Approximate representations of J_0

- For small r:

$$J_0(r) = \sum_{k=0}^{\infty} \frac{(-1)^k}{2^{2k}(k!)^2} r^{2k} \simeq 1 - \frac{1}{4}r^2$$

First two terms are reasonably accurate for $0 \leq r \overset{<}{\sim} 1$.

- As $r \to \infty$:

$$J_0(r) = \simeq \sqrt{\frac{2}{\pi}} r^{-1/2} \cos(r - \pi/4)$$

This term is reasonably accurate for $r \overset{>}{\sim} 1$.

Asymptotic approximations to J₀(r)