Trigonometric Identities

- Memorize this (Euler's formula):

  $$e^{\imath\theta} = \cos\theta + \imath\sin\theta,$$

  in which $\imath = \sqrt{-1}$.

- Also memorize this:

  $$cos^2\theta + \sin^2\theta = 1.$$

From these two formulae, frequently occurring trigonometric identities involving cosine and sine can be derived.

For example, note:

$$e^{\imath 2\theta} = \cos 2\theta + \imath\sin 2\theta.$$

But

$$\begin{aligned} e^{\imath 2\theta} &= \left(e^{\imath\theta}\right)^2 \\ &= (\cos\theta + \imath\sin\theta)^2 \\ &= \cos^2\theta + 2\imath\cos\theta\sin\theta + \imath^2\sin^2\theta \\ &= (\cos^2\theta - \sin^2\theta) + \imath(2\cos\theta\sin\theta). \end{aligned}$$

Comparing real and imaginary parts gives

$$\begin{aligned} \cos 2\theta &= \cos^2\theta - \sin^2\theta, \\ \sin 2\theta &= 2\cos\theta\sin\theta. \end{aligned}$$

Using $cos^2\theta + \sin^2\theta = 1$, we can also write

$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta.$$

Similarly, by noting

$$(\cos(a) + \imath\sin(a))(\cos(b) + \imath\sin(b)) = e^{\imath a}e^{\imath b} = e^{\imath(a+b)} = \cos(a+b) + \imath\sin(a+b),$$

one finds

$$\begin{aligned} \cos(a+b) &= \cos a\cos b - \sin a\sin b \\ \sin(a+b) &= \cos a\sin b + \cos b\sin a \end{aligned}$$