The axisymmetric collapse of a mixed patch and internal wave generation in uniformly stratified fluid

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Laboratory experiments are used to investigate the axisymmetric collapse of a localized mixed region in uniformly stratified ambient. The collapsing fluid forms an intrusion and generates vertically propagating internal gravity waves in the stratified ambient. The speed of the intrusion is found to be \((0.085 \pm 0.001)N_0H_m\) where \(H_m\) is the depth of the mixed patch and \(N_0\) is the buoyancy frequency. Internal wave frequencies are set by the buoyancy frequency, \((\omega \approx 0.8N_0)\), and the effective horizontal wavenumber is set by the radius of the cylinder so that \(k_r = 2R_c\). Vertical displacement amplitudes scale with the depth of the mixed patch according to \(|\xi|/(H_m/2) = 0.032 \pm 0.002\) and we find that about 2% of the available potential energy of the mixed region is extracted by the internal waves. Extrapolation of these results to oceanic circumstances of mixed region collapse beneath a hurricane gives a conservative estimate of the power extracted by internal waves during the lifecycle of the storm is estimated to range from \(6 \times 10^{10}\) to \(4 \times 10^{12}\) W. The corresponding power from all hurricanes averaged over the course of a year can range from \(1 \times 10^9\) to \(8 \times 10^9\) W. © 2010 American Institute of Physics. [doi:10.1063/1.3489124]

I. INTRODUCTION

Dense fluid formed in the North Atlantic and Southern oceans sinks to the ocean floor and flows into the abyssal ocean. To maintain the stable stratification observed in the abyss some of the dense bottom water must mix with the lighter overlying fluid. Turbulent patches generated by breaking of the ocean’s internal wave field are primarily responsible for the estimated 2–3 TW necessary to maintain the observed stratification.1 However, the mechanisms by which internal waves are generated are not well understood. The baroclinic conversion of tidal energy into the internal wave field is estimated to account for as much as 1 TW, while the rest of the required power is attributed to surface processes such as the direct forcing of the wind.2

Tropical cyclones are transient events that occur sporadically, but recent studies show they may be an important source of energetics for deep ocean mixing3–5 and nutrient transport.6 These dynamics will be investigated in detail in a large scale observational study of oceanic response to the passage of a tropical cyclone called the interaction of typhoon and ocean project. The study will provide crucial observational evidence of air-ocean interactions before, during, and after the passage of a tropical storm. In situ observations of such events are practically nonexistent due to the difficulty of taking measurements in such extreme weather.

Emanuel5 demonstrated that a substantial amount of the ocean heating required to drive the poleward heat flux may be accounted for by localized mixing due to tropical cyclones. These storms are responsible for as much as 15% of peak ocean heat transport4 and are capable of driving the meridional overturning circulation.3

Recent studies have attempted to quantify the power that is imparted to surface waves, near-inertial interfacial internal waves, and geostrophic currents by the passage of a hurricane.7,8 These studies focused on the dynamics of the upper ocean response to a hurricane and ignored the physical collapse of the mixed region itself.

Tropical storms can mix the ocean to several 100 m depth9 leaving cold water in their wakes10 that return to near prestorm conditions over a period lasting weeks to months through the process of restratification.3 In the absence of differential warming of near surface waters by the Sun, a mixed region of fluid surrounded by the stably stratified ocean will seek to return to its minimum energy state by flowing horizontally along a surface of neutral buoyancy as an intrusive gravity current or, simply, an intrusion.

Lelong and Sundermeyer11 studied the geostrophic adjustment of a mixed patch of fluid in stratified ambient evolving as a function of inertial period. The numerical simulations were intended to model isolated density anomalies created by, for instance, internal wave breaking. Their numerical simulations showed that small scale, high-frequency waves were the first to be generated during the relaxation of the patch.

Laboratory experiments offer a starting point for investigating the dynamics of mixed region collapse and internal wave generation in the stratified ambient beneath. With the intention of extending our results to hurricane driven mixing we begin by examining internal waves in the idealized case which throws out the complicated dynamics of surface winds, turbulence, and rotation. In the absence of rotation, our study focuses on those high frequency waves which propagate over short time scales. That is, waves generated during the initial collapse phase.

Lock-release experiments have been used to examine the
collapse of a mixed region in uniformly stratified fluid.\textsuperscript{12-14} In these, a small patch of mixed fluid is separated by a gate from an ambient fluid. When the gate is removed the mixed region collapses forming an intrusion that propagates horizontally along an isopycnal of equal density.

Wu\textsuperscript{12} observed internal waves generated from the collapse of a localized patch of fluid situated at mid-depth at one side of a rectangular tank filled with uniformly stratified fluid. He was able to observe the generation of internal waves through the displacement of dyed isopycnal surfaces and inferred that the initial collapse was entirely responsible for the generation of internal waves. He also noted that the energy density seems to be peaked around 0.8 of the buoyancy frequency, but was unable to obtain precise measurements of amplitude, wavelength and frequency.

Other noteworthy experiments have explored the collapse of a localized mixed patch in uniformly stratified ambient both near a boundary\textsuperscript{15,16} and with continuous forcing\textsuperscript{17,18} of the mixed patch. The evolution of a continually forced mixed patch at a pycnocline has also been explored.\textsuperscript{19} These studies focused on the evolution of the intrusion and disturbances, such as internal waves, arising at the level of the intrusion. Desilva and Fernando\textsuperscript{18} identified two distinct mechanisms of internal wave generation in a stratified fluid during the collapse. Waves are generated by the force of the propagating intrusion on the background stratification and, as observed by Wu,\textsuperscript{12} internal waves are excited by the initial collapse.

Sutherland \textit{et al.}\textsuperscript{14} were able to measure the characteristics of waves that propagate downward and away from the mixed region and their associated energy transport using a nonintrusive analysis technique called synthetic schlieren.\textsuperscript{20} These partial-depth lock-release experiments were restricted to rectilinear geometries and did not account for the effects of rotation.

As a step toward understanding the generation of internal waves by the initial collapse, here, we present the first examination of internal waves excited by the axisymmetric collapse of a mixed region in a uniformly stratified ambient. Specifically, we estimate the energy associated with waves which propagate into the stratified ambient beneath the mixed region.

The extension to axisymmetric geometries is nontrivial because as the intrusion advances radially its height must decrease in vertical extent as a result of mass conservation. Furthermore, whereas the measurement of the spanwise-uniform wavefield is relatively straightforward, the measurement of the conical wavefield that results in our experiments requires the application of a recently developed “axisymmetric schlieren” method.\textsuperscript{21}

Details of the experimental setup and image processing are described in Sec. II. The methods used to compute wave frequency, radial wavenumber, and vertical displacement amplitude are explained in Sec. III and the fraction of the system’s available potential energy extracted by the waves is given in Sec. IV. In Sec. V the results are scaled so as to predict wave energetics on oceanic scales.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Schematic of experimental setup showing (a) top view of the tank showing a hollow cylinder of radius $R_c$ inserted at the center of the tank. (b) Front view showing the partial depth, $H_{cyl}$ to which the cylinder was inserted. The conductivity probe traverses the fluid through the center of the cylinder. (c) Density profile measurements before and after the fluid in the cylinder was mixed. The measured height of the mixed region, $H_m$, is indicated.}
\end{figure}

II. EXPERIMENTAL SETUP

A schematic diagram of the laboratory setup is shown in Fig. 1(a). In order to take the best advantage of the schlieren visualization method, the experiments were conducted in a rectangular tank that is $H=48.5$ cm in height with a square base measuring $L=47.5$ cm on each side. The “double-bucket” technique\textsuperscript{22} was used to fill the tank with uniformly salt stratified fluid to a depth of approximately 45 cm.

To capture the small-scale motions of vertically propagating internal waves, a digital camera was situated in front of the tank and a screen of black and white lines was attached to the back. An array of fluorescent lights provided back illumination of the image screen from which light traveled through the tank. The Sony DCR-TRV6 charge-coupled device camera, situated $L_c=400$ cm from the tank, was zoomed in so that the tank filled the field of view of the camera. The corresponding pixel resolution was 0.067 cm.

A hollow cylinder was carefully inserted into the center of the tank to a partial depth, $H_{cyl}$. To examine the effect of varying the mixed region depth, three successive experiments were performed with $H_{cyl}=5, 10$, and 15 cm. The setup is illustrated in Fig. 1(b). Two cylinders of radii $R_c=3.85$ and 5.05 cm were used in the experiments. The fluid within the transparent cylinder was then thoroughly mixed to a depth $H_m$ moderately above $H_{cyl}$ with an oscillating mechanical stirrer. The fluid within the cylinder was then allowed to settle until turbulent motions subsided.

Traversing the fluid vertically, a conductivity probe (Precision Measurement Engineering Model II) provided a measure of the density as a function of depth. Two measurements were taken with the probe inserted through the center of the cylinder. The first was taken before the fluid within the cylinder was mixed and provided a measure of the back-
ground density gradient. The second probe measurement was taken after the fluid in the cylinder was mixed and, from the density profile, the depth of the mixed region, \( H_m \), was determined.

An example of the density profiles for a particular experiment is shown in Fig. 1(c). The background density profile shows that the fluid was uniformly stratified throughout the entire depth of the tank. The profile taken after the fluid in the cylinder was mixed shows that the density remained constant over the depth of the well-mixed region followed by a small transition region below which the fluid was uniformly stratified. The measured depth of the mixed region, \( H_m \), was taken to be the distance between the surface and the top of the transition region.

The background density gradient was calculated by fitting a line to the ambient density profile, \( \bar{\rho}(z) \), below the depth of the bottom of the cylinder. From this we determined the buoyancy frequency \( N_0 = \sqrt{g(r\rho - \rho_0)(d\bar{\rho}/dz)} \), which ranged from 1.2 to 1.6 s\(^{-1}\).

The experiment proceeded by rapidly extracting the cylinder vertically. The mixed fluid collapsed under the force of buoyancy, moving upward and downward toward a level of neutral buoyancy. This motion caused the column of fluid beneath the cylinder to undulate up and down and internal waves immediately began to emerge from the undulating column.

Intruding into the stratified ambient, the mixed fluid flowed horizontally at a depth below the surface approximately equal to half the depth of the mixed region, \( H_m/2 \). A top view of the experiment confirmed that the spreading of the intrusion was axisymmetric, and from the front view of the tank, we observed that the intrusion head thinned as it spread radially. It traveled at a near constant speed for the first two buoyancy periods then slowed over a duration of about one buoyancy period and halted before reaching the edge of the tank.

In a few of the experiments dye was added to the mixed fluid to visualize the intrusion and to determine its speed. Figure 2 shows the evolution of the intrusion over three buoyancy periods in an experiment where \( H_m \approx 4.2 \) cm, \( N_0 \approx 1.4 \) s\(^{-1}\), and \( R_c \approx 5.05 \) cm.

Isopycnal surfaces disturbed by internal waves caused the local density gradient to increase or decrease relative to the background density gradient. Synthetic schlieren makes use of the optical principle that light rays bend more where the refractive index changes rapidly. By measuring vertical displacements of the image behind the tank of black and white lines having thickness (3 mm) and assuming the waves are axisymmetric, a simple matrix inversion is used to determine the vertical gradient of the fluctuation density field, \( \partial_z \rho \).

This field is directly proportional to the change in the squared buoyancy frequency due to the waves \( \Delta N^2(r,z,t) = -(g/\rho_0)\partial_z \rho \). The time rate of change of this field, \( N_t^2 \), is calculated by measuring differences in \( \Delta N^2 \) between successive frames taken \( \Delta t = 0.04 \) s apart and has the effect of filtering slowly evolving variations such as long hydrostatic waves at the level of the intrusion or ambient temperature and lighting changes in the laboratory. The \( N_t^2 \) field enhances changes due to internal waves which occur over relatively fast time-scales.

Figure 3 demonstrates the procedure used to calculate the \( N_t^2 \) field. The digital image shown in Fig. 3(a) was taken after the fluid in the transparent cylinder was mixed and turbulent motions had ceased. It illustrates the thin horizontal lines that make up the schlieren image screen. In this experiment \( H_m \approx 10 \) cm, \( N_0^2 \approx 2.3 \) s\(^{-2}\) and \( R_c \approx 5.05 \) cm. Comparing an image taken two buoyancy periods after the extraction of the cylinder with an image taken \( \Delta t \) earlier we computed the vertical velocity of the apparent displacement, \( \partial_z \) shown in Fig. 3(b).

Finally the \( \partial_z \) field was axisymmetrically inverted to produce the \( N_t^2 \) field shown in Fig. 3(c). Specifically, the image to the right of the centerline was inverted to determine the wavefield to the right. Separately, the left image of the \( \partial_z \) field was used to find the wavefield to the left. Figure 3(c) shows an example of the \( N_t^2 \) field which results from processing each side independently. If the processed images did not exhibit satisfactory reflection symmetry about the vertical center line then the assumption of axisymmetry was shown to be incorrect (presumably because the cylinder was not well extracted vertically) and the experiment was excluded from analysis. For those experiments which did display the requisite symmetry, the entire \( N_t^2 \) field was determined from the right side of the image and used for subsequent analysis. Figure 4 shows the evolution over three buoyancy periods of the \( N_t^2 \) field for the same experiment as that shown in Fig. 3. The images show the downward propagation of...
waves cones emanating from the bottom of the collapsing mixed region. From this field the frequency, radial wavenumber, vertical displacement amplitude, and integrated energy flux were computed.

Reynolds numbers were determined for the intrusive gravity current as well as for the vertically propagating internal waves. For the intrusion, $Re=N_0 H^2/\nu$ ranges from $4.3 \times 10^4$ to $3.8 \times 10^4$. Using a characteristic velocity of $U=\omega/k_r$ and a characteristic length scale of $L=k_z^{-1}$ the Reynolds number for the internal waves ranged from $2.1 \times 10^2$ to $8.3 \times 10^2$. These values were large enough that viscosity did not play a significant role in the evolution of the flow.

III. THEORY AND ANALYSIS

The Navier–Stokes equations for small-amplitude, axisymmetric waves in a Boussinesq, incompressible, and inviscid fluid are used to derive a differential equation describing the evolution of the streamfunction, $\psi(r,z,t)$, in polar coordinates. The streamfunction is defined so that $\vec{u}=\nabla \times (\psi \hat{\theta})$ and its structure may be represented by a superposition of Bessel and complex exponential functions.

The streamfunction and all other fields of interest are related to the vertical displacement, $\xi$, through the polarization relations listed in Table I. Using the equations of motion, these relationships were derived by defining the vertical displacement such that $w=\partial \xi/\partial t$.

At a fixed vertical level $z$, the Fourier–Bessel transform is used to compute internal wave properties from radial time series of the $N^2_r$ field using

\[ N^2_r(r,t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{nm}^2(k_n,\omega_m)J_0(k_r r)e^{-i\omega_m t} \] (1)

in which it is understood that the field is the real part of the sum. Here $k_n=\alpha_n/R$ is the radial wavenumber given in terms of the zeroes $\omega_m$ of $J_0(r)$ where the radial extent $R=20$ cm is just less than half of the width of the tank, $L/2$. We define $\omega_m=(2\pi/T) m$ where $T$ is the duration of the time series. The vertical wavenumber can then be deduced from the dispersion relation $k_z=k_z(n/\omega)^2-1$.

Figure 5 demonstrates the technique used to compute the peak values of frequency and wavenumber using the same experimental data that were used to produce Fig. 3. First, the radial time series shown in Fig. 5(a) was transformed using the Fourier–Bessel series expansion given by Eq. (1) to find the amplitudes $A_{nm}^2$. These were squared in magnitude to create the contour plot shown in Fig. 5(b), which indicates the power associated with each discrete frequency and radial wavenumber. There is some spread in both frequency and wavenumber, yet most of the power is concentrated within a narrow range.

Isolating the dominant frequency involves Fourier transforming several vertical slices of the radial time series taken from the $N^2_r$ field. At fixed $r$, the squared Fourier coefficients are plotted as a function of frequency to create a Fourier power spectrum. The peak of this spectrum, $\omega_*$, determines the central frequency of the wavepacket at each $r$ [Fig. 5(c)]. Similarly, the peak radial wavenumbers, $k_*$, were found for fixed times $t$ from the power determined from successive Bessel transforms of $N^2_r$ [Fig. 5(d)].

Because of the finite domain size, Fourier–Bessel coefficients were determined at discrete values of $k_n$ and $\omega_m$. To increase precision in our determination of the dominant frequencies and wavenumbers a parabola was fit to three points
about the maximum value in each power spectrum and the peak of that parabola was taken to be the peak of the spectrum.

Figure 6(a) shows the peak frequency, \( \omega_0 \), for a range of \( r \). The radial locations were chosen near the center of the tank where the highest amplitude undulations of the fluid occurred and hence where the signal was strongest. Figure 6(b) shows \( k_r \) for several time steps chosen every eighth of a period over two buoyancy periods beginning one buoyancy period after the cylinder was extracted. These plots were averaged to obtain representative values of \( \omega_0 \) and \( k_r \) for waves at each vertical position \( z \), along with their associated error estimates.

Finally, the peak frequencies and peak radial wavenumbers were averaged over a range of heights to arrive at a characteristic wavenumber \( \overline{k}_r \) and frequency \( \overline{\omega}_r \) for each experiment.

The amplitude of the vertical displacement field was derived from the \( N^2 \) field using the relationship between \( A_n \) and \( A_{N^2} \) given by the polarization relations in Table I so that at a fixed level \( z \)

\[ \hat{\xi}(r, t) = \sum_{n=0}^{N} \sum_{m=0}^{M} -A_{N^2}(k_m, \omega_m) \varphi_{0}(k_m r) e^{-i\omega_m t}. \]  

The maximum value of \( |\hat{\xi}| \) was found for several radial time series taken at different vertical locations to obtain the maximum vertical displacement of the waves near \( r=0 \). A single characteristic vertical displacement amplitude was found for each experiment by averaging over \( z \in [H_m-11, H_m-9] \) and the standard deviation of these values was used as a measure of uncertainty.

The range of heights used to evaluate \( |\hat{\xi}| \) were chosen as high as possible to maximize the amount of information that could be acquired before any waves reflected on the tank walls, while ensuring that any erroneous schlieren measurements, due to turbulence in the mixed region, were avoided. For consistency among the experiments, the vertical slices used in the calculation of frequency and wavenumber, amplitude, and energy were taken, in a 2 cm range, approximately 10 cm below the mixed region, \( H_m \).

The flux of energy, \( F_E(z) = \int_0^\infty \int_0^{\infty} \rho_0 \omega \varphi \, dr \, d\theta \), due to the internal waves crossing a fixed vertical level is transient during the process of collapse and restratification. Waves propagate downward and away from the mixed region eventually reaching the side of the tank and reflecting back toward the center. However, experiments show that wave generation was most substantial during the first few buoyancy periods. To capture the majority of energy extracted by the waves while avoiding interference from reflecting waves at late times, \( F_E \) was integrated over two buoyancy periods to obtain the energy as

\[ E = \int_{T_1}^{T_2} F_E |z| \, dt = 2\pi R^2 \rho_0 \sum_{n=0}^{N} \sum_{m=0}^{M} A_{N^2}^2(k_m, \omega_m) J_0(k_m r) e^{-i\omega_m t}. \]  

In most experiments \( T_1 \) was taken to be one buoyancy period, \( T_b = 2\pi / N_0 \), after the start of the experiment. However, when \( H_m \) was small the waves took longer to propagate well below the mixed region so \( T_1 = 2T_b \) was used for those experiments. In either case \( T_2 = T_1 + 2T_b \).

To ensure that the energy estimate was as accurate as possible, the calculation involved summing over all frequencies and wavenumbers of the Fourier–Bessel spectrum. The integrated flux taken over two buoyancy periods decreased with increasing depth below the mixed region. For example, the profile for the experiment shown in Fig. 3 is plotted in Fig. 7. Large values immediately below the mixed region were a consequence of errors in the schlieren measurement which gave unphysical results where the motion was turbulent. Further below, the energy decreased due to the transient nature of the waves that took a longer time to propagate a greater distance below the collapsing mixed patch.

A conservative estimate of the energy flux was therefore taken to be the average value of \( F_E(z) \) over a 2 cm range about a depth of 10 cm below the mixed region. Integration of the mean energy flux in time gives the energy through Eq. (3).

For analysis purposes, the energy is normalized by the initial available potential energy (APE). This is given by

\[ APE = \int_0^{2\pi} \int_0^R \int_0^{H_m} (\overline{\rho} - \overline{\rho_f}) g z r dr dz d\theta \]  

in which \( \overline{\rho} \) and \( \overline{\rho_f} \) are initial and final density profiles, respectively, as determined by probe measurements shown, for example, in Fig. 1. In the absence of mixing, the entire volume of fluid in the cylindrical lock would collapse to form a thin horizontal layer of uniform density fluid.
mental setup, the layer thickness is so small so that the final state is well approximated as a continuously stratified fluid. Thus, an estimate of the APE that ignores mixing may be expressed in terms of explicitly determined experimental parameters by

$$APE = \frac{\pi \rho_0 H_m^2 R_c^2 N_0^2}{12}.$$  \hspace{1cm} (5)

IV. RESULTS

To determine if the waves were excited by the initial collapse as opposed to the propagating intrusion, intrusion speeds, $U$, were found by applying linear regression to radial time series that track the intrusion head at its level of neutral buoyancy. For a series of these experiments, $U$ is plotted against the characteristic velocity scale $N_0 H_m$ (not shown). The slope of the best-fit line passing through the origin is the dimensionless Froude number $Fr=U/N_0 H_m=0.085 \pm 0.001$. This is significantly smaller than the value $Fr=0.13 \pm 0.02$ measured for rectilinear intrusions generated by the collapse of a mixed region in stratified ambient and smaller than the value $Fr=0.125$ predicted by linear theory.26

The axisymmetric geometry used in our experiments may act to slow the intrusion through lateral spreading and consequent reduction of head height. A detailed explanation for the lower and constant observed speed is being examined in research distinct from this study.

If internal waves are excited by the head of the intrusion then the speed of the intrusion should set the phase speed of the waves, $c_p=\omega/k_r$, and the plot of $\omega/N_0$ against $k_r H_m$ should exhibit the same linear relationship as $U$ against $N_0 H_m$. Figure 8 shows this is not the case. Instead, $\omega/N_0$ is relatively constant with respect to $k_r H_m$. Moreover, snapshot images show that lines of constant phase emanate from below the mixed region and not the intrusion front. This further suggests that the oscillating fluid below the collapsing mixed region launches the waves.

The values of $\omega/N_0$, which lie within a narrow range about 0.8, indicate that the frequency of the waves is set by the buoyancy frequency. A narrow range of relative internal wave frequencies was also observed in rectilinear studies of collapsing mixed regions in uniformly stratified fluid for which $\omega/N_0 \in (0.6, 0.8)$. It is at a frequency of $0.8 N_0$ that internal gravity waves most efficiently transport energy and hence exert the greatest feedback on the source that generates them.

We expect that the radius of the cylinder sets the wavelength of the waves. Figure 9 shows that the value of $k_r$ relative to the radius of the cylinder lies in a range of $k_r R_c$ between 1.5 and 3.5, with most values near 2. Using the relationship $k_r R_c = 2$, we find the first zero crossing of $J_0(k_r)$ is $\alpha_0 = 1.2 R_c$. Because this is approximately equal to the radius of the cylinder, it strongly supports the hypothesis that the radius of the cylinder sets the radial wavelength, $k_r$. An asymptotic expansion of the Bessel function shows that the wavelength is $2 \pi/k_r$ from the source with amplitude decaying as $r^{-3/2}$.

The maximum vertical displacement amplitude is plotted against the height of the mixed region in Fig. 10. Although the amplitudes do not vary much from experiment to experiment, over a narrow range the amplitude is observed to increase linearly as a function of the depth of the mixed region. In the absence of a mixed region ($H_m=0$) no internal waves are generated and so $|\xi|=0$. A line passing through the origin is fit to the data resulting in a slope of $|\xi|/(H_m/2) = 0.032 \pm 0.002$. That is, the vertical displacement amplitude

![FIG. 8. Nondimensional plot of $\omega/N_0$ against $k_r H_m$. The dotted line has slope $Fr=0.085 \pm 0.001$ which is the Froude number measured from the intrusion speeds. A characteristic vertical error bar is shown in the lower left hand corner.](image)

![FIG. 9. Relative frequency plotted with relative radial wavenumber for a range of experiments. A characteristic vertical error bar is shown in the lower left hand corner.](image)

![FIG. 10. The maximum amplitude of the vertical displacement field plotted against half the height of the mixed region. A best fit line passes through the origin with given slope.](image)
is 3.2% the half-depth of the mixed region. This small fraction indicates that most of the energy released by the collapse goes into the radial motion of the intrusion and return flows.

Figure 11 shows the potential energy carried away by downward propagating internal waves for each experiment over two buoyancy periods given as a percentage of the APE. Despite the scatter in Fig. 11, the measurements give a useful order-of-magnitude estimate of the energy extracted by the waves. The average amount of potential energy imparted to the internal wavefield from the mixed region is found to be on the order of 2%. This fraction may be considered an underestimate because it measures only the energy associated with the transient waves occurring before they reflect from the boundaries of the tank.

This result is smaller than that found for rectilinear intrusions, in which almost 10% of the APE was extracted by internal waves over two buoyancy periods. The fraction of energy extraction is consistent with a recent study examining internal waves generated by axisymmetric convective plumes, for which the energy associated with the internal waves at the level of neutral buoyancy was around 4% of the plume’s kinetic energy.

An explanation for the relative decrease in internal wave energy is provided by examining the relationship between the collapsing fluid and the internal wavefield. This is mediated by the return flow that moves inward immediately above and below the intrusion in order to replace fluid lost to the outflow. The energy imparted to internal waves is governed by the kinetic energy and ultimately the speed in the return flow which, in turn, is governed by the intrusion’s speed. Because mass conservation dictates that the nose of the intrusion decreases in height as it spreads laterally in an axisymmetric geometry, the return flow is not as fast as the rectilinear case.

V. DISCUSSION AND CONCLUSIONS

The axisymmetric collapse of a localized mixed region in a stratified ambient generates an intrusion and excites vertically propagating internal gravity waves in the underlying stratified fluid. Our analysis suggests that the frequency of these waves is set by the buoyancy frequency of the fluid and the wavelength is set by the radius of the mixed patch. Vertical displacement amplitudes are relatively small and increase as the depth of the patch increases according to the relation $|\mathcal{g}|/(H_m/2) = 0.032 \pm 0.002$. We found the amount of energy transported by the waves over two buoyancy periods to be on the order of 2% of the system’s available potential energy.

In the absence of rotational effects, turbulence, and wind induced currents, adapting our results directly to oceanic circumstances of storms generating internal waves is premature, but is the first step toward quantifying the fraction of energy available for deep ocean mixing from the collapsing mixed region following a moving hurricane. Since the waves scale with the collapsing mixed region, we can conclude that turbulence within the mixed patch does not contribute to the generation of the measured internal waves. Subsequent energy considerations are specifically associated with the collapse of the mixed region as a mechanism for the generation of internal waves which transport energy to the region underlying the mixed region.

Emanuel gives a simple example using parameters characteristic of Hurricane Edouard, which left a cold water wake with a temperature change of 3 °C to a depth $H_m \approx 50$ m, and along-track and cross-track lengths of 2000 and 400 km, respectively.

Using the established relationship between the height of the mixed region and the vertical displacement field, we predict the amplitude of the waves to be $|\mathcal{g}| = 0.8$ m. The area traversed by a hurricane is not radially symmetric, but for predictive purposes we assume that the radius of the mixed patch is at least as big as half the crosstrack dimension, 200 km. Because we found $k_r \approx 2/R_c$ we predict that the radial wavelength of the internal waves would be $\lambda \approx 600$ km.

Buoyancy frequencies observed in the open ocean can vary from about $0.0008$ (Ref. 28) to $0.009$ s$^{-1}$ (Ref. 29) so that the buoyancy period, $T_b$, can range from 10 min to over 2 h. Since $\omega = 0.8N$, corresponding wave frequencies range from $6 \times 10^{-4}$ to $7.2 \times 10^{-3}$ s$^{-1}$.

Liu et al. estimates that the passing hurricane increases the system’s available potential energy at a rate of 0.16 TW so we conservatively assume that $10^{17}$ J of APE contributes to the collapse of the mixed region during the 18 day lifespan of the storm. The amount of energy extracted by the internal waves in our experiments varied from about 1%–5% of the system’s available potential energy which amounts to $10^{15} - 5 \times 10^{15}$ J of wave energy over $2T_b$. This indicates that $6 \times 10^{10} - 4 \times 10^{12}$ W of power is input to the internal wavefield from a single moving hurricane over two buoyancy periods. These values are comparable to the steady and globally distributed power associated with the tides, but it must be emphasized that wave generation by a hurricane is a localized and transient event.

Hurricanes occur infrequently and over short timescales. So a globally averaged annual estimate of the energy extracted by deep ocean internal waves provides a more conservative estimate of the influence of hurricane-generated internal waves relative to those generated by the tides.
There are about 80 tropical cyclones each year\textsuperscript{5} of which about 50 develop into hurricane strength storms. Using an annual average, we find that the power input from tropical cyclones to the internal wavefield from the collapse of the mixed region ranges from $1 \times 10^9$ to $8 \times 10^9$ W. This result is comparable to the 1 GW estimate by Nilsson \textit{et al.}\textsuperscript{8} of the power transported away, horizontally, by near inertial waves that manifest as undulations of the thermocline. Because the internal waves in our study propagate vertically, they would have a more significant impact upon mixing in the oceanic abyss.

By extrapolating the results of our laboratory experiments, we have shown that internal waves left in the wake of a moving hurricane can transport a significant amount of energy through the stable stratification of the deep ocean abyss where they may contribute to deep ocean mixing. Because hurricanes and mixed region collapse evolve on time scales comparable to the earth's rotational period, in the next stage of our research we will extend these experiments to abyssal where they may contribute to deep ocean mixing. Because hurricanes and mixed region collapse evolve on time scales comparable to the earth's rotational period, in the next stage of our research we will extend these experiments to abyssal where they may contribute to deep ocean mixing. Because hurricanes and mixed region collapse evolve on time scales comparable to the earth's rotational period, in the next stage of our research we will extend these experiments to abyssal where they may contribute to deep ocean mixing. 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