

Handy Integrals of Trigonometric Functions

$$\begin{aligned}\int \cos(ax) \, dx &= \frac{1}{a} \sin(ax) + C \\ \int x \cos(ax) \, dx &= \frac{1}{a} x \sin(ax) + \frac{1}{a^2} \cos(ax) + C \\ \int x^2 \cos(ax) \, dx &= \frac{1}{a} x^2 \sin(ax) + \frac{2}{a^2} x \cos(ax) - \frac{2}{a^3} \sin(ax) + C \\ \int \sin(ax) \, dx &= -\frac{1}{a} \cos(ax) + C \\ \int x \sin(ax) \, dx &= -\frac{1}{a} x \cos(ax) + \frac{1}{a^2} \sin(ax) + C \\ \int x^2 \sin(ax) \, dx &= -\frac{1}{a} x^2 \cos(ax) + \frac{2}{a^2} x \sin(ax) + \frac{2}{a^3} \cos(ax) + C\end{aligned}$$

$$\begin{aligned}\int \sin(ax) \sin(bx) \, dx &= \int \frac{1}{2} (-\cos[(a+b)x] + \cos[(a-b)x]) \, dx \\ \int \cos(ax) \cos(bx) \, dx &= \int \frac{1}{2} (\cos[(a+b)x] + \cos[(a-b)x]) \, dx \\ \int \sin(ax) \cos(bx) \, dx &= \int \frac{1}{2} (\sin[(a+b)x] + \sin[(a-b)x]) \, dx\end{aligned}$$

$$\begin{aligned}\int_0^L \sin(n\pi x/L) \sin(m\pi x/L) \, dx &= \begin{cases} 0 & n \neq m \\ L/2 & n = m \end{cases} \\ \int_0^L \cos(n\pi x/L) \cos(m\pi x/L) \, dx &= \begin{cases} 0 & n \neq m \\ L/2 & n = m \end{cases} \\ \int_0^L \sin(n\pi x/L) \cos(m\pi x/L) \, dx &= \begin{cases} \frac{2nL}{\pi(n^2-m^2)} & n \neq m, n+m - \text{odd} \\ 0 & n \neq m, n+m - \text{even} \\ 0 & n = m \end{cases}\end{aligned}$$