Fourier Series: Summary

Range of $f(x)$	Series type	short cuts	formulae
1) $[-L, L]$	a) Fourier series	i) none	$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\pi x/L) + b_n \sin(n\pi x/L)$
			$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos(n\pi x/L)\ dx$, $b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin(n\pi x/L)\ dx$
		ii) $L = \pi$	$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$
			$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)\ dx$, $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)\ dx$
		iii) $f(x)$ even	$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\pi x/L)$
			$a_n = \frac{2}{L} \int_{0}^{L} f(x) \cos(n\pi x/L)\ dx$
		iv) $f(x)$ odd	$f(x) = \sum_{n=1} b_n \sin(n\pi x/L)$
			$b_n = \frac{2}{L} \int_{0}^{L} f(x) \sin(n\pi x/L)\ dx$
	b) complex Fourier series	i) none	$f(x) = \sum_{n=-\infty}^{\infty} c_n \exp(\imath n\pi x/L)$
			$c_n = \frac{1}{2L} \int_{-L}^{L} f(x) \exp(-\imath n\pi x/L)\ dx$
		ii) $L = \pi$	$f(x) = \sum_{n=-\infty}^{\infty} c_n \exp(\imath nx)$
			$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \exp(-\imath nx)\ dx$
		iii) $f(x)$ real	$f(x) = c_0 + \sum_{n=1}^{\infty} c_n \exp(\imath n\pi x/L) + {c_n}^{\star} \exp(-\imath n\pi x/L)$
			$c_{-n} = (c_n)^{\star}$ (complex conjugate)
		iv) $f(x)$ real, even	$f(x) = c_0 + \sum_{n=1}^{\infty} c_n[\exp(\imath n\pi x/L) + \exp(-\imath n\pi x/L)]$
			$= c_0 + \sum_{n=1}^{\infty} 2c_n \cos(n\pi x/L)$
			$c_n = \frac{1}{L} \int_{0}^{L} f(x) \cos(n\pi x/L)\ dx = c_{-n}$
		v) $f(x)$ real, odd	$f(x) = \sum_{n=1}^{\infty} c_n[\exp(\imath n\pi x/L) - \exp(-\imath n\pi x/L)]$
			$= \sum_{n=1}^{\infty} 2\imath c_n \sin(n\pi x/L)$
			$c_n = \frac{-\imath}{L} \int_{0}^{L} f(x) \sin(n\pi x/L)\ dx = -c_{-n}$
2) $[0, L]$	a) Fourier cosine series		$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\pi x/L)$
			$a_n = \frac{2}{L} \int_{0}^{L} f(x) \cos(n\pi x/L)\ dx$
	b) Fourier sine series		$f(x) = \sum_{n=1} b_n \sin(n\pi x/L)$
			$b_n = \frac{2}{L} \int_{0}^{L} f(x) \sin(n\pi x/L)\ dx$
3) $[-\infty, \infty]$	Fourier transform		$f(x) = \int_{-\infty}^{\infty} F(\alpha) \exp(\imath \alpha x)\ d\alpha$
			$F(\alpha) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) \exp(-\imath \alpha x)\ dx$