/(2\alpha_f w_0^*) \).

These considerations suggest the process by which a tornado may form. The increase in swirl surrounding the source can act to laminarize the flow in its vicinity. On the other hand, the concurrent decrease in the plume Rossby number leads to an inhibition of vertical motion. This develops first aloft where the plume Rossby number is already small. As time progresses, \( \text{Ro}_p \) becomes reduced at a progressively closer distance to the source. If reaching the source before the tornado can develop, the source fluid deflects from the vertical, as examined in simulations by Fabregat Tomàs et al. (2016). For a moderately lazy plume, there are two reasons to expect that tornado formation may nonetheless occur. Even in the absence of rotation, turbulent entrainment into a lazy plume is reduced near the source. For a rotating lazy plume, this suggests that the flow leaving the nozzle would be more readily laminarized by swirling flow around the source. Of course, lower entrainment also results in a lower rate of increase in swirl. Nonetheless, what is entrained undergoes a positive vertical strain between the source and the virtual origin situated above the source at \( z = z_0 \) where \( W_0(z_0) = W_{00} \approx w_0^* > w_0 \) (figure 2b). The strain acting upon the (rotating) fluid leaving the nozzle and what fluid is entrained near the source can lead to increased vertical vorticity within the plume on a faster time-scale depending upon the local value of the entrainment \( \alpha_f \). This reasoning suggests that tornado formation is less likely to occur for very lazy plumes (\( \text{Ro}_0 \gg 1 \)) because \( \alpha_f \to 0 \), suppressing entrainment of vorticity from the ambient fluid.

Consideration of the plume alone is insufficient to encapsulate this problem. This is because the inhibition of vertical motion in the plume where \( \text{Ro}_p \sim \text{Ro}_c \) results in radial outflows that affect the evolution of the surrounding ambient fluid. This in turn affects the flow surrounding the source. For example, the radial velocity field surrounding a non-rotating plume is given by

\[
u_r(r, z) = -\alpha \frac{bW_0}{r} \propto \frac{z^{2/3}}{r}.
\]

(2.18)

The inverse radial dependence of the radial velocity follows immediately from (2.10) if one assumes no vertical strain. This flow exhibits vertical shear, which is inhibited in the presence of strong background rotation, as assessed by the spatial- and time-dependent ambient Rossby number

\[
\text{Ro}_a(r,z,t) = \frac{[u_h]}{fr},
\]

(2.19)

in which \( u_h = (u_r, u_\theta) \) and the overline denotes azimuthal averaging. As will be shown, vorticity and vertical velocity in the ambient fluid are negligible compared respectively with \( f \) and \( u_h \), and so are not included in the definition of \( \text{Ro}_a \). Clearly \( \text{Ro}_a \) is smaller with increasing radius \( r \) due both to the presence of \( r \) in the denominator and also due to the decrease in the horizontal velocity with radial distance, as in (2.18). Far from the plume where the flow is slow and the corresponding ambient Rossby number is small, the
flow is expected to be nearly invariant in the vertical. Near the plume, vertical shear is expected due to the differential horizontal entrainment with \( z \), as in (2.18). Furthermore, the radial pressure gradient is not expected to be negligible because it changes in response to Coriolis and centripetal forces. As a consequence of all these effects, we will show that there is vertical strain in the ambient fluid, which modifies the power law dependence of \( u_r \) upon \( r \) from the inverse relationship in (2.18). We will also show that the magnitude of \( u_r \) well outside the plume at fixed \( r \) and \( z \) increases linearly in time, in contrast with non-rotating plumes for which \( u_r \) is time-independent. Hence, from (2.12) the ambient azimuthal velocity increases quadratically in time, in contrast with the linear increase in time of the azimuthal velocity within the plume, as predicted by (2.16).

2.3. Vortex stability

While rotation influences the early time evolution of the plume and ambient fluid possibly preconditioning the flow to transition into a tornado, another consideration for whether a tornado persists is its stability. If a tornado forms, then there is necessarily an axial as well as an azimuthal flow within the tornado as a consequence of buoyant fluid from the source continuing to rise through the column. We assess the persistence of a tornado by considering the stability of such a flow.

There have been several studies of the stability of a columnar vortex having both azimuthal and axial flow (though assuming uniform density fluid inside and outside the vortex). These investigations were motivated originally by the study of trailing vortices behind an airplane wing (Batchelor 1964), and more recently in consideration of stretched vortices in turbulent flow (Delbende et al. 2002). Following Batchelor (1964), the azimuthal velocity field is represented by

\[
\begin{align*}
  u_{\theta t} &= \frac{\Gamma}{2\pi r} \left[ 1 - \exp\left( -\frac{r^2}{b_t^2} \right) \right].
\end{align*}
\] (2.20)

Here, \( b_t \) is a measure of the radius of the vortex, and \( \Gamma \) represents the (constant) circulation outside the tornado. The corresponding maximum azimuthal velocity is (Lessen et al. 1974)

\[
U_{\theta t} \simeq 0.639 \frac{\Gamma}{2\pi b_t}.
\] (2.21)

Taking the axial direction to be vertical, both the vertical vorticity and vertical velocity have a Gaussian structure in \( r \). In particular, the vertical velocity has radial structure given by \( w_t = W_{0t} \exp(-r^2/b_t^2) \). The ratio of the characteristic azimuthal velocity, \( \Gamma/(2\pi b_t) \), to \( W_{0t} \) defines the swirl of the vortex:

\[
q = \frac{\Gamma}{2\pi b_t W_{0t}}.
\] (2.22)

Alternately, the swirl can be represented in terms of the maximum azimuthal velocity by

\[
q^* \equiv \frac{U_{\theta t}}{W_{0t}} \simeq 0.639q.
\] (2.23)

For the purposes of our analyses below, the latter definition is more convenient because, unlike (2.20), we find that the far field azimuthal velocity does not vary inversely with \( r \).

Numerous theoretical studies of the temporal and absolute/convective instability of the Batchelor vortex with and without viscous effects have shown that the vortex is stable if \( q \gtrsim 1.5 \), hence \( q^* \gtrsim 1 \) (e.g. see Lessen et al. (1974); Lessen & Paillet (1974); Stewartson & Leibovich (1987); Delbende et al. (1998)). Thus, for a tornado to persist, the azimuthal velocity should be at least on the order of the vertical velocity.
3. Laboratory Experiments

Here we describe the setup, analysis methods and results of laboratory experiments. The experiments were performed in four institutions (designated by ‘A’ - U. Alberta, ‘C’ - U. Cambridge, ‘L’ - ENS de Lyon, and ‘M’ - U. Aix-Marseille) using tanks with different geometries, as listed in table 1. The set-up of experiments in each institution provided different capabilities in terms of examining the dependence of the plume evolution upon ambient fluid depth and horizontal tank geometry. The L- and M-experiments used particle image velocimetry (PIV) to measure ambient flow velocities in horizontal and vertical laser light sheets cutting through the plume. One interesting feature of the experiments is that for certain experimental parameters the plume was found sometimes to transform into a tornado. An example of the formation of a tornado and a counterpart experiment in which a tornado did not form is shown in figure 1. The manifestation of a tornado was found to be repeatable particularly in the L-experiments. This repeatability was partially a consequence of the long (2 hour) spin-up times and the cylindrical inner tank geometry, both of which ensured solid body rotation was achieved well before the start of an experiment. That said, the analyses that follow show that even if a tornado can develop, it may not do so as a consequence of rotationally influenced random turbulent motions that act against its formation.

3.1. Setup of experiments

Although the theory above (and simulations which follow) described an upward propagating plume of buoyant fluid, in laboratory experiments it was convenient to inject negatively buoyant fluid downward. Under the Boussinesq approximation, the dynamics governing the plume evolution are the same. In all cases, a plume of saline water from a reservoir was injected downward through a nozzle into an ambient fluid of fresh water, as indicated in the schematic shown in figure 3. The nozzle consisted of an expansion chamber behind a small opening of radius $b_0$ covered with a fine mesh (Hunt & Linden 2001), which ensured that fluid leaving the nozzle was turbulent upon exiting the source. In the C-experiments, the nozzle radius was $b_0 = 0.375 \text{ cm}$, whereas in the other experiments $b_0 = 0.2 \text{ cm}$. In some experiments (M), the reservoir was suspended above the tank with fluid passing through a valve that controlled the flow rate. Typically the surface of the reservoir was at least 1 m above the location of the nozzle tip. During the course of the experiment, the depth of reservoir fluid dropped by at most 1 cm such that the gravity-driven flow rate could be taken as approximately constant. In the C-experiments, the reservoir was a constant-head tank situated above the surface of the ambient fluid. In the other experiments (A, L), the reservoir was situated next to the base of the tank and the flow was driven by a peristaltic pump.

All experiments were performed with the tank and reservoir on a rotating table. Except in a small number of control experiments with no rotation, after filling the tank and reservoir, and priming the fluid between the reservoir and nozzle, the table was set to rotate at a prescribed rate, typically between $|\Omega| = 0.1$ and $0.5 \text{ s}^{-1}$ (in radians per second), although some of the C-experiments had rotation rates as large as $1.9 \text{ s}^{-1}$. The spin-up time was at least 1 hour and up to 4 hours depending upon the fluid depth.

Of the nearly 300 experiments that were performed, a tornado was found to develop in 56 instances. Table 2 lists the 33 distinct parameters of these experiments. The other 23 experiments, which had identical parameters to some of those listed in table 2, were performed as a test of repeatability.
Table 1. Experiment location, geometry and parameters, including the side-to-side tank width/diameter ($L_T$), distance between source and tank bottom ($H_0$), magnitude of the rotation rate ($|\Omega|$), reservoir density ($\rho_0$), and source volume flux ($Q_0$). The last column lists the visualization methods used.

| Location          | Base Geometry | $L_T \ [\text{cm}]$ | $H_0 \ [\text{cm}]$ | $|\Omega| \ [\text{s}^{-1}]$ | $Q_0 \ [\text{cm}^3/\text{s}]$ | $\rho_0 \ [\text{g/cm}^3]$ | Visualization |
|-------------------|---------------|----------------------|----------------------|-----------------------------|-----------------------------|---------------------|---------------|
| U. Alberta (A)    | Square        | 50                   | 30-35                | 0.1-1.1                     | 1.067-1.072                 | dye                 |               |
| U. Cambridge (C)  | Octagonal     | 100                  | 10-110               | 0.1-1.9                     | 1.009-1.055                 | dye                 |               |
|                   | Circular      | 74                   | 40                   | 0.2-1.0                     | 1.002-1.055                 | dye                 |               |
| ENS de Lyon (L)   | Circular      | 90                   | 21                   | 0-0.5                        | 1.066-1.13                  | PIV                 |               |
| Aix-Marseille (M) | Square        | 50                   | 22-63                | 0.3-0.4                      | 1.047-1.132                 | dye+PIV             |               |

Figure 3. Schematic showing the setup of the laboratory experiments and indicating symbols used to represent experiment parameters. The reservoir position here corresponds to the M and C experiments; for the A and L experiments, the reservoir was instead situated below the tank and fluid was fed by means of a peristaltic pump.

3.2. Analysis methods

For all experiments, a camera that rotated with the table was set to look through one of the side-walls of the tank. In the A- and C-experiments, back-lighting passing through a translucent mylar sheet was situated opposite this sideview camera, and the plume itself was dyed with food colouring. This enabled clear visualization of the precession of the rotating plume (Frank et al. 2017) and of the formation of the tornado when it occurred, as shown for example in figure 1(a).

In the L- and M-experiments a vertical laser light sheet shone through the plume centreline in a plane perpendicular to the line-of-sight of the side camera. In both sets of experiments, the tank was seeded with hollow glass microspheres used to visualize and measure the ambient fluid motion through PIV. For the M-experiments the plume fluid was dyed with Rhodamine-B so that the boundary between the turbulent plume and ambient fluid was readily visualized. Images from two M-experiments with and without
rotation are shown in figure 4. These reveal a surprising feature of the rotating plume experiments. Whereas in the non-rotating case (figure 4a) the ambient flow is primarily horizontal toward the plume as expected, the flow in the rotating case for which no tornado developed (figure 4b) exhibits strong vertical circulations associated with the deflection of the plume from the vertical. At the time shown in figure 4b the plume is deflected leftward and behind the laser light sheet. In the vertical plane of the light sheet, the ambient flow is carried leftward and upward along the right flank of the deflected plume. Even though the PIV particles transiently passed through the light sheet, because

| Expt | $H_0$ [cm] | $|\Omega|$ [s$^{-1}$] | $w_0$ [cm/s] | $g_{\theta 0}$ [cm/s$^2$] | $R_{00}$ | $R_{01}$ | $H_f$ [cm] | $T_i$ [s] | $r_c$ [cm] | $U_{\theta t}$ [cm/s] | $b_{\theta t}$ |
|------|-----------|-----------------|-------------|-----------------|--------|--------|---------|-------|--------|-----------------|--------|
| A1†  | 30        | 0.2             | 8.1         | 68              | 101    | 2.6    | 162     | 32    | 18     | --              | --     |
| A2†  | 32        | 0.3             | 4.7         | 72              | 39     | 8.1    | 94      | 21    | 8      | --              | --     |
| A3   | 30        | 0.3             | 5.7         | 62              | 48     | 4.7    | 115     | 21    | 12     | --              | --     |
| C1   | 110       | 0.4             | 15.5        | 51              | 97     | 1.0    | 580     | 28    | 16     | --              | --     |
| C2   | 110       | 1.5             | 16.2        | 55              | 27     | 1.0    | 609     | 11    | 2      | --              | --     |
| C3   | 110       | 1.7             | 16.2        | 55              | 24     | 1.0    | 609     | 10    | 1      | --              | --     |
| C4   | 110       | 1.9             | 16.2        | 57              | 21     | 1.0    | 609     | 9     | 1      | --              | --     |
| L1   | 21        | 0.1             | 5.7         | 67              | 143    | 5.1    | 115     | 49    | 7      | 0.8             | 1.8    |
| L2   | 21        | 0.1             | 5.7         | 130             | 143    | 9.9    | 115     | 58    | 20     | 0.3             | 1.5    |
| L3   | 21        | 0.1             | 11.5        | 130             | 286    | 2.5    | 229     | 69    | 18     | 0.8             | 2.0    |
| L4*  | 21        | 0.1             | 14.3        | 130             | 358    | 1.6    | 287     | 73    | 19     | --              | --     |
| L5   | 21        | 0.2             | 5.7         | 67              | 72     | 5.1    | 115     | 29    | --     | 1.3             | 3.0    |
| L6   | 21        | 0.2             | 14.3        | 130             | 179    | 1.6    | 287     | 43    | 10     | 0.4             | 2.1    |
| L7*  | 21        | 0.2             | 17.2        | 130             | 215    | 1.1    | 344     | 45    | 17     | 1.6             | 2.7    |
| L8*  | 21        | 0.2             | 22.9        | 130             | 286    | 0.6    | 458     | 48    | 17     | 0.9             | 1.7    |
| L9   | 20        | 0.3             | 2.9         | 67              | 24     | 20.3   | 57      | 18    | --     | 1.3             | 2.0    |
| L10  | 21        | 0.3             | 5.7         | 67              | 48     | 5.1    | 115     | 21    | 9      | 1.6             | 2.3    |
| L11  | 21        | 0.3             | 5.7         | 130             | 48     | 9.9    | 115     | 25    | 6      | 0.7             | 1.5    |
| L12  | 20        | 0.3             | 8.6         | 67              | 72     | 2.3    | 172     | 24    | --     | 2.2             | 3.3    |
| L13  | 21        | 0.3             | 8.6         | 130             | 72     | 4.4    | 172     | 28    | 9      | 0.3             | 1.6    |
| L14  | 21        | 0.3             | 11.5        | 130             | 95     | 2.5    | 229     | 30    | 11     | 0.6             | 2.0    |
| L15  | 21        | 0.3             | 14.3        | 130             | 119    | 1.6    | 287     | 32    | 10     | 0.3             | 2.5    |
| L16* | 21        | 0.3             | 22.9        | 130             | 191    | 0.6    | 458     | 36    | 10     | 3.0             | 1.8    |
| L17  | 21        | 0.4             | 2.9         | 67              | 18     | 20.3   | 57      | 15    | --     | 1.4             | 1.6    |
| L18  | 21        | 0.4             | 5.7         | 67              | 36     | 5.1    | 115     | 17    | 8      | --              | --     |
| L19  | 21        | 0.4             | 5.7         | 130             | 36     | 9.9    | 115     | 20    | 12     | 1.0             | 1.7    |
| L20  | 21        | 0.4             | 8.6         | 29              | 54     | 1.0    | 172     | 16    | 7      | 1.3             | 1.1    |
| L21  | 21        | 0.4             | 8.6         | 130             | 54     | 4.4    | 172     | 23    | 9      | 0.4             | 1.3    |
| L22* | 21        | 0.4             | 11.5        | 130             | 72     | 2.5    | 229     | 24    | 8      | 0.5             | 1.6    |
| L23* | 21        | 0.4             | 14.3        | 130             | 90     | 1.6    | 287     | 26    | 10     | --              | --     |
| L24* | 21        | 0.4             | 22.9        | 130             | 143    | 0.6    | 458     | 29    | 12     | 2.8             | 1.9    |
| L25* | 21        | 0.5             | 22.9        | 130             | 115    | 0.6    | 458     | 24    | 8      | 0.9             | 1.2    |

Table 2. Parameters and analysis results for experiments in which a tornado was observed: fluid depth below nozzle ($H_0$), background rotation ($|\Omega|$), source mean velocity ($w_0$), source reduced gravity ($g_{\theta 0}$), source Rossby number ($R_{00}$), source Richardson number ($R_{01}$), source Reynolds number ($R_{02}$), depth predicted by (2.15) where rotation directly influences the corresponding pure plume ($H_f$), time for onset of tornado ($T_i$), distance of tornado centroid from z-axis ($r_c$), maximum azimuthally-averaged azimuthal velocity of tornado ($U_{\theta t}$) and radius from centroid where $U_{\theta t}$ is largest ($b_{\theta t}$). In starred experiments, the tornado developed only briefly before being deflected off axis and devolving back into a turbulent flow. In the daggered experiments, the plume descends into a two-layer fluid with a fresh water upper layer depth $H_1 = 8$ cm and saline fluid below. Dashes indicate that measurements were unavailable.
Figure 4. Side view of M-experiments with $H_0 = 21.5$ cm, $Q_0 = 0.26$ cm$^3$/s, $\rho_0 = 1.067$ g/cm$^3$ and a) with no background rotation at $t = 60$ s and b) with $\Omega = 0.33$ s$^{-1}$ at $t = 20$ s. The left panels show a vertical cross-section through the plume illuminated by a laser light sheet. Velocity vectors computed by PIV are superimposed provided the speed was less than 1 cm/s.

The right panels show particle streak images composed by averaging successive frames over time (top: $50 \leq t \leq 60$ s; bottom: $20 \leq t \leq 25$ s). The horizontal band near $z = -6$ cm in each plot is the remnant of the horizontal laser light sheet whose image was mostly removed by a filter on the side-view camera.

As an example, figure 5 shows the PIV-computed vertical and horizontal velocity fields from three L-experiments. Because the plume was not dyed in the L-experiments, the sheet was approximately 1 mm thick, they passed within the plane long enough to be captured by the camera at its frame rate of 60 fps, as confirmed by the streak images in the right panels of figure 4.

Also for the L- and M-experiments, a second laser created a horizontal light sheet situated 6-8 cm below the nozzle opening. This was viewed with a second camera. In the M-experiments the camera was situated $\approx 1$ m above the tank bottom; in the L-experiments the camera was located below and to the side of the tank and viewed the horizontal motion through an angled mirror situated below the tank. The horizontal and vertical laser light sheets had different colours, and filters were applied to the corresponding viewing cameras so as selectively to block one of the colours. Thus simultaneous horizontal and vertical measurements of velocity around the plume were made possible. PIV analysis was performed using the software “UVMAT” (a free Matlab toolbox available at servforge.legi.grenoble-inp.fr/projects/soft-uvmat).

As an example, figure 5 shows the PIV-computed vertical and horizontal velocity fields from three L-experiments.
Figure 5. Velocities measured using PIV in three L-experiments showing a) no tornado formation in weak background rotation and moderate volume flux (left column), b) tornado formation in moderate background rotation and moderate volume flux (Expt L10, middle column) and c) no tornado formation in moderate background rotation and large volume flux (right column). In all experiments $H_0 = 21 \text{ cm}$ and $\rho_0 = 1.066 \text{ g/cm}^3$. The top row shows velocity (green arrows) and speed (gray scale) from vertical cross-sections at $t = 25 \text{s}$ after the start of an experiment. The velocity magnitude and speed for all three panels are indicated at the bottom of the top-middle plot. Arrows are plotted only if their magnitude is less than $0.6 \text{ cm/s}$. Likewise, the middle row shows velocity and speed from horizontal cross-sections $6 \text{ cm}$ below the source at $t = 25 \text{s}$. These plots are shifted so that the velocity is plotted about the centroid of the speed at $(x_c, y_c)$. The bottom row shows radial time series of the azimuthal velocity which is azimuthally averaged about the centroid. For $0 \leq r \lesssim 1 \text{ cm}$, zero values are assigned to data where the standard deviation of the azimuthal average exceeds the mean value.

location of the plume in the vertical (top row) and horizontal (middle row) is instead visualised by a grayscale showing the measured speed of the flow. Arrows indicate the motion of the surrounding ambient fluid.

The time evolution of the horizontal flow was examined first by locating the centroid, $\mathbf{x}_c = (x_c, y_c)$, of the speed measured with horizontal PIV. These measurements were unreliable within $1 \text{ cm}$ of the centre of plume primarily due to the fast vertical transport of particles across the horizontal laser light sheet. Ignoring this region, the speed in the
horizontal plane was found to be approximately annular, and the measured centroid, $x_c$, of the annulus was observed to well-represent the center of the flow field. With respect to the position of the centroid, the horizontal velocity field was decomposed into the azimuthal and radial components, and these were then azimuthally averaged to form time series of $u_\theta$ and $u_r$, respectively. The accuracy in the measurement of $u_\theta$ and $u_r$ was assessed by computing the standard deviation in the azimuthal average at each time. If the mean was less than a standard deviation, then the field was set to zero. In practice, we found that the mean radial velocity was less than or comparable to the standard deviation, making such measurements unreliable. However, beyond 1-2 cm from the centroid, the azimuthal velocity exhibited a sufficiently coherent flow to provide a good signal-to-noise ratio. Radial time series of the azimuthal velocity are shown in the bottom row of figure 5 for each of the three experiments.

In the experiment with relatively low rotation (figure 5(a)), the plume at $t = 25$ s was in the process of being deflected from the vertical axis with strong vertical and radial motions being evident near the source. At a distance 6 cm from the source, the horizontal flow around the plume remained approximately axisymmetric, although the radial time series of the azimuthal flow (bottom row) shows that its radial extent broadened substantially after the plume was deflected off-axis shortly after 25 s. In contrast, in the experiment with the same source volume flux but three times the background rotation, the plume transformed into a tornado after 17 s. The maintenance of large azimuthal vorticity associated with the tornado is evident in the radial time series of $u_\theta$ (figure 5(b), bottom): after 25 s the radial extent of the peak in $u_\theta$ remains approximately constant while the peak value increases moderately in time. Despite the coherent structure of the vortex, strong radial and vertical circulations are evident in the vertical cross-section at $t = 25$ s. Eventually these motions resulted in the breakdown of the tornado. In the experiment with the same background rotation as that in figure 5(b), but with double the source vertical velocity, the plume became significantly deflected from the vertical axis at $t = 25$ s. While the ambient motion in the vertical plane appeared to be turbulent, there remained a coherent azimuthal flow around the centroid of the plume, although the radial time series (figure 5(c), bottom) shows the flow was relatively weak and broadened radially over time.

### 3.3. Analysis of experiments

In experiments for which a tornado occurred, analyses were performed, with results given in table 2. It was often observed that the fluid from the source was deflected off-axis shortly before the formation of the tornado and that when the tornado did form, its axis was displaced from the $z$-axis overlying the source. Generally the time, $T_t$, for formation of a tornado was found to be longer in experiments with slower rotation. Although there was some variation, in part due to the somewhat subjective assessment of $T_t$, generally we found that the tornado formation time was approximately a half period of background rotation: $T_t \sim 3/\Omega$.

The radial displacement, $r_c = |x_c|$, of the tornado from the $z$-axis above the source was measured by locating the centroid of the speed measured by horizontal PIV. This value was averaged over times between 5 and 10 seconds after the unambiguous formation of the tornado. Typical displacements were found to be on the order $r_c \sim 1$ cm = $5b_0$. Once formed, however, the tornado exhibited little horizontal variation in its location. The strength of the tornado was assessed by the maximum azimuthal velocity, $U_\theta$. In some experiments, particularly those with small $r_c$, the strength increased approximately linearly in time for tens of seconds. In experiments that ran for long times, a tornado typically persisted more than a minute before collapsing to form a turbulent plume.
deflected from the vertical axis. Tornados that formed at distances larger than $r_c = 1$ cm from the $z$-axis typically had smaller maximum azimuthal velocity, $U_{\theta t}$ and larger radius, $b_{\theta t}$.

Although these diagnostics provide some qualitative insight into the properties of the tornados, they are limited by the measurements which were too noisy near the tornado core to extract reliable information about the radial structure of the azimuthal velocity. As shown in numerical simulations below, the actual radius, $b_{\theta t}$, of the tornado was closer to that of the nozzle radius, and the radial displacement of the tornado axis from the $z$-axis above the source was less than $3b_0$.

Although identical experiments could be run with a tornado appearing in one and not in the other, there appeared to be a “sweet-spot” of parameters for which a tornado was more likely to occur. Figure 6 shows regime diagrams indicating parameters resulting in tornado formation in at least one experiment (circles) or not at all (crosses). Both for moderate plume density (figure 6(a)) and high plume density (figure 6(b)), it appeared as though a tornado was less likely to occur if the source vertical velocity or background rotation was too large. Of course, no tornado occurred if there was no background rotation.

More information is required to elucidate the processes that may or may not lead to tornado formation. For this reason, we turn to the analysis of numerical simulations.

4. Numerical simulations

Here we describe the details of the numerical model. We then present snapshots and qualitative analyses of three simulations run with the same parameters as the experiments shown in figure 5. Quantitative analyses are performed of these and other simulations, with data given in table 3.

4.1. Setup and analysis methods

Numerical simulations were run using the open-source software OpenFOAM, version 19.06, which employed a finite-volume scheme written in C++. The code has previously been applied to investigations of plumes (Wang et al. 2011; Kumar & Dewan 2014; Suzuki
et al. 2016) through the use of a pre-built solver “buoyantBoussinesqPimpleFoam”. This was further adapted to include the addition of the Coriolis force to the momentum equations. Because the flow within the plume was turbulent, the code was run as a large-eddy simulation (LES) using a pre-built subgrid-scale one-equation eddy viscosity model (Yoshizawa 1986), “kEqn”. This model solved a transport equation to compute the subgrid scale kinetic energy, and subsequently obtained the subgrid-scale eddy viscosity. The LES simulation was thus improved by overcoming the shortcomings of the local equilibrium assumption (Smagorinsky 1963) in high Reynolds number flows and/or flows simulated with coarse resolution (Huang & Li 2009).

In most simulations the main computational domain was prescribed as a cylinder of radius \( R_d = 20 \text{ cm} \) and height \( H_d = 30 \text{ cm} \). A buoyant plume originated from the bottom of the main domain (at \( z = 0 \text{ cm} \)) after passing through an expansion chamber similar in geometry to the turbulent plume nozzle used in experiments. Specifically, the fluid entered the chamber through a circular opening of radius \( 0.2 \text{ cm} \), flowed through a lower cylindrical neck region of height \( 0.1 \text{ cm} \), then passed through a cylindrical chamber of radius \( 0.4 \text{ cm} \) and height \( 2 \text{ cm} \), and finally passed through another cylindrical neck region of radius \( b_0 = 0.2 \text{ cm} \) and height \( 0.1 \text{ cm} \) before entering the bottom of the main domain. No-slip boundary conditions were prescribed within the chamber and neck regions to enhance the turbulent character of the flow entering the main domain. The top of the domain was prescribed as an outlet ensuring that the volume flux leaving the top matched the volume flux entering through the inlet. Consequently a fraction of the plume fluid reaching the top of the domain passed vertically through it while the majority spread radially toward the side walls. The bottom and side walls of the main domain were prescribed either with no-slip or free-slip boundary conditions, though the choice of boundary conditions was found insignificantly to affect the plume dynamics.

Given this geometry, the interior mesh was created with the open-source software, Gmsh. In each horizontal plane, this created a tessellated grid with resolution \( 0.1 \text{ cm} \) around the circumference of the inlet and chamber. Within the main domain the circumferential resolution increased linearly from \( 0.1 \text{ cm} \) to \( 0.2 \text{ cm} \) between the circular opening at the bottom of the main domain (centred at \( r = 0 \)) and a radius of \( 9 \text{ cm} \). Between \( 9 \text{ cm} \) and the radius \( R_d = 20 \text{ cm} \) of the side walls the circumferential resolution increased linearly from \( 0.2 \text{ cm} \) to \( 2.0 \text{ cm} \). The tessellated grid was extruded vertically to maintain a vertical resolution of \( 0.1 \text{ cm} \). The high resolution near \( r = 0 \) was found to give sufficient resolution for the LES of the plume, whereas the coarse resolution near the side walls (where the flow was relatively slow and large scale) allowed for faster runtime of the code. Running in parallel on 64 CPUs with time steps no greater than 0.001 s (ensuring a Courant number no greater than 0.5), the code took approximately 22 hours to simulate 30 s of the plume and ambient fluid evolution.

Simulations were run with parameters similar to those of the L-experiments. The buoyancy of the plume was set by prescribing its temperature, \( T \), and using the linear relation \( \rho = 1 - \alpha_T (T - T_{\text{ref}}) \) to compute the corresponding nondimensional density, in which \( \alpha_T = 0.0002 \text{ K}^{-1} \) is the thermal expansion coefficient and \( T_{\text{ref}} = 293 \text{ K} \). Initially the fluid in the main domain was set to have temperature \( T_{\text{ref}} \). The fluid in the “nozzle” was set initially to have temperature \( T \), which was also the temperature of fluid entering the inlet during the course of a simulation. In most simulations, we set \( T = 625 \text{ K} \), corresponding to a density difference between the ambient and source fluid of 0.0664 g/cm\(^3\), and a source reduced gravity \( |g'_{00}| = 65 \text{ cm/s}^2 \), similar to that in experiments having a saline source with density 1.067 g/cm\(^3\) injected downward into fresh water. Notably, in one simulation the source temperature was set to be the same as that of the ambient so as to examine a rotating jet.
The vertical velocity, $w_0$, was prescribed at the inlet. In addition, the intensity of turbulence at the source was modified using a precursor method (Tabor & Baba-Ahmadi 2010) in which the source velocity randomly fluctuated typically by 10% of its mean value. The laminar kinematic viscosity of the fluid was $\nu = 0.0004 \text{ cm}^2/\text{s}$, which is smaller than that of water in order to enhance turbulent flow leaving the source. This was done to compensate for the lack of a turbulence-generating screen, which was situated over the nozzle opening in laboratory experiments. Once the flow was rendered turbulent, the specific value of the laminar viscosity insignificantly affected the flow evolution because of the large source Reynolds number, $Re_0 \equiv w_0s_0/\nu$. The diffusivity, $\kappa$, for temperature was defined by $\nu/Pr$ in which the laminar Prandtl number was set to be $Pr = 7$. For numerical efficiency, this is much smaller than the corresponding Schmidt number (characterising relative salt diffusivity) of the laboratory experiments. However, it is anticipated the turbulent flow is insensitive to the value of the laminar diffusivity. The corresponding turbulent Prandtl number, $Pr_t$, was chosen to be 0.7, consistent with the simulations of van Reeuwijk et al. (2016).

The third key parameter explored in simulations was the background rotation. As a control, some simulations had no background rotation. Otherwise $\Omega$ was set, as in the L-experiments, to range between $\Omega = 0.1$ and $0.5 \text{s}^{-1}$.

In all simulations the plume rose to the top of the domain and then spread radially outward, though some fluid passed through the top of the domain so as to match the volume flux at the inlet. The radial spread of the plume did not influence the dynamics near the source over the duration of the simulations, as confirmed by simulations performed with double the domain height.

As a test of the quantitative accuracy of the numerical results, a simulation was performed with no background rotation of a pure plume having source Richardson number $R_i_0 = 1$. To analyze the plume structure, the azimuthally-averaged perturbation density field was temporally averaged between 4 and 8 s and then the radial profiles at each height between the source and $z = 10 \text{ cm}$ were fit to a Gaussian of the form $\rho_p = \rho_e \exp(-r^2/b^2)$. Consistent with expectations, the radial profiles were well-represented by a Gaussian, and the width $b(z)$ increased linearly with height sufficiently far from the source. Extrapolating downward to where the linear fit to $b$ was zero, put the virtual origin of the plume at $z_0 \simeq 1.6 (\pm 0.6) \text{ cm}$ above the source. The upward displacement of $z_0$ is due to the fluid leaving the nozzle not being fully turbulent. As expected for a pure plume, the momentum and volume fluxes were found to decrease with height above $z_0$ with power laws of $-1.3 (\pm 0.1)$ and $-1.7 (\pm 0.1)$, respectively, consistent with the theoretical power laws of $-4/3$ and $-5/3$ (not shown). Because most experiments showed tornado formation only for lazy plumes ($R_i_0 > 1$), we also performed a simulation with no rotation and with $w_0 = 5.7 \text{ cm/s}$ and $|g_0'| = 65 \text{ cm/s}^2$ (hence $R_i_0 = 5.0$), similar to the source velocity and reduced gravity of experiment L10, shown in figure 5(b). In this case the virtual origin was found to lie $z_0 \simeq 2.4 \pm 0.4 \text{ cm}$ above the source and the plume width was nearly constant between $z = 0$ and $2 \text{ cm}$ before increasing linearly with height above the virtual origin. The upward shift in $z_0$ compared to the case with $R_i_0 = 1$ is expected for a lazy plume.

Because most experiments and simulations were performed for lazy plumes, for consistency the results presented here are scaled with the effective source parameters, in particular the effective source vertical velocity $w_0s_0$, measured according to (2.8). In the simulation discussed above with no rotation and $R_i_0 = 5.0$, we found $w_0s_0 \simeq 13.4 \text{ cm/s}$, more than twice the source velocity, $w_0$. Measurements of the actual centerline vertical velocity averaged over times between 5 and 10 s had a maximum of 13.3 cm/s at $z =$
2.9 cm. This value and its location were comparable to \( w_{0s} \) and the measured virtual origin, \( z_0 \), respectively.

In simulations with rotation, the radius of the plume likewise was calculated by fitting a Gaussian to the azimuthally and temporally averaged density field, except in the case of a rotating jet simulation for which the vertical velocity field was used to find a Gaussian to the azimuthally and temporally averaged density field, except in the case (figure 7(a), bottom) as a consequence of entrainment drawing the rotating ambient motion was predominantly horizontal with a cyclonic circulation around the plume near the turbulent eddies in the plume and near the top of the domain, the ambient motion was qualitatively similar to that in non-rotating fluid: the plume remained centred about the vertical axis and widened as the buoyancy decreased with height due to ambient fluid entrainment. Except near the turbulent eddies in the plume and near the top of the domain, the ambient motion was predominantly horizontal with a cycloidal circulation around the plume (figure 7(a), bottom) as a consequence of entrainment drawing the rotating ambient

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<th>( \Omega ) ( [s^{-1}] )</th>
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<th>( R_{10} )</th>
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<td>0</td>
<td>48</td>
<td>0</td>
<td>2850</td>
<td>-2.4</td>
<td>6.6</td>
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Table 3. Parameters and analyses of simulations, giving the background rotation (\( \Omega \)), source velocity (\( w_0 \)) and reduced gravity (\( \theta_0 \)), source Rossby (\( R_{00} \)), Richardson (\( R_{10} \)) and Reynolds (\( R_{00} \)) numbers, location of the virtual origin of a pure plume above the source (\( z_0 \)), effective vertical velocity of the pure plume (\( w_{0s} \)), power law exponents for radial decay at \( t = 10s \) of the radial (\( p_r \)) and azimuthal (\( p_\theta \)) velocity respectively given by (4.1) and (4.2), normalized rate of change of radial velocity at \( (z, r) = (2, 3) \text{cm} \) defined in (4.3) (\( \dot{c}_r \)), normalized second time derivative of azimuthal velocity at \( (z, r) = (2, 3) \text{cm} \) defined in (4.4) (\( \ddot{\theta}_{90} \)), normalized rate of change of azimuthal velocity within the plume at \( z = 2 \text{cm} \) defined in (4.6) (\( \dot{\theta}_{90} \)), time when plume is first deflected off-axis (\( T_d \)), and the time of initial formation of the tornado (\( T_2 \)) if it occurs.

4.2. Results for three simulations

We begin with a detailed examination of three simulations with parameters identical to the experiments presented in figure 5. As in the experiments with no tornado formation evident (figure 5a,c), the simulations resulted in the plume eventually being deflected off-axis and remaining turbulent if \( \Omega = 0.1 \text{s}^{-1} \) and \( w_0 = 5.7 \text{cm/s} \) (S1) and if \( \Omega = 0.3 \text{s}^{-1} \) and \( w_0 = 11.4 \text{cm/s} \) (S10). A tornado formed in the simulation with \( \Omega = 0.3 \text{s}^{-1} \) and \( w_0 = 5.7 \text{cm/s} \) (S7). For each simulation, figure 7 shows vertical cross-sections through the plume of the in-plane velocity and density perturbation, and it shows horizontal cross-sections of horizontal velocity and vertical vorticity. While the full vertical extent of the domain is shown, only half the lateral extent is plotted.

In simulation S1 (figure 7(a)), which had source Rossby number \( \text{Ro}_0 = 143 \), the structure of the plume at \( t = 25s \) (0.4 of a rotational period) was qualitatively similar to that in non-rotating fluid: the plume remained centred about the vertical axis and widened as the buoyancy decreased with height due to ambient fluid entrainment. Except near the turbulent eddies in the plume and near the top of the domain, the ambient motion was predominantly horizontal with a cycloidal circulation around the plume (figure 7(a), bottom) as a consequence of entrainment drawing the rotating ambient
Figure 7. Corresponding to the experiments shown in figure 5, snapshots at $t = 25$ s from simulations a) S1, b) S7 and c) S10 having $|\mathbf{g}'_0| = 65 $ cm$^2$/s$^2$ and rotation and source velocity as indicated in the top row of plots: (top row) in-plane velocity $(v, w)$ (arrows) and perturbation density (colour) in the $(y, z)$-plane; (bottom row) horizontal velocity (arrows) and vertical component of vorticity, $\zeta$, (colour) in a horizontal plane at $z = 2$ cm (a height chosen to be close to the effective virtual origin for lazy plumes). In all cases, arrows are shown only if their magnitude is less than 1 cm/s. The scale of the arrows in all plots is indicated above the bottom-left plot.

fluid radially inward. In this simulation, the plume was found to deflect from the vertical axis at $T_d = 28$ s.

In stark contrast, simulation S7 (figure 7(b)) shows that the plume transformed into a tornado, doing so at time $T_t \approx 17$ s. This simulation was run with the same parameters as experiment L10 (figure 5(b)) except that in the simulation the background rotation is counterclockwise ($\Omega = +0.3$) and source is positively buoyant originating from the bottom. In that case a tornado also formed, but around 20 s. The simulated tornado was characterized by a tight, vertically extended core of fluid whose density changed little with height up to $z \approx 15$ cm. The vertical cross-section shows that the ambient velocity is primarily horizontal up to $z \approx 5$ cm from the source at $t = 25$ s, with an apparent inward radial component over the bottom $\approx 5$ cm, occurring in part because some entrainment continues to take place, but also because the core of the vortex is displaced from the $z$-axis above the source. Despite the presence of moderate rotation, there are significant vertical as well as horizontal motions in the ambient fluid well outside the tornado above $z \approx 5$ cm. The horizontal cross-section (7(b), bottom) shows strong cyclonic motion...
surrounding a localized core of positive vertical vorticity having a radius $b_{\theta t} \simeq 0.3$ cm. The maximum vorticity is $67 \text{s}^{-1} = 112 f$.

In the simulation with the same background rotation but with twice the vertical velocity at the source (S10), the plume first deflected significantly off-axis at $T_d = 14$ s and remained deflected at $t = 25$ s, as shown in figure 7(c). Although the ambient flow outside the plume is primarily horizontal below $z \simeq 5$ cm, the $y$-velocity is positive on either side of the plume as a consequence of the centroid of the disturbance being shifted to the second quadrant. The vertical vorticity at $z = 2$ cm is no longer coherent and single-signed, although a near-axisymmetric azimuthal flow surrounds the centroid of the plume.

In order to gain insight into the dynamics governing the plume and ambient fluid evolution, vertical time series were constructed of the density normalized dynamic pressure, $P_0$, and vertical velocity, $W_0$, at the centroid of the flow, and of the maximum azimuthally-averaged azimuthal velocity, $U_\theta$, about the centroid. The centroid itself, which can change position with height and time, was determined from the magnitude of the vertical vorticity field associated with the flow. After constructing the time series, MatLab’s “rlowess” method was used to smooth the data by averaging over 1 s in time and 1 cm in the vertical. This procedure helped to filter out fast- and fine-scale motions so as to focus on the statistically quasi-steady evolution of the fields. The results, corresponding to the three simulations in figure 7, are shown in figure 8. These fields are normalized using the effective source velocity, $w_0$.

The vertical time series of $P_0$ show the front of the starting plume rising past $z = 15$ cm in the first two or three seconds of the simulation (the rise time depending upon the source buoyancy). The pressure decreased rapidly in the vertical from the source to $z \simeq 2$ cm, close to the location of the lazy plume virtual origin. It is over this distance that the relatively low momentum of the lazy plume adjusts to become closer to that of a pure plume. Above the virtual origin the pressure increased with height corresponding to the decrease in the vertical velocity, as expected for a pure plume. The right and upward streaks in the pressure time series correspond to unfiltered turbulent eddies rising upward through the ascending plume. Unlike a plume with no background rotation, as time progressed the vertical extent of the low-pressure region above the virtual origin became smaller, as is evident, for example, in figure 8(b) by the converging dashed lines indicating where the pressure is half the minimum pressure at $t = 1$ s. The vertical extent of the low pressure region decreased more rapidly in simulations S7 and S10, which had faster background rotation (figure 8b,c). The corresponding increasing dominance of an adverse vertical pressure gradient approaching the source in time was likewise noted in the simulations of rotating plumes in (non-uniformly) stratified fluid by Fabregat Tomàs et al. (2016).

The low-pressure region vanished altogether at the critical time $T_d \simeq 28$ s in simulation S1 (figure 8a), at $T_d \simeq 14$ s in simulation S7 (figure 8b), and at $T_d \simeq 13$ s in simulation S10 (figure 8c). At these times the plumes deflected from the vertical axis at the source. After these times, in the first and last cases the pressure became somewhat homogeneous with height, corresponding to the source fluid being deflected significantly away from the vertical. In the middle case (figure 8(b)), the low-pressure region also vanished at $T_d \simeq 14$ s. However, at $t \simeq 17$ s the pressure near the source decreased rapidly once more and the vertical extent of the low pressure region then extended well above the source after time $T_1 \simeq 18$ s corresponding to the formation of a tornado having azimuthal velocity in cyclostrophic balance with the radial pressure gradient.

The descent toward the source over time of relatively higher pressure coincided with a decrease in the vertical velocity within the plume (figures 8d-f). In particular, the
positive vertical strain near the source vanished at the deflection time $T_d$, at which time the vertical velocity near the virtual origin was approximately $0.5w_{0\star}$. These dynamics lie in stark contrast with non-rotating plume theory. Not only was the evolution of the rotating plume unsteady, but the system evolved from one having decreasing vertical velocity with height above the virtual origin to one in which the vertical velocity was near uniform with height at the time when the pressure low vanished.

The change in the centreline pressure and vertical velocity can be attributed to the
increase of azimuthal velocity in the plume, as shown by the time series of $U_\theta$ in figure 8(g-i). Entrainment of the rotating ambient fluid surrounding the plume efficiently increased the azimuthal velocity within the plume by way of angular momentum conservation. As $U_\theta$ increased, a greater vertical strain was required in order to change both the radial and azimuthal velocities. If the buoyant flow possessed insufficient potential energy to supply the requisite kinetic energy, then vertical motion became inhibited.

As a measure of the importance of azimuthal flows in influencing the plume dynamics, figure 8(j-l) shows time series of the plume Rossby number, $R_{op}$, defined by (2.17). Associated with the growth of $U_\theta$ in the plume is the increase in vertical vorticity $\zeta \sim 2U_\theta/b_\theta$. Even though at early times the vertical velocity was much larger than $U_\theta$, $\zeta$ quickly became much larger than $f$, so that $R_{op} \approx W_0/U_\theta$ rapidly reduced. This reduction occurred most rapidly well above the source because ambient fluid was carried radially inward over a larger distance, $\sim b$, resulting in a larger spin-up due to angular momentum conservation.

As well as the decrease in the plume Rossby number aloft, also evident in our simulations of lazy plumes was a more rapid decrease in the plume Rossby number close to the source as compared with the change in the plume Rossby number moderately above the height of the virtual origin (around $z \approx 2$ cm). This was particularly evident in simulation S7 (indicated by the arrow below figure 8k), which had higher source Richardson number ($Ri_0 = 5.0$) than that in simulation S10 with the same background rotation ($Ri_0 = 1.3$). The vertical strain associated with the acceleration of fluid leaving the source of a lazy plume enhanced the vertical vorticity and corresponding azimuthal velocity. The reduction in $R_{op}$ near the source is associated with vertical motion being inhibited, which could act to deflect the plume from the vertical axis. However, it is also associated with greater swirl at the source that could laminarize the flow and lead to tornado formation. Indeed, because entrainment is reduced for lazy plumes near the source, laminarization may occur more readily for lazy plumes.

The evolution of the ambient fluid motion around the plume is examined by constructing vertical time series of the radial, azimuthal and vertical velocities that are azimuthally averaged around a radius $r_a = 3$ cm from the centroid of the flow. This radius was chosen to be close to the boundary at $z = 15$ cm between the turbulent plume and ambient fluid at early times in the simulations. The resulting time series are shown in figure 9 for the same three simulations with snapshots shown in figure 7.

As the front of the starting plume rose, the ambient fluid was first pushed radially outward around its head, but the flow then reversed to be drawn into the plume itself. The inward radial flow was generally faster further above the source and fluctuated as successive eddies rose upward through the plume. Of course, theory predicts the radial entrainment velocity $\alpha W_0$ is smaller with increasing height above the virtual origin. However, at a fixed radius, $r_a$, outside the plume the radial inflow is larger with height because the boundary of the plume is closer to $r_a$.

In all simulations, radial and azimuthal velocities initially exhibited vertical shear. However, approaching the time, $T_d$, at which the plume was deflected from the vertical, azimuthal velocity contours became nearly vertical for $z \gtrsim 0$. Contours of the ambient Rossby number, $R_{oa}$ (defined by (2.19)), are superimposed on the vertical time series of $u_\theta$ (figure 9(d-f)). If $u_r$ was neglected in the definition of $R_{oa}$, the contours would coincide with lines of constant $u_\theta$. It is clear that well above the source ($z \gtrsim 5$ cm) at early times the radial velocity contributes significantly to increase the ambient Rossby number beyond 0.1 (thick yellow contour in figure 9(d-f)). Close to the source, the Rossby number remained smaller for longer times leading to the vertical shear being reduced more rapidly there.
Figure 9. Vertical time series corresponding to the three simulations shown in figure 7 (S1 left, S7 middle, S10 right), showing azimuthally-averaged normalized velocities at \( r_a = 3 \) cm: (top row) radial velocity, (middle row) azimuthal velocity and (bottom row) vertical velocity. In the plots of \( u_r/w_0 \), the thick contour indicates where the radial velocity is zero and solid and dashed contours are plotted at intervals of 0.01. In the plots of \( u_\theta/w_0 \), black contours are plotted at intervals of 0.2; the thick contours indicate where the azimuthal velocity is zero. The yellow contours in this plot indicate where the ambient Rossby number is 0.1 (solid), 0 (thick solid) and 0.01 (dashed). In the plots of \( w/w_0 \), contours are plotted at \(-0.1\) (dashed), 0 (thick solid) and 0.01 (solid). Note that the range of time in the first column of plots is twice that of the second and third columns.

These considerations lead to a complicated evolution of the ambient vertical velocity field (figure 9(g-i)). As the front of the starting plume passed a given height it induced a downward velocity in the surrounding ambient. Thereafter, below \( z \approx 5 \) cm a net upward flow developed, increasing upward from zero at \( z = 0 \), reaching a maximum and then decreasing again. The corresponding vertical strain near \( z = 0 \) acted to reduce the vertical shear of \( u_r \) and \( u_\theta \).

4.3. Plume and ambient fluid analyses

In order to quantify better the physical processes revealed by the vertical time series within the plume and surrounding ambient, here we develop empirical diagnostics used to characterise the forcing and temporal evolution of flow within and surrounding the plume at times leading up to the deflection of the plume from the vertical axis.

Corresponding to the three simulations examined above, figure 10a-c plots the radial dependence at \( z = 2 \) cm (close to the virtual origin at \( z_0 \)) and \( t = 10 \) s of the most significant forcing terms in the radial and azimuthal momentum equations (2.11) and (2.12). Specifically, these are the radial pressure gradient \((\partial P/\partial r)\), the Coriolis accel-
erations \( (fu_\theta \text{ and } -fu_r) \) and the centripetal acceleration \( (u_\theta^2/r) \). For simulation S1 \( (\Omega = 0.1 \text{s}^{-1}) \), the radial pressure gradient dominated over the Coriolis and centripetal accelerations at \( t = 10 \text{s} \) (0.16 of a rotational period) so that the inward radial flow increased in time. Conversely, in the simulations with \( \Omega = 0.3 \text{s}^{-1} \), the acceleration due to the radial pressure gradient was in near balance with the Coriolis acceleration \( (\text{Ro}_a \ll 1) \) for \( r > 3 \text{ cm} \), and with the centripetal acceleration \( (\text{Ro}_a \gg 1) \) for \( r \lesssim 1 \text{ cm} \). Due to the vertical strain in the ambient fluid, the azimuthally-averaged radial velocity field did not decay with radius as \( r^{-1} \), as would be required by (2.10) with \( \partial w/\partial z = 0 \). Instead, it decayed as

\[
 u_r \propto r^{-p_r} \tag{4.1}
\]

with \( p_r \simeq 0.33 \) in simulation S1 and \( p_r \simeq 0.5 \) in simulations S7 and S10. The radial profile of azimuthal velocity also exhibited power law behaviour near the plume according to

\[
 u_\theta \propto r^{-p_\theta} \tag{4.2}
\]

although \( p_\theta \) was typically larger than \( p_r \). The power law exponents for all simulations are given in table 3. Generally \( p_r < 1 \) and \( p_\theta < 1 \) except in the simulation of a jet (S4) and the simulations with the fastest rotation (S11 and S12), for which \( p_r \simeq 1 \).

This analysis was also performed by extracting at \( t = 10 \text{s} \) radial profiles taken at \( z = 1 \text{ cm} \) and \( z = 3 \text{ cm} \) (not shown). The results showed that the computed power law exponents, \( p_r \) and \( p_\theta \), did not vary significantly with height for \( z = 2 \pm (1 \text{ cm}) \). However, the magnitude of the radial and azimuthal flows did increase in time. The time evolution of the azimuthally-averaged ambient flow at \( r = r_a = 3 \text{ cm} \) and \( z = 2 \text{ cm} \) (close to the height of the lazy plume virtual origin) is plotted in figure 10(d-f) for the three simulations considered above. These show that the inward radial flow increases linearly in time according to

\[
 u_r(r_a = 3 \text{ cm}, t)/w_{0*} \simeq -\dot{c}_r \quad ft, \tag{4.3}
\]

in which \( \dot{c}_r \) is an empirically determined constant. In all simulations (see table 3), we find \( \dot{c}_r \simeq 0.0020 \pm 0.0005 \). Because the Coriolis term, \(-fu_r\), is the dominant driving force in the azimuthal momentum equation (2.12), the azimuthal velocity is expected to increase quadratically in time according to

\[
 u_\theta(r_a = 3 \text{ cm}, t)/w_{0*} \simeq \ddot{c}_\theta (ft)^2/2, \tag{4.4}
\]

in which \( \ddot{c}_\theta \) is an empirically determined constant. This indeed is found to be comparable to \( \dot{c}_r \), as expected (see table 3).

Combining these results, a crude representation of the evolution of the ambient flow around \( z = 2 \text{ cm} \) (near the virtual origin) is

\[
 u_r(r, t)/w_{0*} \simeq -\dot{c}_r \left(\frac{r}{r_a}\right)^{-p_r}, \quad u_\theta(r, t)/w_{0*} \simeq \frac{1}{2} \ddot{c}_\theta (ft)^2 \left(\frac{r}{r_a}\right)^{-p_\theta}. \tag{4.5}
\]

Figure 10(g-i) examines the time evolution of the maximum vertical vorticity and the maximum vertical and azimuthal velocities within the plume at \( z = 2 \text{ cm} \). Particularly for lazy plumes, until close to the time of plume deflection the maximum azimuthal velocity was found to increase approximately linearly in time according to

\[
 U_\theta/w_{0*} \simeq \dot{c}_{\theta 0} \quad ft. \tag{4.6}
\]

In most simulations, \( \dot{c}_{\theta 0} = 0.03(\pm 0.01) \). The largest value \( \dot{c}_{\theta 0} \simeq 0.04 \) was measured in the simulation with largest source Richardson number \( (\text{Ri}_0 = 9.9) \) and the smallest value \( \dot{c}_{\theta 0} \simeq 0.012 \) was measured in the simulation of a jet \( (\text{Ri}_0 = 0) \). This suggests that the
Figure 10. For the three simulations shown in figure 7, analyses showing the following: (top row) log-log plots of the azimuthally-averaged radial profiles at $t = 10 \text{s}$ and $z = 2 \text{cm}$ of the density normalized pressure gradient (thick blue), centripetal acceleration (dashed), the Coriolis acceleration in the radial momentum equation (solid black) and the Coriolis acceleration in the azimuthal momentum equation (solid red); (middle row) time evolution at $r_a = 3 \text{cm}$ and at three different heights (as indicated in (d)) of the normalized azimuthally-averaged radial and azimuthal velocities; (bottom row) time evolution within the plume at $z = 2 \text{cm}$ of normalized maximum vertical vorticity (black), maximum vertical velocity (red), and the maximum azimuthal velocity (green). In the top row dotted lines are the offset best-fit lines to $f u_\theta$ (black) and $f u_r$ (red) with numbers indicating the slope and associated error. In the middle row the red dotted lines show the offset best-fit line to $u_r(t; z = 2 \text{cm}, r = 3 \text{cm})/w_0$ over the time range indicated, with numbers giving the slope. In (d-i), the crosses and circles on the time axes respectively indicate when the plume first deflected and when it first transformed into a tornado.

linear increase in time of $U_\theta$ within the plume, as opposed to the quadratic increase in the surrounding ambient, was strongly influenced by the reduction in entrainment into lazy plumes, with entrainment being further reduced as the surrounding fluid acquired greater azimuthal velocity. The positive vertical strain near the source of lazy plumes acted to increase the absolute vorticity of what fluid was entrained as well as the fluid leaving the source. Consequently, $U_\theta \sim b_\theta \zeta/2$ increased in time. Plumes that were closer to being pure near the source had zero or negative vertical strain associated with the decrease in mean vertical velocity with height. So, while their relative entrainment of the surrounding azimuthal flow was larger, the associated vorticity within the plume was reduced. Although there is no consistent trend among all simulations performed, these
results suggest that tornado formation is more effective for lazy plumes, which are more readily laminarized by the build-up of swirl in the ambient and which nonetheless develop vorticity within the core due to vortex stretching by positive vertical strain.

In all cases, the vertical velocity in the plume at \( z = 2 \) cm began to decrease when \( w_0^\star/(\zeta b_0) \sim 4 \), which nearly coincides with the critical Rossby number, \( \text{Ro}_c = 3.4 \), at which rotation was found in experiments by Fernando et al. (1998) to constrain vertical motion in a plume. The plume deflected from the vertical axis when the vertical and azimuthal velocities were comparable. In simulations S1 and S10 (figures 10(g,i)), the sideways deflection of the plume significantly reduced the vorticity and vertical velocity. In contrast, in simulation S7 (figure 10(h)), the vorticity and azimuthal velocity continued to grow after deflection until \( ft \sim 10.2 \) when a tornado developed, this being associated with significantly increased vorticity, vertical velocity and azimuthal velocity.

4.4. Tornado analyses

In the four simulations for which a stable tornado formed, the flow and structure of the tornado were measured at a time 5 s after the tornado first began to develop such that it extended beyond 10 cm above the source. At each height the velocity fields were computed from an azimuthal average around the centroid of the vertical vorticity field associated with the flow, from which were extracted at each height the maximum vertical velocity, \( W_{ot} \), the maximum azimuthal velocity, \( U_{ot} \), and the width, \( b_{ot} \), where the azimuthal velocity was largest. The resulting vertical profiles are plotted in figure 11.

In all four cases \( U_{ot} \) moderately decreased with height between 1 and 10 cm above the source and, consistent with the suppression of three dimensional turbulent motions, \( U_{ot} \) was smaller than \( w_0^\star \), the effective source vertical velocity. The radial extent of the tornado, \( b_{ot} \), was approximately twice the source radius, \( b_0 \), showing an increase with height that was much smaller than that associated with a proper plume. The tornado extent measured in experiments was significantly larger, but this is attributed to large uncertainties in the measurement of azimuthal velocities near the plume where PIV particles passed too quickly through the horizontal laser light sheet and due to blurring of the image associated with significant changes in the index of refraction between the ambient fluid and fluid within the tornado.

The maximum vertical velocity generally increased with height at least up to \( z = 5 \) cm, consistent with the vortex being stretched. Between the four simulations with a tornado there was significant variation in typical values of \( W_{ot} \) (figure 11c). This can be explained by the plot in figure 11(d) showing the distance, \( r_c \), of the centroid of the tornado from the \( z \)-axis as a function of height. In simulations where the tornado was most closely aligned above the source such that \( r_c \approx b_0 \), the maximum azimuthal and vertical velocities were largest and the radius of the tornado was smallest. However, in simulations where the fluid from the source was deflected by a significant radial distance before the tornado formed (\( r_c \approx 2b_0 \)), the radius was larger and the velocities smaller. A similar observation was made for tornados that formed in laboratory experiments.

Taking the ratio of \( U_{ot} \) to \( W_{ot} \) gives the swirl, defined by (2.23). Vertical profiles of this ratio are plotted in figure 12. The swirl for all tornados was greater than 1 at least below \( z = 5 \) cm, consistent with these being stable. The tornado with the largest swirl was that deflected most from the vertical axis. It is reasonable to suppose that the further a tornado is situated away from the \( z \)-axis, the less likely it is to remain stable due to the turbulent processes taking place between the source and the significantly radially offset base of the tornado, which act to provide an irregular flux of fluid feeding the tornado. Greater swirl, which has a stabilizing effect, is therefore necessary for tornados that are deflected further from the \( z \)-axis. Indeed, the corresponding simulation snapshot shown
Figure 11. Vertical profiles of normalized fields associated with a tornado in simulations for which a stable tornado appears showing (a) maximum azimuthal velocity, (b) radial extent (c) centreline vertical velocity, and (d) radial distance of the centroid from the z-axis. The parameters for each simulation, with values in cgs units, are indicated in (a).

Figure 12. Vertical profiles of the swirl associated with the tornados observed in four simulations. The line colours correspond to those shown in figure 11(a).

in figure 7(b) exhibits significant axial variations in the perturbation density field along the core of the tornado. A movie of this simulation likewise shows the tornado on the verge of breaking up at $t = 28 \text{s}$ before reforming and finally beginning to break down entirely at $t \approx 38 \text{s}$.

The results in figures 11 and 12 indicate that the properties of the tornado are sensitive to the dynamics of the chaotic flow developing near the source that tend to deflect the plume from the vertical around the time that sufficient vorticity builds up near the source. Consequently, it is impossible to make quantitative predictions about the tornado structure or even its formation based upon source conditions. Nonetheless, the observations that a tornado that is more closely aligned above the source has maximum azimuthal and vertical velocities close to $w_0$, suggests that heuristic arguments can be developed to predict whether it is possible for a tornado to occur. This is discussed below.
5. Discussion

The culmination of the analyses above demonstrates processes that tend to drive the plume away from the vertical as well as processes that strengthen the swirl of the plume near the source, possibly leading to tornado formation.

Regarding processes that lead to deflection, we have shown that the reduction in the plume Rossby number aloft inhibits vertical motion which drives significant turbulent motion in the surrounding ambient, a consequence of the rising fluid from the plume being deflected radially outward. This chaotic flow surrounding the plume descends toward the height of the source, being associated with the decrease of the height within the plume where the vertical velocity is reduced. Even below this chaotic flow the azimuthal velocity in the ambient fluid surrounding the plume near the source increases quadratically in time. Near the plume the ambient flow is in near-cyclostrophic balance so that the radial pressure gradient is approximately equal to the centripetal acceleration, \( u_\theta^2/r \). Hence the pressure surrounding the plume near the source decreases as a fourth power of time. Using (4.5), a semi-empirical estimate of the pressure surrounding the plume around the virtual origin is given by

\[
\frac{P}{w_0^2} \sim -\frac{\dot{c}_\theta^2}{4(2p_\theta + 1)}(ft)^4 \left( \frac{r}{r_a} \right)^{-2p_\theta},
\]

in which \( r_a = 3 \text{ cm} \) and, based on measurements of lazy plume simulations, we estimate \( p_\theta \approx 0.7 \) and \( \dot{c}_\theta \approx 0.003 \). Around the time of a half-rotation period, for which \( ft \approx 6 \), this expression evaluated at \( r = b_0 \) gives \( P/w_0^2 \approx -0.05 \), which is comparable to the pressure at the source. This suggests the plume deflection occurs not just due to the build-up of an adverse vertical pressure gradient but also because the ambient flow ultimately reverses the radial pressure gradient near the source on a time of the order \( T_d \sim 6/f \).

Regarding processes that lead to possible tornado formation, we have shown that the vertical strain near the source associated with lazy plumes leads to a linear increase in time of the vorticity and maximum azimuthal flow near the source. Entrainment is reduced for lazy plumes, suggesting that this swirl can act more effectively to laminarize this flow for which turbulence is already suppressed. Using (4.6) with \( \dot{c}_{\theta 0} \approx 0.03 \) (a typical value measured in lazy plume simulations), the azimuthal velocity near the virtual origin at \( ft = 6 \) is estimated to be \( U_\theta \approx 0.2w_0 \). Due to the suppression of vertical motion aloft descending toward the source, the azimuthal flow in the plume near the source is comparable to the vertical velocity close to the deflection time, thus establishing the condition for sufficiently large swirl to allow a tornado to form and persist.

Just because a tornado can occur, does not mean that it does. In laboratory experiments, a tornado sometimes formed and other times did not even though the experiment parameters were ostensibly identical. Likewise, the manifestation of a tornado in simulations was sensitive to the magnitude of the noise imposed on the vertical velocity of the source at the inlet (ranging in different simulations between 0.05 and 0.2 the mean vertical velocity). The analyses above demonstrate that the conditions for a tornado to form are coincident with the plume being deflected. If the randomness associated with the chaotic flow around the plume is such that the flow gets pushed too far off-axis, then a tornado will not form. On the other hand, if for a sufficiently long time after \( ft \sim 6 \) axisymmetry is maintained or the plume is not deflected too far (with the centroid displaced no more than about \( 2b_0 \)), then a tornado can form. While this makes an accurate prediction for the occurrence of a tornado challenging, the analyses above suggest that the source Rossby number, \( R_{00} \), and source Richardson number, \( R_{\theta 0} \), best assess conditions favorable to tornado formation, as shown by the regime diagram in figure 13.
Certainly, it is necessary for the source Rossby number $Ro_0$ to be much greater than the critical value $Ro_c \simeq 3.4$, otherwise vertical motion would be inhibited immediately upon leaving the source. Consistent with the regime diagram in figure 13, it is reasonable to suppose a lower bound of $Ro_0 \gtrsim 10$. There appears to be no strict upper bound on $Ro_0$ because the time for plume deflection and for tornado formation are both comparable to a half-period of rotation. That said, if the background rotation is very small, then the domain would have to be very tall and wide to ensure that the flow in the far field does not significantly influence the flow near the source.

A tornado is more likely to form if the plume is lazy, with the regime diagram in figure 13 suggesting it is necessary to have $Ri_0 \gtrsim 1$ for stable tornados to form. Another possible constraint on the formation of a stable tornado is the condition that the swirl parameter $q^*\star$ be larger than unity. Because the tornado forms near the source, its radius, $b_t$, should not be much larger than that of the source itself, with simulations (figure 11(b)) suggesting a radius $b_t = b_{0t} \sim 2b_0$. Some entrainment may occur between the source and base of the tornado, particularly if the source flow is deflected off-axis. Hence, because the somewhat larger volume flux rises through a vortex core of somewhat larger radius than the source, the vertical velocity within the tornado should be of similar order to the effective vertical velocity, $w_0\star$, of the plume. In simulations of lazy plumes that transform into tornados (figure 11(c)) measurements indeed suggest $W_{0t} \lesssim w_{0\star}$ if the tornado is not displaced too far from the vertical axis through the source. Because the ambient fluid has uniform density, the buoyancy flux between the source and base of the tornado is constant. This permits an estimate of the reduced gravity of the fluid entering the base of the tornado to be

$$g'_{0t} \sim \frac{b_0^2}{b_{0t}^2} \frac{w_0}{W_{0t}} g'_{00} \gtrsim 0.3 g'_{00} \frac{(Ri_0 - 1)^{1/10}}{Ri_0^{1/2}},$$

in which we have used (2.7) to give an explicit estimate of $w_{0\star} \simeq W_{00}$. Hence, using (2.5), the Richardson number, $Ri_t$, associated with the flow entering the base of the tornado
Sutherland et al. satisfies,

\[ \text{Ri}_t = \frac{5}{4\alpha} \frac{\nu''_t b_t}{W_{0t}^2} \gtrsim \frac{(\text{Ri}_0 - 1)^{3/10}}{\text{Ri}_0^{1/2}} \simeq \text{Ri}_0^{-1/5}, \]  

(5.3)

in which the last expression holds if \( \text{Ri}_0 \gg 1 \). Hence, if the plume is very lazy at the source, then the flow entering the base of the tornado would have an excess of momentum over buoyancy. As buoyancy is necessary to provide the axial strain that maintains a tornado, it is reasonable to assume that a tornado can form if the plume is lazy, but not so lazy that entrainment into the plume near the source is entirely suppressed. Experiments and simulations suggest \( 1 \lesssim \text{Ri}_0 \lesssim 20 \).

6. Conclusions

In our analysis of experimental and simulated rotating plumes we have shown that significant chaotic vertical and horizontal motion is induced in the ambient fluid surrounding the plume. Aloft, this is a consequence of the vertical motion in the plume being redirected outward at heights where the plume Rossby number decreases below unity, a consequence of efficient entrainment of the surrounding rotating ambient fluid. Vertical strain is likewise induced in the surrounding ambient fluid near the level of the source so as to reduce the vertical shear imposed by differential radial entrainment with height. This leads to a linear increase in time of the inflow velocity and a corresponding quadratic increase in time of the ambient azimuthal velocity that builds a negative pressure quartically in time, ultimately acting to deflect the plume from its vertical axis. For lazy plumes, the vertical strain near the source results in a linear increase in time of vorticity, increasing the swirl that has the potential to laminarize the flow and transform the plume into a tornado. The time for both plume deflection and tornado formation occurs on a scale of half a period of the background rotation.

Whether or not a tornado forms depends somewhat randomly on the nature of the turbulent fluctuations surrounding the plume when it becomes deflected. However, analyses suggest a tornado is more likely to form if the source Richardson number lies in the range \( 1 \lesssim \text{Ri}_0 \lesssim 20 \). A lower bound on the source Rossby number is \( \text{Ro}_0 \simeq 10 \). In theory there is no upper bound on the source Rossby number except for limitations posed by the domain size and by ambient motion occurring independent of the plume that may cause it to deflect before a half-period of rotation.

For example, consider the oil plume of the 2010 Deepwater Horizon accident. Estimates have been made for the source volume flux, \( Q_0 \sim 0.2 \text{ m}^3/\text{s} \), and buoyancy flux, \( B_0 \simeq 1 \text{ m}^4/\text{s}^3 \), emanating from the pipe of radius 0.238 m (Sovolofsky et al. 2011). From these, the source Richardson number is estimated to be \( \text{Ri}_0 \simeq 12 \): the plume was moderately lazy. However, the source Rossby number was \( \text{Ro}_0 \simeq 6 \times 10^4 \), and so it is likely that bottom currents and the action of the barotropic tide would deflect the plume before a tornado could form on the time scale of \( 6/f \sim 24 \text{ hours} \) at \( 28.7^\circ \text{N} \).

In the experiments and simulations presented here, the ambient fluid had uniform density. However, tornado formation has also been observed in laboratory experiments of a rotating plume impinging downward upon a density interface in a two-layer fluid (Ma 2018). Whereas the initially turbulent plume spread at the interface because turbulent entrainment reduced its density to be lighter than the lower layer fluid, once a tornado formed, laminarization reduced entrainment so that the vortex could efficiently penetrate through the interface and carry nearly undiluted fluid from the source to depth. If sufficiently quiescent ambient conditions could exist and if the plumes were sufficiently lazy, this could have important implications for the vertical transport of pollutants from
effluent released at depth and of nutrients and heat released from abyssal geothermal vents, in which case tornado formation would result in the near-undiluted vertical transport of the source fluid.

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Declaration of Interests. The authors report no conflict of interest.

REFERENCES

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