Generation and Trapping of Gravity Waves from Convection with Comparison to Parameterization

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Abstract. This work quantifies the wave generation and wave trapping mechanisms active in a mesoscale model of waves in the vicinity of convection. The results depend on the details of the latent heating in our model, which is derived from precipitation radar measurements, and on the wind shear and stability profiles of the background state. Three simulations with different background wind profiles are compared to illuminate the dependence of the results on the background wind profile. We further compare the waves in our nonlinear mesoscale model to linear calculations of wave generation, propagation, and trapping, and run two additional nonlinear simulations with larger wave forcing to examine nonlinear effects. The comparisons to the linear results are then used to test and improve a source parameterization for convectively generated gravity waves.
1. Introduction

Convection generates waves, and waves generate convection. The link is inherent in theoretical models of tropical wave generation and convection initiation. The investigation of these links began with large scale waves in concert with observational evidence for the properties of large-scale tropical wave modes and theoretical understanding of their nature [Matsuno, 1966; Holton, 1972; Lindzen, 1974; Salby and Garcia, 1987; Bergman and Salby, 1994; Wheeler and Kiladis, 1999]. More recently, attention has drifted to smaller scale waves and their interaction with mesoscale properties of convection, again in concert with observational evidence for the properties of smaller scale waves and the finer-scale structure of convection [Mapes, 1993; Shige, 2001].

In our previous work we have focused on wave generation by convection and the subsequent wave propagation and interaction with the background atmosphere [Alexander and Holton, 1997; Alexander et al., 2004; Beres et al., 2005]. When waves break or otherwise dissipate, they can drive global-scale circulations. For gravity waves, these effects on the circulation are most apparent in the middle atmosphere where they have a first order effect [Holton, 1982; Giorgetta et al., 2002; Scaife et al., 2002]. The global-scale effects are widely parameterized in global models, but the parameterizations require many unavailable details of the wave properties to have sufficient realism for predictive power. Parameterizations describing convectively generated gravity waves have been developed [Chun and Baik, 1998, 2002; Beres et al., 2004], but still contain necessary simplifying assumptions that are poorly validated to date.

We have developed a model for wave generation for case study analysis that is particularly well suited for testing/validation of the parameterization assumptions and method:
We force waves using radar-observed precipitation structure within convection and study their properties and propagation in a dry version of a mesoscale cloud-resolving model. The parameterizations must assume simple properties of the heat sources within the global model grid box, and are based on linear theory [Chun and Baik, 1998, 2002; Beres et al., 2004]. Our model, in contrast resolves the convection within an area similar to a grid box scale, and it includes a realistic complex assortment of convective heating cells and the full nonlinear response to the convective forcing. This allows us to test many of the necessary simplifying assumptions in the parameterizations.

In previous work, we compared the modeled wave properties to those observed during the Darwin Area Wave Experiment (DAWEX) [Alexander et al., 2004; Hamilton et al., 2004; Vincent et al., 2004; Tsuda et al., 2004], and found good agreement in timing and general properties of the waves, but found the strength of the heating in our model was underestimated by a factor of roughly 3-5. We therefore test the wave response in the model to our new heating strength estimates here to examine the degree of nonlinearity in the response. In the course of our previous work we also noticed wave trapping in the troposphere which we investigate further here. The trapped waves may play a role in subsequent organization of a squall-line observed in our DAWEX case study. We also quantify the wave generation mechanisms and their spectral signatures, and compare the modeled waves to the Beres et al. [2004] parameterization of convectively generated gravity waves as it has been applied in Beres et al. [2005]. These model results suggest some improvements for future applications of the parameterization.
1.1. Mesoscale Model Simulations

Our mesoscale model and method of forcing with observations from precipitation radar is described in detail in Alexander et al. [2004]. Here we summarize only a few essential details. The model is that of Durran and Klemp [1983] with more recent modifications described in Piani et al. [2000] and Beres et al. [2002]. In the present application, we turn off all moist processes in the model, and instead provide wave forcing with observed spatial and temporal variations in convective latent heating derived from precipitation radar [Keenan et al., 1998; May et al., 2002]. The radar observations provide 3-dimensional volumetric retrievals of reflectivity with 2-km horizontal resolution, 1-km vertical resolution, and 10-minute time resolution. The conversion of reflectivity to heating rates contain considerable uncertainties in both magnitude and the shape of the vertical profile. In Alexander et al. [2004], we therefore simplified the conversion by assuming the column heating is proportional to the column reflectivity on the resolved horizontal grid, and we then redistribute the column heating in a half-sine vertical profile between the 900 mb level and the altitude of the highest significant radar echo. This preserves essential information on the depth of the heating, but de-emphasizes details in the vertical profile of reflectivity that do not well represent the heating profile.

We assume a horizontally uniform background atmosphere with vertical variations derived from the 5-day mean of 3-hourly DAWEX radiosonde profiles for the November campaign period. The heating is input in a circular area of 256-km diameter centered on the radar site, and this is centered within the 400-km square mesoscale model domain. The top and 4 sides of the domain have wave permeable boundary conditions [Durran et al., 1993]. The model resolution is run at the same 2-km horizontal resolution as the
radar data, but at higher 0.25-km vertical resolution to resolve the important wave responses. The radar reflectivity is retrieved up to 19-km altitude, and the model top is 25 km. The buoyancy frequency $N$ profile is shown in Alexander et al. [2004], and has a tropopause at 17 km with constant $N = 0.027 \text{ s}^{-1}$ in the stratosphere and variable $N$ in the troposphere (average $N \sim 0.011 \text{ s}^{-1}$ between 0-12 km, a layer of low stability $N \sim 0.007 \text{ s}^{-1}$ 12-15 km, then rapid increase to stratospheric values above 15 km).

1.1.1. Spectral analysis of stratospheric waves.

The top panels of Figure 1 show a spectrum of stratospheric wave momentum flux from the first 4 hours of the simulation along with vertical profiles of zonal and meridional wind for this case, which are identical to Alexander et al. [2004]. The contours in Fig. 1 are logarithmic to show the range of amplitudes and phase speeds in the model. The prominent result for this previously reported case is the strong response for waves propagating in the northeast (NE) direction with slow phase speeds, $\sim 5 - 10 \text{ m s}^{-1}$. This spectrum also shows the effect of wind filtering via the arc of low wave fluxes centered on the wind direction. (The stratospheric wind speed as a function of direction is also overplotted as a dashed line.) The center row of panels show the same result for a simulation with only the tropospheric portion of the background modified to $\sim 0$ winds below 10 km, as shown center right. The spectrum for this second case is nearly identical to the first, indicating that the winds in the troposphere below 10 km have virtually no effect on the stratospheric wave spectrum. The bottom panels of Fig. 1 show a third simulation, this time with constant near-zero winds at all altitudes. The wave spectrum in this case is dramatically altered. The spectrum is now nearly isotropic. The high-phase speed wave
amplitudes are imperceptively altered, but the low NE phase speed peak has disappeared, and the arc of wind filtering is also gone.

These results clearly indicate that the wind shear between 10-17 km is of primary importance to the shape of the resulting wave spectrum, and that the winds below that level have minimal effect.

1.1.2. Trapped waves in the troposphere.

Examination of the model winds in the troposphere reveal trapped waves with a SE preference in their propagation direction (Figure 2). Trapped waves generated by convection have been implicated in remote initiation of subsequent convection in model studies [Mapes, 1993; Shige, 2001]. In the 2-dimensional tropical squall line model study of Shige [2001], a preference for westward trapped wave propagation was found. The upper level shear in their model was westward, in the same sense as in our case study. The authors suggested the westward preference was caused by wave trapping due to an over-reflection mechanism [Lindzen and Tung, 1976]. In our study, we find a preference for trapping of waves propagating in the opposite direction, towards the east. The trapping in our case is best described by an internal reflection mechanism [Lighthill, 1978].

Trapped waves carry no momentum flux vertically, so the cospectra of horizontal and vertical wind perturbations \((u'w', v'w')\) associated with these waves in the model are nearly zero. To examine the properties of the trapped waves, we look at the power spectrum of vertical velocity in the troposphere. To see the spectrum of waves in the troposphere in the spaces between strong vertical velocities occurring within the forced updrafts, the latter must be masked prior to taking the Fourier transform. We chose a height-dependent mask, applying \(|w'| < w_{\text{max}}(z)\), with an empirical profile \(w_{\text{max}}(z)\)
chosen to be just larger than the largest wave perturbations at each height. The value of $w_{\text{max}}(z)$ varies smoothly from 0.375 m s$^{-1}$ at 3 km to 0.85 m s$^{-1}$ at 12 km. Figure 3 shows the power spectra computed from the space-time Fourier cospectra of the masked vertical velocities in the troposphere 3-12 km, averaged over the 4 quadrants, and plotted vs horizontal wavenumber and frequency. The dashed lines are lines of constant phase speed as marked. The features in these spectra show the combined effects of the wave generation mechanisms and trapping.

The high phase speed lobes in each panel in Figure 3 are associated with a tropospheric vertical wavelength of $\sim 33$ km, the response expected from deep convective heating $\sim 16.5$ km. Figure 4 shows the distribution of heating depths input to the model, which in fact peak at 16 km. The second lobe with phase speeds $\sim 20-40$ m s$^{-1}$ is likely associated with enhanced wave trapping. The lowest phase speed lobe in the NE quadrant is a remnant of the imperfect masking of the forced updrafts, not trapped waves.

The peaks in the trapped wave spectrum from the first 4 hrs of the simulation occur for SE propagating waves with $(\omega, k) = (3.2 \text{ hr}^{-1}, 2.3 \times 10^{-2} \text{ km}^{-1})$ and $(0.8 \text{ hr}^{-1}, 1.0 \times 10^{-2} \text{ km}^{-1})$. Using the nonhydrostatic dispersion relation, both of these modes have a ground-relative horizontal group speed of 25 m s$^{-1}$ to the SE. Such trapped waves generated by the Hector in the early afternoon would travel 225 km SE in the 2.5-hr time between the peak Hector and the later development of the squall line to the SE [Alexander et al., 2004], placing these wave perturbations in the right place, at the right time, and with the right orientation to aid in the organization and initiation of the squall line. We estimate the low-level vertical wind perturbations due to these trapped waves to have a magnitude
of $\sim 1 - 1.5$ m s$^{-1}$. We next investigate the spectral selectivity of wave trapping with linear theory in section 2.

2. Wave Transmission and Reflection

To investigate wave trapping and its spectral and directional preference in our model, we examine wave transmission and reflection with linear theory. The form of the Taylor-Goldstein equation for a compressible hydrostatic atmosphere [Lindzen and Tung, 1976] can be solved for non-zero perturbations to give:

$$m^2 = \frac{H^2 N^2}{\hat{c}^2} + \frac{H^2 U_{zz} + HU_z}{\hat{c}} - \frac{1}{4},$$

where $m$ is the vertical wavenumber $U_z$ and $U_{zz}$ are the first and second vertical derivatives of the background wind profile in the direction of wave propagation, $H$ is the pressure scale height, and $N$ the buoyancy frequency. Setting $m = 0$ and solving for $\hat{c}$ gives the critical intrinsic phase speed at which wave reflection should take place $\hat{c}_{\text{crit}}$. (Note: This theory assumes the WKB approximation such that $m^{-1}$ much larger than the vertical background variations, e.g. $U_z/U$, etc.) A plot of $\hat{c}_{\text{crit}}$ vs wave propagation direction and height is shown in Figure 5. White regions are where $\hat{c}_{\text{crit}} > 100$ m s$^{-1}$, and are therefore regions of free propagation for the waves in our model. In regions with smaller values, waves with $\hat{c} > \hat{c}_{\text{crit}}$ should suffer reflection. This simplified theory suggests there may be a region in the SE quadrant ($270-350^\circ$ azimuth) where many waves may be trapped in the troposphere below $\sim 12$ by reflection in the 12-15 km layer. However, shallow layers of low $\hat{c}_{\text{crit}}$ also exist in the troposphere in the SE quadrant, and it is unclear from this analysis whether these layers will be reflecting or transmitting; their scale violates the WKB approximation.
As with any approximate theory, caution must be taken when applying the above. Internal waves can penetrate beyond reflection and critical levels if the background veers with height [Shutts, 1995, 1998], if it varies in time [Eckermann, 1997; Buckley et al., 1999], or if the wavepackets are of sufficiently large amplitude [Sutherland, 2000].

Even for small-amplitude waves in steady, two-dimensional flows, transmission across a reflection level can occur if the depth of the region over which the waves are evanescent is sufficiently small. This circumstance was examined analytically by Sutherland and Yewchuk [2004] for the case of waves propagating in an unsheared background flow with stratification prescribed by piecewise-constant buoyancy frequency. More recently, Nault and Sutherland [2005] computed the structure of small-amplitude waves propagating in an arbitrarily-specified profiles of background stratification, $N^2(z)$ and mean flow, $U(z)$.

To examine the spectral dependence of wave transmission and reflection occurring over vertical distances much smaller than the pressure scale height, we next proceed by integrating the incompressible Boussinesq [Spiegel and Veronis, 1960] form of the Taylor-Goldstein equation:

$$\phi_{zz} - k^2 \left[ -\frac{N^2}{(\omega - kU)^2} - \frac{U_{zz}}{\omega - kU} + 1 \right] \phi = 0, \quad (2)$$

where subscript $z$ denotes vertical derivatives, $\phi(z)$ is the streamfunction amplitude and $k$ and $\omega$ are prescribed values, respectively, of horizontal wavenumber and intrinsic frequency. As we are primarily interested in wave reflection, we focus our examination upon waves with $\omega$ and $k$ chosen so that they do not encounter a critical level. That is, we restrict the phase speed $\omega/k$ so that nowhere does it equal $U$ over the domain of integration and therefore (2) is not singular. In these regions of spectral space, the transmission
coefficient is assumed to be zero, but flagged with a value of $-1$ to distinguish from perfect reflection.

At some vertical level, $z_i$, $\phi_1$ is specified as the superposition of an incident wave propagating in one direction and a guess at the amplitude and phase of a reflected wave that propagates in the opposite direction. Integrating (2) to a vertical level $z_f$ gives the value of $\phi_1(z_f)$ and its derivative. The procedure is repeated first by specifying $\phi_2$ as the superposition of the same incident wave with a different guess at the structure of the reflected wave and then by integrating (2) to find $\phi_2(z_f)$.

Invoking causality, one then determines a linear combination of $\phi_1(z_f)$ and $\phi_2(z_f)$ that ensures the resulting transmitted wave propagates solely in the same direction as the incident wave. Using this linear combination and normalizing, $\phi$ can be constructed everywhere between $z_i$ and $z_f$ so that it describes the structure of an incident wave with amplitude $A_i$ that partially reflects with amplitude $A_r$ and partially transmits with amplitude $A_t$.

Here partial transmission occurs as a consequence of wave propagation in nonuniformly stratified fluid while being Doppler-shifted by background winds. Consistent with corresponding calculations in optics and quantum mechanics, we define the transmission coefficient to be $T \equiv |A_t/A_i|^2$, which represents the fraction of energy that is transported to $z_f$ from waves incident at $z_i$.

Figure 6 shows transmittance for the no wind case (bottom panels of Fig. 1) vs horizontal wavenumber $k$ and frequency $\omega$. A lobe of low transmission occurs centered along a line of constant tropospheric vertical wavelength of 24 km (center dashed line). This represents a wave with approximately twice the wavelength of the depth of the stability
duct, marked by a region of low stability in the upper troposphere between 12-15 km. We also show lines of constant vertical wavelength of 48 km, corresponding to the 1/4 wavelength ducted mode described in Lindzen and Tung [1976] (upper dashed line), and also 17 km corresponding to the depth of the troposphere and the approximate depth of the strongest latent heating cells (lower dashed line). Also plotted for reference is a line of constant phase speed $c = 30 \text{ m s}^{-1}$ which roughly corresponds to the minimum in transmittance.

To show the additional effects of the winds on wave transmission and reflection, we compute transmittance vs phase speed averaged over horizontal wavelength in Figure 7. The no wind case is shown with the dotted line. Wind effects along the diagonal coordinates SW-NE are shown with the gray line, and NW-SE with the solid black line. The upper tropospheric wind shear causes critical level removal of low phase speed westward propagating waves, reduces transmission for eastward propagating waves, and increases transmission for high phase speed westward propagating waves.

The effects of wind reflection can be isolated by computing

$$\Delta = 1 - \frac{T_U}{T_n}$$

where $T_n$ is the transmittance for the no wind case, and $T_U$ the transmittance with wind effects included. The fractional change in transmittance $\Delta$ is shown in the right panel of Figure 6 for both NE and SE propagating waves. The wind effects enhance wave reflection for high phase speed waves $> 50 \text{ m s}^{-1}$.

These results suggest the SE propagating trapped waves in our model troposphere are trapped via a total internal reflection mechanism which is most effective at higher phase speeds. The directional preference of trapping is opposite to that inferred by Shige.
[2001], and occurs via a different mechanism than the over-reflection mechanism they hypothesized. The differences between our results may be explained by differences in the phase speeds and directions of the waves generated in the two models. In the 2-dimensional squall line case modeled by Shige [2001], there is a well-known preference for westward wave generation opposite to the direction of the fast-moving eastward storm propagation (e.g. Fovell et al. [1992]). This likely created an absence of eastward propagating waves that might be trapped via total internal reflection, and placed the westward wave phase speeds relative to the ground in the range of upper level wind speeds in their model that could be trapped via the over-reflection mechanism. The wave generation in our model is instead associated with short-lived, slow-moving, deep heating cells that generate waves in the troposphere with fast phase speeds relative to the ground. (The slow phase speeds in our model are generated above the upper tropospheric shear layer.) Only a small fraction of the tropospheric wave spectrum has a critical level in the upper troposphere. Instead, the relatively high phase speeds lead to a preference for reflection of waves propagating opposite to the upper level winds via the total internal reflection mechanism.

2.1. Quantifying the “Obstacle Effect” Mechanism of Wave Generation

The three experiments shown in Fig. 1 suggested that the upper level wind shear had a pronounced effect on the stratospheric wave spectrum, most notably through generation of low phase speed waves via an obstacle type effect of wave generation, but also through wind filtering effects such as critical level interactions and wave reflection processes. To separate the wave generation and wind filtering effects, we filter the spectrum from the no wind case with the wind transmittance effects show in Fig. 7. We then subtract this
spectrum from the full-wind case spectrum thus isolating the wave generation effects of the shear on the stratospheric wave spectrum.

Figure 8 shows momentum flux spectra vs phase speed integrated over the 4 quadrants of propagation direction. The solid line is the flux from the full wind model, which is the same as that reported in Alexander et al. [2004]. The dotted line shows the flux from the no-wind model times the factor $T_U/T_n$ which accounts for the wind reflection effects, and removes waves that would have critical levels. Except for critical level removal, the wind filtering effects are fairly small, at most $20 - 40\%$, but are included for completeness.

Although differences for westward propagating waves (NW & SW) with phase speeds 10-30 m s$^{-1}$ look large in Fig. 8, it should be noted that the varying ordinate scale between panels exaggerates these effects because the westward fluxes are quite weak. The full wind model westward fluxes at 10-30 m s$^{-1}$ are weaker than the wind-filtered no-wind model because wave refraction to short vertical scales coupled with model diffusion removes portions of the spectrum beyond the phase speeds that encounter critical levels (see also Alexander and Holton [1997]; Beres et al. [2002]. The primary differences occur for NE propagating waves at low phase speeds, as seen in the difference plot Figure 9.

Figure 9 shows the differences between the solid and dotted lines in Figure 8 representing wave fluxes generated via the upper troposphere obstacle effect. Fluxes in the NE direction with phase speeds 5-10 m s$^{-1}$ are by far the largest with a smaller fraction in the SE direction, and additional very small fluxes at near-zero phase speeds in the NW and SW quadrants. The phase speeds of these obstacle effect waves match the speeds and directions of the latent heating cells which peak at 5 m s$^{-1}$ in azimuth 70°, but which
form a distribution of phase speeds $\sim 0 - 15 \text{ m s}^{-1}$ in a range of azimuths from SE to N (Figure 10).

### 2.2. Parameterization of Convectively Generated Gravity Waves

The properties of waves generated by convection are known to be sensitive to the upper troposphere winds and to any motion of the convective heating cells relative to the ground. To apply a parameterization, one must reduce the potentially complex distribution of latent heating cell properties within a grid box area to a few parameters. In the Beres et al. [2005] parameterization application, the motion of the heating cells is assumed to be equal to the large-scale vector wind at 700 mb. The horizontal scales and depths of the heating are set to single parameters, and the time scales are represented by a broad reddened frequency spectrum. Wave reflection effects are neglected. This approach generates a realistic and broad spectrum a gravity wave phase speeds that are important to the most prominent tropical middle atmosphere wind oscillations like the QBO and SAO. The Beres et al. [2005] application did not include wave forcing associated with upper-level wind shear relative to the motion of the heating cells, the so called “obstacle effect”. Although a formulation for wave generation via the obstacle effect is included in Beres et al. [2004], the practical application has large uncertainties. Chun and Baik [1998] and Chun and Baik [2002] also formulated a parameterization for wave generation via the obstacle effect, and it has similar uncertainties. The main issue is that the heating depth is not a single parameter, as must be assumed for parameterization, but a distribution of depths, and the relevant wind speed flowing over the heat cell obstacle can depend sensitively on the depth of the cell. The momentum fluxes generated via this obstacle effect mechanism are quite sensitive to the wind speeds relative to the heating cell [Beres
The relevant wind speed will depend on how deeply the heat cell penetrates into the overlying shear zone, and if the shear is strong, these uncertainties are large, so the resulting uncertainties in the fluxes will be large. For Chun and Baik [1998], the uncertainty is embodied in the simple parameters $\bar{U}_2$, and $a$ for wind and heating depth since it is only the stationary waves relative to the heating that are included in their parameterization. But for the Beres et al. [2004] approach, an additional issue of the momentum flux in this stationary component relative to the momentum flux in the nonstationary waves would add complexity. Although this approach might be more realistic, Beres et al. [2005] chose to omit this complexity because of the large uncertainties.

Figure 11 shows the momentum flux vs phase speed and propagation direction for three sets of parameterization settings described in Table 1. Case W uses the parameterization settings applicable to the assumptions used in Beres et al. [2005] for the WACCM model application. The heating cells are assumed to travel with the speed and direction $(u_c, v_c)$ of either the 700 mb background wind, or this value minus 10 m s$^{-1}$ if the speed exceeds 10 m s$^{-1}$. The cell depth is computed from the half-sine wave fit to the average heating profile in the domain and the heating rate is the average over the domain. The mean wind in the heating region $(U_h, V_h)$ is the average background wind over the depth of the heating. These settings describe the nonstationary wave generation.

The middle panel of Fig. 11 shows case D parameter settings, which have been modified to reflect the known details of the resolved heating in the DAWEX radar domain. Wave reflection from our calculations in section 2 is also included. The primary difference between cases W and D are the effects of wave reflection, with the motion of the heating.
cells \((u_c, v_c)\) having additional small effects. Other differences are negligible. As for case W, case D also describes only the nonstationary wave generation.

Case S describes stationary wave generation, those waves generated via the obstacle effect. The heat cell motions are assumed to fill a distribution given by the shape of the spectrum in Fig. 10. The distribution peaks at \((u_c, v_c) = 5 \text{ m s}^{-1} 70^\circ \text{ north of east}.\) The sum over the distribution in the stationary wave parameterization is normalized to unity, so the distribution then also describes the intermittency of \((u_c, v_c)\) for the parameterized momentum flux calculations. The background wind speed and direction over the top of the obstacle is also a distribution with values spanning the range of winds at 12 km to 15 km. This roughly describes the variations in cell penetration depth into the 12-15 km shear zone. Since these waves are generated above the upper tropospheric shear zone, wave reflection effects are not applicable to this case.

The bottom panel in Fig. 11 shows the sum of stationary and nonstationary wave fluxes cases S + D, which can be compared to the DAWEX model results in the top panel of Fig. 1. The comparison shows that the modified parameterization captures many of the important details of the model fluxes quite well considering the gross simplifications required for the parameterization. Differences include: (1) The critical level filtering extends to larger phase speeds in the model, carving out a larger arc of zero fluxes. The cause is the numerical diffusion in the model, which effectively removes short vertical wavelength waves. (2) The width of the phase speed distribution is somewhat broader in the parameterization than in the model, and the nonstationary wave phase speed peak occurs at higher values in the parameterization \((\sim 25 \text{ m s}^{-1} \text{ vs } \sim 20 \text{ m s}^{-1})\). (3) The
magnitudes of the parameterized fluxes are somewhat larger as expected, 20% larger for NE propagating waves. Nonlinear effects are explored further in the next section.

2.3. Quanitifying Nonlinear Effects

Song et al. [2003] called attention to nonlinear effects in gravity wave generation by convection. They compared wave spectra generated in a 2-dimensional cloud-resolving model to waves generated in a linearized model forced with the latent heating from the full simulation. Their results showed that the linear model generated very similar wave properties to those generated in the full model, but the linear assumption tends to over-predict the wave fluxes, particularly at slower phase speeds.

Our model is nonlinear, so if the magnitude of our input latent heating is accurate, the nonlinear terms missing in the Song et al. [2003] study should be included. However, in Alexander et al. [2004] we found that our input heating rates were likely a factor of 3–5\times too small when we compared the waves to observational constraints.

We therefore examine nonlinear corrections to our model by increasing the strength of the heating by factors of 3 and 5 in two additional simulations, and compare the results in Figure 12. The solid line is the same as in Fig. 8 while dotted and dashed curves show wave fluxes for the 3\times and 5\times simulations divided respectively by factors of 9 and 25. This comparison shows the wave fluxes in the simulations do not scale linearly over this range of a factor of 5, but nonlinear effects are evident, particularly at the lower phase speeds. Nonlinear effects are essentially absent at phase speeds larger than \sim 20 m s^{-1}. The low phase speed nonlinear effects are in the same sense as those reported in Song et al. [2003], but our effects are very much smaller, likely because their linearization was a factor of 10,000\times compared to our factor of 3–5\times. The linear assumption is however a
prominent feature of parameterizations [Chun and Baik, 1998, 2002; Beres et al., 2004], so these should be corrected for nonlinear effects.

3. Discussion

To quantify the differences between wave fluxes in the nonlinear model and the parameterization, we quote here values for NE propagating waves. The nonstationary wave fluxes are 0.27 mPa in the baseline DAWEX model and 0.34 mPa in the linear parameterization. The corresponding stationary wave fluxes are 0.19 mPa in the model and 0.22 mPa in the parameterization.

In the simulations with 3× and 5× heating rates, the nonstationary wave component increases approximately linearly, while the stationary waves increase less than the linear assumption predicts. For the 1×, 3×, and 5× heating cases, the total fluxes in the model are 0.46, 3.6 and 8.1 mPa respectively. (These increases are less than linear, since in the linear assumption, the increase would be by factors of $3^2 = 9$ and $5^2 = 25$.) The deviations from linearity occur in the stationary wave components.

The linear parameterization (cases D+S) predicts 1×, 3×, and 5× heating cases would generate fluxes of 0.56, 5.0 and 14 mPa respectively. For the most likely 3× heating rate values based on the DAWEX study [Alexander et al., 2004], the parameterization overpredicts the fluxes by 39%. The agreement would be significantly worse using the case W parameterization. According to the comparison in our case study, the parameterization benefits most from inclusion of two effects: (1) wave reflection and trapping of a fraction of the high phase speed wave energy in the troposphere, and (2) the generation of waves via the obstacle effect. The importance of including these processes in the parameterization will clearly vary depending on the depths of the heating cells, and on the wind shear and
stability profiles. In this paper, we have described approximate linearized methods that may aid in developing improved versions of the parameterization in future applications, although some additional cases studies would be valuable for this purpose.

4. Conclusions

In our simulations of waves generated by deep heating, internal reflection traps $\sim 60 - 70\%$ of eastward propagating wave energy at phase speeds of $25-50 \text{ m s}^{-1}$. Reflection further traps $50 - 60\%$ of SE propagating waves with phase speed $> 50 \text{ m s}^{-1}$.

An “obstacle effect” is the mechanism responsible for generating waves with low phase speeds and a preference for NE propagation in the model stratosphere. The “obstacles” are the deep heating cells that penetrate into the region of strong westward shear at $z \sim 12 - 15 \text{ km}$.

Comparison to the Beres et al. [2004] parameterization shows good agreement particularly when reflection of high phase speed waves is included as well as stationary waves generated via the obstacle effect.

The parameterization [Beres et al., 2004], based on linear theory, now quantitatively compares well to the model’s higher phase speed, nonstationary wave flux spectrum, but overpredicts the dependence of the low phase speed wave fluxes on the strength of the heating. These increase linearly in the parameterization but less than linearly in the model. The Chun and Baik [2002] parameterization is also based on linear theory, and it neglects nonstationary wave generation, including only the stationary component of the forcing, so it will suffer from similar errors in its dependence on the strength of the heating.
Parameterization of stationary waves in our DAWEX example is also very sensitive to the depth of the heating cells' penetration into the upper level shear zone as well as the motion of the heating cells relative to the ground. These details are not generally known in GCM applications. The parameterization of the stationary wave component therefore remains a challenging problem with large uncertainties.

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Figure 1. Wave momentum flux vs propagation direction and phase speed (left panels) for the three simulations described in the text and their associated wind profiles (right panels) with zonal wind $U$ in black, and meridional wind $V$ in gray. The dashed white line in the top two panels shows the vector stratospheric wind which serves to filter an arc-shaped region of phase speeds. Flux units are $10^{-6}$ kg m$^{-2}$ s$^{-1}$.
**Figure 2.** Vertical velocity at 6-km altitude and 19:03 LT in the model when the forcing is no longer active, and only unforced wave motions remain. A train of trapped SE propagating wave motions are apparent more than $\sim 100$ km from their source. These waves are trapped in the troposphere and propagate horizontally with only weak attenuation.
Figure 3. Vertical velocity power spectra in the troposphere showing the properties of waves in the spaces between the forced convective updrafts. NE, NW, SW, and SE refer to the four quadrants of wave propagation direction, northeastward, northwestward, southwestward, and southeastward, respectively. Dashed lines are constant phase speeds labeled in m s$^{-1}$.
Figure 4. Heating magnitude-weighted distribution of heating depths input into the model. The distributions peak at a heating depth of 16 km.

Figure 5. Contours of $\hat{c}_{\text{crit}}$ versus propagation angle and height.
Figure 6. Transmittance for the no wind case as a function of horizontal wavenumber and frequency with lines of constant vertical wavelengths = 17, 24, 48 km overplotted (dashed) for buoyancy frequency $N = 0.011 \text{ s}^{-1}$ representing the value in the tropospheric duct. A line of constant phase speed = 30 m s$^{-1}$ is also overplotted (dotted) for reference.
Figure 7. Left Panel: Transmittance vs phase speed averaged over horizontal wavelength for the no wind case (dotted) and for the full wind case averaged along the SW-NE line (gray) and along the NW-SE line (solid). Positive phase speeds have an eastward component, and negative westward. Right Panel: The effect of wind reflection vs phase speed for NE (gray) and SE (solid) propagating waves.
Figure 8. Momentum fluxes vs phase speed from the model run with full winds (solid) and that run with no wind filtered with the wave transmittances shown in Figure 6 (dotted). Note the change in ordinate scale between panels: The fluxes are largest by far in the NE propagation direction.
Figure 9. Differences between the full wind and filtered no wind cases (solid minus dashed lines in Fig. 6) showing the momentum flux vs phase speed associated with wave generation via the obstacle effect. Note the change in ordinate scale between panels: The fluxes are largest by far in the NE propagation direction.
**Figure 10.** Convective updraft spectrum vs propagation azimuth and phase speed averaged over altitudes 12.25-15.25 km. The distribution of power shows average speeds and directions of the deep convective cells which move primarily NE at 5 m s$^{-1}$. 
Figure 11. Wave momentum flux vs propagation direction and phase speed for three different sets of parameterization settings described in Table 1: The top panel (case W) uses the Beres et al. [2005] settings that were applied in WACCM. The middle panel (case D) uses settings modified to include the known properties of the resolved convection in the radar domain and wave reflection effects. The bottom panel (cases D + S) adds the “obstacle effect” wave generation (case S) to case D. Flux units are $10^{-6}$ kg m$^{-2}$ s$^{-1}$. 
Table 1. Parameterization settings

<table>
<thead>
<tr>
<th>Case</th>
<th>W</th>
<th>D</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>Beres et al. [2005] modified DAWEX</td>
<td>modified DAWEX</td>
<td>stationary waves</td>
</tr>
<tr>
<td>Heating depth</td>
<td>12 km</td>
<td>12.6 km</td>
<td>12.6 km</td>
</tr>
<tr>
<td>Cell speed ((u_c, v_c))</td>
<td>((-1.47, 0.7) \text{ m s}^{-1})</td>
<td>((1.7, 4.7) \text{ m s}^{-1})</td>
<td>distribution</td>
</tr>
<tr>
<td>Wind ((U_h, V_h))</td>
<td>((-5.19, 1.48) \text{ m s}^{-1})</td>
<td>((-5.19, 1.48) \text{ m s}^{-1})</td>
<td>((-5.0, -5.0) - (-14.8, -2.1) \text{ m s}^{-1})</td>
</tr>
<tr>
<td>Heating rate</td>
<td>5.94 K day(^{-1})</td>
<td>5.94 K day(^{-1})</td>
<td>5.94 K day(^{-1})</td>
</tr>
<tr>
<td>Wave Reflection</td>
<td>neglected</td>
<td>included</td>
<td>n/a</td>
</tr>
</tbody>
</table>

\(\dagger\) For the stationary waves, this is the wind at the top of the obstacle.

Figure 12. Stratospheric wave momentum flux spectrum averaged over the 4 propagation direction quadrants normalized to illustrate nonlinear effects. The solid line is the same as in Fig. 7. The dotted line shows the wave flux generated in a simulation identical except for input heating magnitudes all multiplied by 3, and wave fluxes divided by 9. The dashed line is for 5\(\times\) input heating amplitudes and wave fluxes divided by 25.