ENPH 131 Assignment #3

Solutions

Problem 12.72

The velocity of a particle is \( v = [3 \, i + (6 - 2 \, t) \, j] \, m/s \), where \( t \) is in seconds. If \( r = 0 \) when \( t = 0 \), determine the displacement of the particle during the time interval \( t = 2 \, s \) to \( t = 6 \, s \).

Use eqn 12-7:

\[
\begin{align*}
\mathbf{v} \, dt &= \mathbf{d} \mathbf{r} \\
\int_0^t [3 \, i + (6 - 2 \, t) \, j] \, dt &= \int_0^t \mathbf{d} \mathbf{r} \\
r &= 3 \, i + (6 \, t - t^2) \, j \\
\Delta x &= r(t = 6) - r(t = 2) \\
\Delta x &= (18 \, i + (36 - 36) \, j) - (6 \, i + (12 - 4) \, j) \\
\Delta x &= 12 \, i - 8 \, j \\
\Delta x &= 12 \, m \\
\Delta y &= -8 \, m
\end{align*}
\]

Tutorial Problem (Curvilinear Motion: Rectangular Components)

A car drives on a curved road that goes down a hill. The car’s position is defined by the position vector, \( \mathbf{r} = [-30.0 \, \cos\left(\frac{\pi}{100} \, t\right) \, i + 30.0 \, \sin\left(\frac{\pi}{100} \, t\right) \, j + A_z \, t \, k] \) ft, where \( A_z = 11.0 \, \text{ft/s} \).

a) The image shows the system projected onto the x-y plane. What are the car’s velocity and acceleration vectors at this position?
b) What is the magnitude, \( v \), of the car’s velocity, \( v \), at \( t = 1.00 \) s?

\[
v = \frac{dr}{dt}
\]
\[
v = \frac{dx}{dt} \left[ -30.0 \cos\left( \frac{\pi}{10.0} \right) \right] i + \left[ 30.0 \sin\left( \frac{\pi}{10.0} \right) \right] j - (A_z) k
\]
\[
v = \left[ 3 \pi \sin\left( \frac{\pi}{10.0} \right) \right] i + \left[ 3 \pi \cos\left( \frac{\pi}{10.0} \right) \right] j - A_z k
\]

At \( t = 1.00 \) s

\[
v = |v| = (v_x^2 + v_y^2 + v_z^2)^{\frac{1}{2}}
\]
\[
v = \left( (3 \pi \sin\left( \frac{\pi}{10.0} \right))^2 + (3 \pi \cos\left( \frac{\pi}{10.0} \right))^2 + v_z^2 \right)^{\frac{1}{2}}
\]
\[
v = (3 \pi^2 + 11)^{\frac{1}{2}}
\]
\[
v = 14.5 \text{ ft/s}
\]

c) What is the magnitude, \( a \), of the car’s acceleration, \( a \), at \( t = 1.00 \) s?

From above

\[
v = \left[ 3 \pi \sin\left( \frac{\pi}{10.0} \right) \right] i + \left[ 3 \pi \cos\left( \frac{\pi}{10.0} \right) \right] j - A_z k
\]

\[
a = \frac{dv}{dt}
\]
\[
a = \frac{d}{dt} \left[ \left[ 3 \pi \sin\left( \frac{\pi}{10.0} \right) \right] i + \left[ 3 \pi \cos\left( \frac{\pi}{10.0} \right) \right] j - A_z k \right]
\]
\[
a = \left[ \frac{3}{10} \pi^2 \cos\left( \frac{\pi}{10.0} \right) \right] i - \left[ \frac{3}{10} \pi^2 \sin\left( \frac{\pi}{10.0} \right) \right] j
\]

At \( t = 1.00 \) s

\[
a = |a| = (a_x^2 + a_y^2 + a_z^2)^{\frac{1}{2}}
\]
\[
a = \left( \left( \frac{3}{10} \pi^2 \cos\left( \frac{\pi}{10} \right) \right)^2 + \left( \frac{3}{10} \pi^2 \sin\left( \frac{\pi}{10} \right) \right)^2 + 0 \right)^{\frac{1}{2}}
\]
\[
a = \frac{3}{10} \pi^2
\]
\[
a = 2.96 \text{ ft/s}^2
Problem 12.78

Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link.

\[ \frac{x^2}{4} + y^2 = 1 \]  

(1)

Therefore, the velocity of the peg is given by the derivative of eqn (1):

\[ \frac{2x}{4} v_x + y v_y = 0 \]

where \( x \) and \( y \) are \( v_x \) and \( v_y \), respectively.

\[ \frac{1}{2} x v_x + y v_y = 0 \]  

(2)

\( v_x = 10.0 \text{ m/s} \) and is constant, \( x = 1 \text{ m} \), and \( y \) is given by eqn (1):

\[ y^2 = 1 - \frac{x^2}{4} \]

\[ y^2 = 1 - \frac{1}{4} \]

\[ y = \left( \frac{3}{4} \right)^{\frac{1}{2}} \text{ m} \]

Substitute these values into eqn (2):

\[ \frac{1}{2} x v_x + y v_y = 0 \]

\[ \frac{1}{2} \left(1\right) \left(10\right) + \left(\frac{3}{4}\right)^{\frac{1}{2}} v_y = 0 \]

\[ v_y = -\frac{5}{\sqrt{3}} \text{ m/s} \]

Finally, solve for the magnitude of the velocity vector:
\[ v = v_x i + v_y j + v_z k \]
\[ v = |v| = (v_x^2 + v_y^2 + v_z^2)^{\frac{1}{2}} \]
\[ v = \left( 10^2 + \frac{3 \sqrt{3}}{v} \right)^{\frac{1}{2}} \]
\[ v = 10.4 \text{ m/s} \]

b) Determine the magnitude of the acceleration of peg A when \( x = 1 \text{ m} \).

The acceleration of the peg is given by the derivative of eqn (2):
\[ \frac{1}{2} \left( x \ddot{x} + \dot{x} \dot{x} \right) + 2 \left( y \ddot{y} + \dot{y} \dot{y} \right) = 0 \]
where \( \dot{x}, \dot{y}, \ddot{x}, \) and \( \ddot{y} \) are \( v_x, v_y, a_x, \) and \( a_y, \) respectively.
\[ \frac{1}{2} \left( x a_x + v_x^2 \right) + 2 \left( y a_y + v_y^2 \right) = 0 \]
\( v_x = 10.0 \text{ m/s} \) and is constant, therefore \( a_x = 0, x = 1 \text{ m}, y = \frac{\sqrt{3}}{2} \text{ m}, v_y = \frac{\sqrt{3}}{2} \text{ m/s}: \)
\[ \frac{1}{2} \left( 0 + 10^2 \right) + 2 \left( \frac{\sqrt{3}}{2} a_y + \left( \frac{\sqrt{3}}{2} \right)^2 \right) = 0 \]
\( a_y = -38.5 \text{ m/s}^2 \)
And since \( a_x = a_z = 0 \)
\[ a = |a| = |a_y| = 38.5 \text{ m/s}^2 \]

**Problem 12.82**

A car travels east 4km for 7 minutes, then north 5km for 8 minutes, and then west 5km for 9 minutes.

a) Determine the total distance travelled.
\[ s_{\text{total}} = s_1 + s_2 + s_3 \]
\[ s_{\text{total}} = 4 + 5 + 5 \]
\[ s_{\text{total}} = 14.0 \text{ km} \]

b) Determine the magnitude of the displacement of the car.

Take EAST to be the +x-direction, and NORTH to be the +y-direction. Therefore
\[ r = 4i + 5j - 5i \]
\[ r = -1i + 5j \]
\[ \Delta r = |r| = \left( r_x^2 + r_y^2 + r_z^2 \right)^{\frac{1}{2}} \]
\[ \Delta r = \left( (-1)^2 + 5^2 + 0 \right)^{\frac{1}{2}} \]
\[ \Delta r = 5.10 \text{ km} \]
c) What is the magnitude of the average velocity?

*Average velocity* is given by

\[ v_{\text{avg}} = \frac{\text{total displacement}}{\text{total time}} \]

\[ v_{\text{avg}} = \frac{5.10 \text{ km}}{(7+8+9) \text{ min}} \]

\[ v_{\text{avg}} = 0.212 \text{ km/min} = 12.7 \text{ km/hr} \]

d) What is the average speed?

*Average speed* is given by

\[ v_{\text{avg}} = \frac{\text{total distance}}{\text{total time}} \]

\[ v_{\text{avg}} = \frac{14 \text{ km}}{(7+8+9) \text{ min}} \]

\[ v_{\text{avg}} = 0.583 \text{ km/min} = 35.0 \text{ km/hr} \]

### Problem 12.85

A particle moves along the curve \( y = x - \left( \frac{x^2}{400} \right) \), where \( x \) and \( y \) are in ft.

a) If the velocity component in the \( x \)-direction is 2 ft/s and remains constant, determine the magnitude of the velocity when \( x = 17 \) ft.

The *position* of the particle is given by

\[ y = x - \left( \frac{x^2}{400} \right) \]  

(1)

The *velocity* of the particle is given by the derivative of eqn (1):

\[ \dot{y} = x - \left( \frac{x \dot{x}}{200} \right) \]

\[ v_y = v_x - \left( \frac{x v_x}{200} \right) \]  

(2)

\( x = 17 \) ft, \( v_x = 2 \) ft/s:

\[ v_y = 2 - \left( \frac{17 \cdot 2}{200} \right) \]

\[ v_y = 1.83 \text{ m/s} \]

\[ v = \sqrt{v_x^2 + v_y^2 + v_z^2} \]

\[ v = \sqrt{2^2 + 1.83^2 + 0} \]

\[ v = 2.71 \text{ ft/s} \]
b) Determine the magnitude of the acceleration when \( x = 17 \) ft.

The acceleration of the particle is given by the derivative of eqn (2):

\[
\ddot{y} = \ddot{x} - \frac{1}{200} (x \ddot{x} + \dot{x}^2)
\]
\[
a_y = a_x - \frac{1}{200} (x a_x + v_x^2)
\]

\( v_x = 2 \text{ ft/s} \) and is constant, therefore \( a_x = 0 \), \( x = 17 \) ft:

\[
a_y = 0 - \frac{1}{200} (0 + 2^2)
\]
\[
a_y = -0.020 \text{ ft/s}^2
\]

Since \( a_x = 0 \), the magnitude of the particle’s acceleration is

\[
a = |a| = |a_y| = 0.020 \text{ ft/s}^2
\]