PHYS 310: Handy Definitions and Formulae

First Law of Thermodynamics

$$\begin{aligned} dU &= \delta Q - P\,dV = T\,dS - P\,dV \\ dH &= \delta Q + V\,dP = T\,dS + V\,dP \\ T &= \left(\frac{\partial U}{\partial S}\right)_V \\ P &= -\left(\frac{\partial U}{\partial V}\right)_S \end{aligned}$$

Heat Capacities

$$\begin{aligned} C_v &\equiv \left(\frac{\partial U}{\partial T}\right)_V \\ C_p &\equiv \left(\frac{\partial H}{\partial T}\right)_P \end{aligned}$$

Thermal Expansivity and Isothermal Compressibility

$$\begin{aligned} \alpha_T &\equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \\ \alpha_p &\equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T \end{aligned}$$

Equations for an Ideal Gas

$$\begin{aligned} P &= \frac{1}{V}\,nRT = \frac{1}{V}\,NkT = \rho R_g T \quad (R_g \text{ is constant for a particular gas; } R_g = R_a \text{ for air}) \\ C_p &= C_v + nR, \quad c_p = c_v + R_g \\ \kappa &= 1 - 1/\gamma, \quad \text{where} \quad \gamma \equiv C_p/C_v = c_p/c_v \ \text{ and } \ \kappa \equiv nR/C_p = R_g/c_p \end{aligned}$$

Formulae for Adiabatic Changes to an Ideal Gas

$$\begin{aligned} P &\propto V^{-\gamma} \propto \rho^{\gamma} \\ T &\propto V^{1-\gamma} \propto P^{\kappa} \\ \theta &\equiv T(P/P_{00})^{-\kappa} \end{aligned}$$

Scale Heights and the Adiabatic Lapse Rate

$$\begin{aligned} H_p &= -\left(\frac{1}{\bar{p}}\frac{d\bar{p}}{dz}\right)^{-1} = \frac{R_g}{g}\bar{T} \\ H_\rho &= -\left(\frac{1}{\bar{\rho}}\frac{d\bar{\rho}}{dz}\right)^{-1} = \left[\frac{1}{\bar{T}}\left(\frac{d\bar{T}}{dz} + \frac{1}{\kappa}\Gamma\right)\right]^{-1} \\ H_\theta &= \left(\frac{1}{\bar{\theta}}\frac{d\bar{\theta}}{dz}\right)^{-1} = \left[\frac{1}{\bar{T}}\left(\frac{d\bar{T}}{dz} + \Gamma\right)\right]^{-1} \\ \Gamma &= \frac{g\kappa}{R_g} = \frac{g}{c_p} \end{aligned}$$

Speed of Sound

$$c_s = \left(\frac{dP}{d\rho}\right)^{1/2} \quad (= \sqrt{\gamma R_g T} \text{ for ideal gas})$$

Van der Waal's Equation

$$\left(P + a\frac{N^2}{V^2}\right)(V - bN) = NkT$$

Hydrostatic Balance

$$\frac{d\bar{p}}{dz} = -\bar{\rho} g$$

Handy Taylor Series Expansions (for $|\epsilon| \ll 1$)

$$\begin{aligned} &(1+\epsilon)^p \simeq 1 + p\,\epsilon + [p(p-1)/2]\,\epsilon^2 \\ &\exp(\epsilon) \simeq 1 + \epsilon + (1/2)\,\epsilon^2 \\ &\ln(1+\epsilon) \simeq \epsilon - (1/2)\,\epsilon^2 \end{aligned}$$