Eigenfunctions Determined from J_0

The bounded set of eigenfunction solutions of

$$r^2R'' + rR' + \lambda r^2 R = 0$$

with $R(a) = 0$ is given by

$$\{R_n(r) = J_0(\alpha_n r/a) \,|\, n = 0, 1, 2, \ldots\}$$

corresponding to eigenvalues $\lambda_n = (\alpha_n/a)^2$. Here, α_n are the zeros of $J_0(r)$ (*i.e.* $J_0(\alpha_n) = 0$). Some values are $\alpha_0 \simeq 2.405$, $\alpha_1 \simeq 5.520$, $\alpha_2 \simeq 8.654$.

The figure below shows the first three eigenfunctions in the case $a = 1$.