1. [4 marks] The 40-kg boy has taken a running jump from the upper surface and lands on his 5-kg skateboard shown.
   (a) [2 marks] What is the common speed \( v \) of the boy and the skateboard along the horizontal surface immediately after the impact?
   (b) [2 marks] Neglecting the impulse of weight, what is the impulse of the normal force exerted by the surface on the skateboard wheels during the impact?

Solution:

(a) Linear momentum conservation in the horizontal direction:

\[
m_b v_b \cos 30^\circ = (m_b + m_s) v.
\]

\[
\Rightarrow \quad v = \frac{m_b v_b \cos 30^\circ}{m_b + m_s} = \frac{(40)(5) \cos 30^\circ}{40 + 5} = 3.85 \text{ m/s}.
\]

(b) In the vertical direction (let upward be positive):

Linear momentum of the system (boy + skateboard) before impact is

\[
-m_b v_b \sin 30^\circ = -100 \text{ N} \cdot \text{s}
\]

Linear momentum of the system after impact is 0. [0.5 marks]

Neglecting the impulse of the weight and applying the linear impulse and momentum principle in the vertical direction, the impulse of the normal force is

\[
I = 0 - (-100) = 100 \text{ N} \cdot \text{s}.
\]

2. [7 marks] The cue gives the cue ball A a velocity parallel to the y axis. The cue ball hits ball B and knocks it straight into the corner pocket. If the speed of the cue ball just before the impact is 2 m/s and the coefficient of restitution is \( e = 1 \), what are the velocity vectors of the two balls just after the impact? The balls are of equal mass.

Solution:

Oblique impact: momentum of B only changes in y' – direction, therefore y' – direction is the line of impact. [1 mark]

In x' – direction (perpendicular to line of impact):

\[
v_{x_B/2} = v_{x_A/2} = 2 \cos 45^\circ = 1.41 \text{ m/s}.
\]

\[
v_{x_B/1} = v_{x_A/1} = 0
\]

In y' – direction (along line of impact):

\[
v_{y_B/2} + v_{y_B/2} = v_{y_A/2} + v_{y_A/2} = 2 \sin 45^\circ = 1.41
\]

\[
e = 1 = \frac{v_{y_B/2} - v_{y_B/2}}{v_{y_A/2} - v_{y_A/2}}
\]

Solving the two equations together gives

\[
v_{y_B/2} = 0, \quad v_{y_B/2} = 1.41 \text{ m/s}
\]

Therefore,

\[
\vec{v}_{x_B/2} = 1.41 \text{ m/s} \quad (45^\circ \text{ above horizontal})
\]

\[
\vec{v}_{y_B/2} = 1.41 \text{ m/s} \quad (45^\circ \text{ above horizontal})
\]
3. [9 marks] A 0.3-kg collar A is released from rest, slides down a frictionless rod, and strikes a 0.8-kg collar B which is at rest and supported by a spring of constant 500 N/m. The velocity of collar A is zero immediately after impact. Determine:
(a) [4 marks] the coefficient of restitution between the two collars; 
(b) [1 mark] the energy lost during the impact; 
(c) [4 marks] the maximum distance collar B moves down the rod after impact.

Solution:
(a) Energy conservation for A: the speed of collar A before impact is 
\[ v_{A1} = \sqrt{2gd\sin 30^\circ} = 3.13 \text{ m/s} \] [1 mark]

Central impact: linear momentum of A+B is conserved along the rod:
\[ m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2} \Rightarrow (0.3)(3.13) + 0 = 0 + (0.8)v_{B2} \Rightarrow v_{B2} = 1.17 \text{ m/s} \] [2 marks]

Coefficient of restitution: 
\[ e = \frac{v_{B2} - v_{A2}}{v_{A1} - v_{B1}} = \frac{1.17 - 0}{3.13 - 0} = 0.37 \] [1 mark]

(b) Energy lost = 
\[ \frac{1}{2}m_A v_{A1}^2 - \frac{1}{2}m_B v_{B2}^2 = 0.922 \text{ J} \] [1 mark]

(c) Before impact, forces on B are balanced. From FBD of B, the initial compression of the spring can be determined as 
\[ s = \frac{m_B g \sin 30^\circ}{k} = 0.0078 \text{ m} \] [1 mark]

Energy conservation for B during its downward motion:
\[ \frac{1}{2}m_B v_{B2}^2 + \frac{1}{2}k\delta^2 = \frac{1}{2}k(s + \delta)^2 - m_B g \delta \sin 30^\circ, \] [2 marks]

where \( \delta \) is the maximum compression of the spring. This results in the following equation for \( \delta \):
\[ 250\delta^2 - 0.024\delta - 0.548 = 0 \Rightarrow \delta = 0.0469 \text{ m} \] [1 mark]