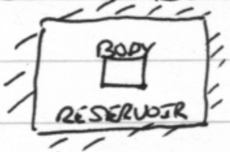
⑧ APPLICATIONS OF GIBBS FREE ENERGY

1] THE 2^{ND} LAW AND STABILITY

AS IN A RECENT EXAMPLE, CONSIDER A BODY IN THERMAL CONTACT WITH A LARGE RESERVOIR THAT CAN ABORBS LOTS OF ENERGY WITHOUT SIGNIFICANT CHANGE IN TEMPERATURE.

ASSUME PRESSURE AND TEMPERATURE OF THE BODY AND RESERVOIR ARE THE SAME AND FIXED.



FOR BODY $dH = TdS$ (SINCE $dP=0$)

FOR RESERVOIR $\left\{\begin{array}{ccc} dH_R & = & -dH \\ \shortparallel & & \shortparallel \\ TdS_R & & -TdS \end{array}\right.$ (BY ENERGY CONSERVATION)

SO TOTAL ENTROPY CHANGE IS

$$0 \underset{\substack{\uparrow \\ 2^{ND}\text{ LAW}}}{\leq} dS_{TOTAL} = dS + dS_R = dS + \left(-\frac{1}{T}dH\right)$$
$$= -\frac{1}{T}\left(dH - TdS\right)$$

BUT GIBBS FREE ENERGY IS $G \equiv H - TS$

$$\Rightarrow dG = dH - TdS - SdT \overset{dT=0}{=\!=} dH - TdS$$

SO $0 \leq dS_{TOTAL} = -\frac{1}{T}dG$ (ISOTHERMAL, ISOBARIC)

THAT IS TO SAY, WHILE ENTROPY ALWAYS TENDS TO INCREASE, THE GIBBS FREE ENERGY ALWAYS TENDS TO DECREASE.

A SYSTEM WILL ALWAYS EVOLVE TO MINIMIZE G.

IF G IS NOT AT ITS MINIMUM IT IS "UNSTABLE".

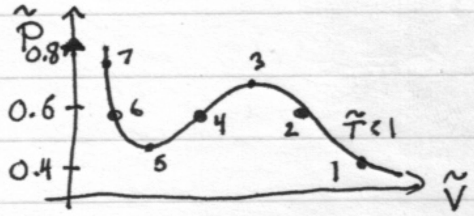
2] VAN DER WAALS EQ$^{\underline{N}}$ REVISITED

RECALL $\left(P + a\frac{N^2}{V^2}\right)(V - bN) = NkT$ EQ$^{\underline{N}}$ STATE FOR LIQUID & GAS (IDEALIZED)

WITH CRITICAL POINT AT $V_c = 3bN$, $kT_c = \frac{8}{27}\frac{a}{b}$, $P_c = \frac{1}{27}\frac{a}{b^2}$

IN NON-DIMENSIONAL FORM WITH $\tilde{P} = \frac{P}{P_c}$, $\tilde{V} = \frac{V}{V_c}$, $\tilde{T} = \frac{T}{T_c}$ GET

$$① \quad \frac{P}{P_c} = \tilde{P} = \frac{8\tilde{T}}{(3\tilde{V}-1)} - \frac{3}{\tilde{V}^2}$$



IF $\tilde{T} < 1$, THE FLUID CAN EXIST WITH 3 POSSIBLE VOLUMES IF $\tilde{P}$ IS IN A CERTAIN RANGE (e.g. $0.5 \lesssim \tilde{P} \lesssim 0.7$ ABOVE).

DETERMINE THE STABILITY OF EACH STATE USING GIBBS FREE ENERGY

ALONG ISOTHERM T, $dG = -SdT + VdP \overset{dT=0}{=} VdP$

$$\Rightarrow \left(\frac{\partial G}{\partial V}\right)_T = V\left(\frac{\partial P}{\partial V}\right)_T = P_c\left[-\frac{24\tilde{T}}{(3\tilde{V}-1)^2}\tilde{V} + \frac{6}{\tilde{V}^2}\right]$$

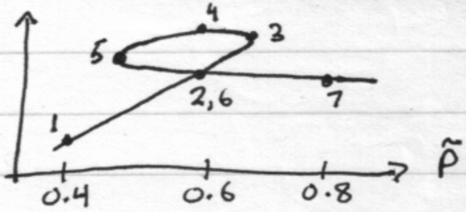
USE $\left(\frac{\partial G}{\partial V}\right)_T = \frac{1}{V_c}\left(\frac{\partial G}{\partial \tilde{V}}\right)_{\tilde{T}}$ AND INTEGRATE BOTH SIDES WRT $\tilde{V}$

$$\Rightarrow G = P_c V_c\left[\int \frac{-24\tilde{T}\tilde{V}}{(3\tilde{V}-1)^2} + \frac{6}{\tilde{V}^2}\, d\tilde{V}\right]$$

$$\Rightarrow G = P_c V_c\left[-\frac{8\tilde{T}}{3}\ln(3\tilde{V}-1) + \frac{8\tilde{T}}{3}\frac{1}{(3\tilde{V}-1)} - \frac{\text{[illegible]}}{\tilde{V}} + \underset{\text{CONSTANT}}{\overset{\text{TEMPERATURE-DEP.}}{\downarrow}} \mathcal{C}(\tilde{T})\right] ②$$

NUMERICALLY AT LEAST, ONE CAN USE ① TO RECAST $G(T,\tilde{V})$ AS $G(T,\tilde{P})$ IN ②.

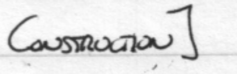
FOR $\tilde{T} < 1$, A TYPICAL PLOT IS:



BUT WE HAVE SEEN G ALWAYS TENDS TO ITS MINIMUM VALUE WITH $\tilde{T}, \tilde{P}$ FIXED. SO THE SYSTEM WILL EVOLVE STRAIGHT FROM STATE $2 \leftrightarrow 6$, SKIPPING 3,4,5 etc

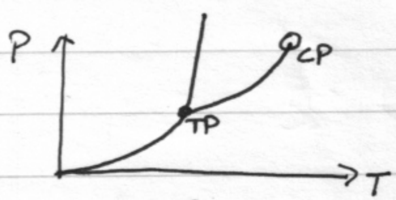
DURING PHASE CHANGE $dT = dP = 0 \Rightarrow dG = -SdT + VdP = 0$

SO $0 = \int_{2\to3\to4\to5\to6} dG = \int_{2\to\cdots\to6}\left(\frac{\partial G}{\partial P}\right)_T dP = \int_{2\to\cdots\to6} V\,dP$



SO IN PV-DIAGRAM AREA $|\overline{2-3-4}|$ = AREA $|\overline{4-5-6}|$ [MAXWELL'S CONSTRUCTION]

3] THE CLAUSIUS - CLAPEYRON EQUATION



RECALL PT-DIAGRAM SHOWING PHASE TRANSITIONS

WILL USE GIBBS FREE ENERGY TO GAIN INSIGHT INTO PHASE BOUNDARIES

AT BOUNDARY BETWEEN 2 PHASES (e.g. LIQUID & GAS) THE SUBSTANCES ARE EQUALLY STABLE AT SAME P & T, SO G IS THE SAME.

$$dG = -S_l\,dT + V_l\,dP = -S_g\,dT + V_g\,dP$$

$$\Rightarrow \boxed{\frac{dP}{dT} = \frac{S_g - S_l}{V_g - V_l}} = \frac{\Delta S}{\Delta V} \leftarrow \text{COULD ALSO BE SOLID}\leftrightarrow\text{LIQUID, SOLID}\leftrightarrow\text{GAS}$$

IT IS MORE CONVENIENT TO WRITE $\Delta S = \frac{\Delta Q}{T} = \frac{l M}{T}$, IN WHICH l IS THE LATENT HEAT AND M THE MASS OF SUBSTANCE

$$① \quad \Rightarrow \boxed{\frac{dP}{dT} = \frac{l M}{T\,\Delta V}}$$

CLAUSIUS - CLAPEYRON EQUATION

(GIVES SLOPE OF PHASE-CHANGE LINES IN PT-DIAGRAM)

SPECIFICALLY FOR WATER ⇔ VAPOUR TRANSITION

TAKE $l \equiv l_v \simeq 2.26 \times 10^6\ \mathrm{J/kg}$ - CONSTANT (FOR $T \approx 100°C$)

ALSO, SINCE $V_g >> V_l$, TAKE $\Delta V \simeq V_g$

FROM IDEAL GAS LAW $\frac{M}{V_g} = \rho = \frac{P}{R_v T}$

HERE R_v IS GAS CONSTANT FOR H_2O: $R_v = \frac{R}{M_{H_2O}} \simeq \frac{8.315\,\frac{J}{Mol\cdot K}}{0.018\ kg/mol} \simeq 462\,\frac{J}{kg\,K}$

$$\text{So ① } \Rightarrow \frac{dP}{dT} \simeq \frac{l_v}{T}\left(\frac{P}{R_v T}\right) = \frac{l_v}{R_v}\,\frac{P}{T^2}$$

$$\text{SOLVING} \Rightarrow P = \mathcal{C}\,\mathrm{EXP}\left[-\frac{l_v}{R_v}\frac{1}{T}\right] \qquad \text{WITH } \mathcal{C} \text{ CONSTANT}$$

TO FIND CONSTANT, WE KNOW $T = T_B \simeq \overset{\swarrow 100°C}{373\,K}$ AT $P = P_0 = 1\,\mathrm{ATM}$

$$\Rightarrow \boxed{P(T) \simeq P_0\,\mathrm{EXP}\left[\frac{l_v}{R_v}\left(\frac{1}{T_B} - \frac{1}{T}\right)\right]}$$

THE VAPOUR PRESSURE EQUATION

(GOOD, ASSUMING $l_v \approx$ CONSTANT)

3] (CONT'D)

EXAMPLE: AT WHAT TEMPERATURE DOES WATER BOIL AS YOU GO TO HIGHER ALTITUDES?

SOL

IGNORING STORM SYSTEMS, etc, WE WILL RELATE ALTITUDE TO PRESSURE USING HYDROSTATIC BALANCE: $P \simeq P_0 e^{-z/H_p}$

IN WHICH WE WILL TAKE $H_p \simeq 8\,\mathrm{km}$

INVERTING THE VAPOUR PRESSURE EQUATION TO WRITE T IN TERMS OF P GIVES

$$T(P) \simeq \left[\frac{1}{T_B} - \frac{R_v}{\ell_v} \ln\left(\frac{P}{P_0}\right)\right]^{-1}$$

$$\simeq \left[\frac{1}{T_B} + \frac{R_v}{\ell_v}\frac{z}{H_p}\right]^{-1}$$

$$\Rightarrow \quad T(\quad) = T_B \left(1 + z/\mathcal{H}\right)^{-1}$$

IN WHICH $\mathcal{H} \equiv H_p \ell_v / (R_v T_B)$

$$\simeq (8000\,\mathrm{m})(2.26 \times 10^6 \tfrac{\mathrm{J}}{\mathrm{kg}}) / [(462 \tfrac{\mathrm{J}}{\mathrm{kgK}})(373\,\mathrm{K})]$$

$$\simeq 1.05 \times 10^5\,\mathrm{m} \simeq 105\,\mathrm{km}$$

EVEN GOING UP TO THE TOP OF EVEREST $z \ll \mathcal{H}$.
SO APPROXIMATE $\left(1 + \frac{z}{\mathcal{H}}\right)^{-1} \simeq 1 - \frac{z}{\mathcal{H}}$

$$\Rightarrow \quad T(z) \simeq T_B \left(1 - \frac{z}{\mathcal{H}}\right) \simeq (373\,\mathrm{K})\left[1 - \frac{z}{105\,\mathrm{km}}\right] //$$

IN PARTICULAR, IN JASPER (≈ 1000 m ABOVE SEA LEVEL) WATER BOILS AT $373\left(1 - \frac{1}{105}\right) \simeq 369\,\mathrm{K} \simeq 96^\circ\mathrm{C}$
(SO THINGS TAKE A LITTLE LONGER TO COOK.)

4] VAPOUR PRESSURE OF WATER IN AIR.

THE CLAUSIUS - CLAPEYRON EQUATION CAN ALSO BE APPLIED FOR MIXTURES OF WATER VAPOUR AND AIR.

LET $\nu \equiv \frac{\#\text{MOLECULES } H_2O}{\#\text{MOLECULES } H_2O + \text{AIR}}$ BE THE "VOLUME MIXING RATIO"

DEFINE $e \equiv \nu p$ TO BE THE "PARTIAL PRESSURE" OF WATER VAPOUR, I.e. THE PART OF PRESSURE DUE TO H_2O.

LET e_s BE THE "SATURATION VAPOUR PRESSURE", WHICH MEASURES THE MOST AMOUNT OF WATER VAPOUR THAT AIR CAN HOLD AT PRESSURE, P.

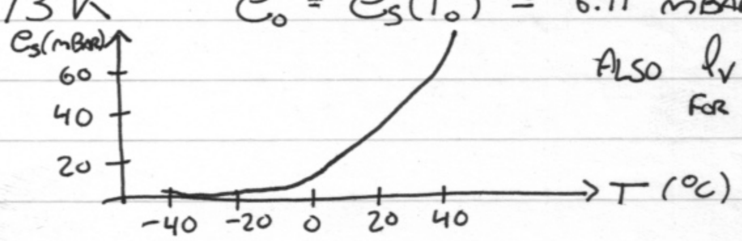
AS WITH PURE H_2O, e_s SATISFIES THE CLAUSIUS - CLAPEYRON EQ$^{\underline{N}}$:

$$\frac{de_s}{dT} = \frac{l_v}{R_v}\frac{e_s}{T^2} \Rightarrow \boxed{e_s(T) = e_0 \,\text{EXP}\left[\frac{l_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]} \quad ①$$

IN WHICH e_0 & T_0 ARE DETERMINED BY MEASUREMENTS.

E.g. AT $T_0 = 273\,K$ $\quad e_0 = e_s(T_0) \simeq 6.11$ mBAR.

GRAPHING ① GIVES



ALSO $l_v \approx 2.50\times10^6 \frac{J}{kg}$ FOR $T \approx 0°C$

(NOTE AIR CAN HOLD MUCH MORE WATER VAPOUR AT HIGH TEMPERATURES)

KNOWING e_s FROM ①, WE CAN DEFINE THE "RELATIVE HUMIDITY"

$$R.H. \equiv e/e_s = \nu/\nu_s = (\#\text{MOLECULES } H_2O)/(\#\text{MOLECULES } H_2O \text{ AT SATURATION})$$

IF m_v IS MASS OF VAPOUR IN MASS m_d OF DRY AIR, ITS "MIXING RATIO" IS $W \equiv \frac{m_v}{m_d}$.

IN TERMS OF PARTIAL PRESSURE: $e = \nu P = \left(\frac{m_v/M_{H_2O}}{m_d/M_{AIR} + m_v/M_{H_2O}}\right)P = \frac{W}{W+\varepsilon}P \simeq \frac{W}{\varepsilon}P$

WHERE $\varepsilon \equiv \frac{M_{H_2O}}{M_{AIR}} \simeq \frac{18.02}{28.97} \approx 0.622$, AND USED $W \ll \varepsilon$ (TYP. $W \sim \frac{5g\,H_2O}{kg\,AIR}$)

SO $\frac{W}{W_s} = \frac{e}{e_s}$.

"DEW POINT" OCCURS AT TEMP T_d WITH $R.H.=1 \Rightarrow W = W_s \simeq \overbrace{\frac{\varepsilon}{P}e_0}^{W_0} \text{EXP}\left[\frac{l_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T_d}\right)\right]$

4] (CONT'D)

EXAMPLE ①:

AIR CONTAINS WATER VAPOUR WITH A MIXING RATIO OF $W = 5.5\,g/kg$ AND THE PRESSURE IS $1026.8\,hPa$ AND $T = 20^\circ C$.
WHAT IS THE VAPOUR PRESSURE AND RELATIVE HUMIDITY?

SOLN

$$e = \nu P = \left(\frac{5.5\times10^{-3}}{5.5\times10^{-3} + 0.622}\right)(1026.8\,hPa) \simeq 9.0\,hPa \quad (9.0\,\text{mBAR})$$

AT $20^\circ C$, SATURATION VAPOUR PRESSURE IS

$$e_s \simeq \underbrace{(6.11\,\text{mBAR})}_{e_0 = e_s(0^\circ C)}\,\text{EXP}\left[\frac{2.50\times10^{6}\,J/kg}{462\,J/(kg\,K)}\left(\frac{1}{273K} - \frac{1}{293K}\right)\right] \simeq 23.6\,\text{mBAR}$$

SO RELATIVE HUMIDITY IS $\frac{e}{e_s} \simeq \frac{9.0}{23.7} \simeq 0.38$
i.e 38% HUMIDITY //

EXAMPLE ②:

FOR THE ABOVE PROBLEM, WHAT IS THE DEW POINT?

SOLN

WANT T_d SUCH THAT $W = W_0\,\text{EXP}\left[\frac{\ell_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T_d}\right)\right]$

WHERE $W_0 = \frac{\varepsilon}{P}e_0$ IS MIXING RATIO AT $T = T_0$

$$\Rightarrow T_d = \left[\frac{1}{T_0} - \frac{R_v}{\ell_v}\,\text{Ln}\left(\frac{W}{W_0}\right)\right]^{-1}$$

$$\simeq \left[\frac{1}{273K} - \frac{462\,\frac{J}{kg\,K}}{2.50\times10^{6}\,\frac{J}{kg}}\,\text{Ln}\overbrace{\left(\frac{0.0055}{(0.622)(6.11\,\text{mBAR})/(1026.8\,\text{mBAR})}\right)}^{=\frac{0.0055}{0.00370}}\right]^{-1}$$

$$\simeq 273K\left[1 - 5.48\times10^{-2}\,\text{Ln}(1.49)\right]^{-1}$$

$$\simeq 273K\,[1.02]$$

$$\simeq 279K$$

SO DEW POINT IS ~6°C //

5] EQUIVALENT POTENTIAL TEMPERATURE

AS MOISTURE-LADEN AIR RISES AND COOLS, SOME OF THE WATER VAPOUR CONDENSES (FORMING CLOUDS) WHICH RELEASES HEAT INTO THE AIR. IF THE DROPS ARE SO MINUTE THAT THEY CONTINUE TO RISE WITH THE AIR, THE PROCESS IS ADIABATIC. BUT IF THE DROPS LITERALLY RAIN OUT, THE PROCESS IS "PSEUDOADIABATIC".

HOW THE TEMPERATURE OF A SATURATED (100% HUMIDITY) AIR PARCEL CHANGES AS IT MOVES THROUGH DECREASING PRESSURE ALL THE WHILE LOSING MOISTURE IS GIVEN BY THE EQUIVALENT POTENTIAL TEMPERATURE, θ_e

USE $S = c_p \ln\theta$ AND $dS = \frac{\delta q}{T} = \frac{1}{T}\left(-l_v \frac{dm_v}{m_{AIR}}\right) \approx \frac{1}{T}\left(-l_v\, dw_s\right)$

IN WHICH w_s $\left(= w_0 \,\mathrm{EXP}\left[\frac{l_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T_d}\right)\right]\right)$ IS SATURATION MIXING RATIO: $\frac{m_{v,SAT}}{m_d}$

$$\text{So} \quad d\ln\theta = -\frac{1}{c_p T} l_v\, dw_s = d\left(-\frac{l_v w_s}{c_p T}\right) - \frac{l_v w_s}{c_p}\frac{1}{T^2}dT$$

NOW ASSUME RELATIVE CHANGE OF TEMPERATURE IS TINY COMPARED TO RELATIVE CHANGE OF w_s: $\frac{dT}{T} \ll \frac{dw_s}{w_s}$

$$\Rightarrow \quad d\ln\theta \approx d\left(-\frac{l_v w_s}{c_p T}\right)$$

$$\Rightarrow \quad \ln(\theta/\theta_e) \simeq -l_v w_s/(c_p T)$$ WHERE θ_e IS INTEGRATION CONSTANT

$$\Rightarrow \quad \boxed{\theta_e = \theta \,\mathrm{EXP}\left[\frac{l_v w_s}{c_p T}\right]}$$

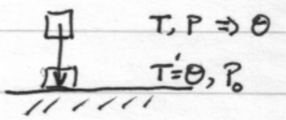
θ_e IS DEFINED SO $\theta \to \theta_e$ AS $\frac{w_s}{T} \to 0$.

SO θ_e IS THE POTENTIAL TEMPERATURE OF A PARCEL OF AIR AFTER ALL THE WATER VAPOUR HAS CONDENSED AND THE SATURATION MIXING RATIO (w_s) IS ZERO.

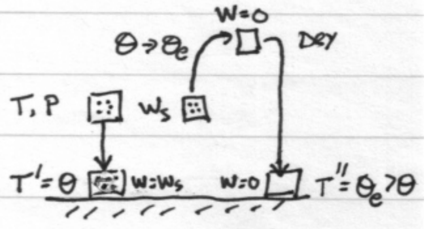
5] (CONT'D)

TO ELUCIDATE THE INTERPRETATION OF θ_e, CONSIDER THE FOLLOWING 3 SCENARIOS

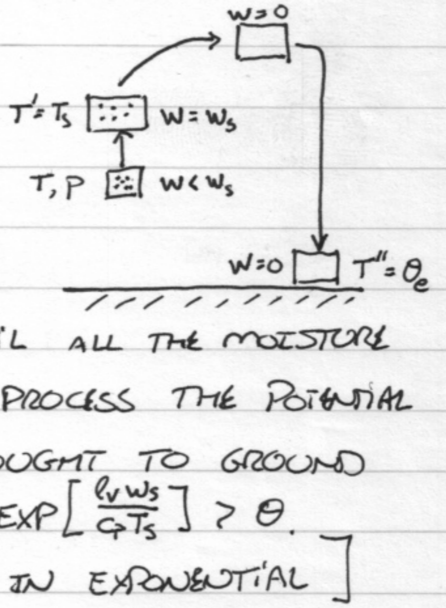
1) A PARCEL OF DRY AIR AT SOME HEIGHT HAS POTENTIAL TEMPERATURE θ. THIS MEANS IS WILL HAVE TEMPERATURE $T'=\theta$ WHEN BROUGHT TO THE GROUND.



2) A PARCEL OF SATURATE AIR (WITH $W=W_s$) HAS POTENTIAL TEMPERATURE θ. AS ABOVE, IF BROUGHT STRAIGHT TO GROUND ITS TEMPERATURE WILL BE $T'=\theta$ (NO CONDENSATION, SO NO HEAT RELEASE). HOWEVER, IF BROUGHT UPWARDS UNTIL IT DRYS OUT, ITS POTENTIAL TEMPERATURE BECOMES $\theta_e = \theta \, \mathrm{EXP}\left[\frac{\ell_v W_s}{C_p T}\right] > \theta$. IF THE PARCEL THEN DESCENDS TO GROUND REMAINING DRY, ITS TEMPERATURE WILL BE $T'' = \theta_e > \theta = T'$. (LATENT HEAT RELEASE HAS WARMED THE AIR)



3) A PARCEL OF MOIST (BUT UNSATURATED) HAS POTENTIAL TEMPERATURE θ. IT IS BROUGHT UPWARDS UNTIL IT HAS COOLED TO TEMPERATURE T_s AT WHICH IT IS SATURATED ($W=W_s$). THEN IT CONTINUES TO RISE UNTIL ALL THE MOISTURE CONDENSES AND RAINS OUT. DURING THIS PROCESS THE POTENTIAL TEMPERATURE INCREASES TO θ_e. IF BROUGHT TO GROUND THE TEMPERATURE WILL BE $T'' = \theta_e = \theta \, \mathrm{EXP}\left[\frac{\ell_v W_s}{C_p T_s}\right] > \theta$.
[NOTE T_s, NOT STARTING TEMPERATURE T IN EXPONENTIAL]



THE LAST SCENARIO GIVES A PARTIAL EXPLANATION FOR THE WARMTH OF CHINOOK WINDS. AS MOISTURE LADEN AIR FROM WEST IS PUSHED UPWARD OVER MOUNTAINS, RAIN FALLS OUT. WHEN DESCENDING ON THE OTHER SIDE, THE AIR IS RELATIVELY WARMER.

5] (CONT'D)

EXAMPLE ①: AIR AT $T_1 = 14°C$ AND $P_1 = 950$ mBAR HAS A MIXING RATIO OF $w_1 = 8$ g/kg. IT RISES ADIABATICALLY UNTIL IT BECOMES SATURATED. FIND AN IMPLICIT EQUATION FOR THE TEMPERATURE, T_S, AT SATURATION.

SOL^N/

FIRST NOTE THAT THE SATURATION VAPOUR PRESSURE IS

$$e_{S1} = e_0 \text{ EXP}\left[\frac{l_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T_1}\right)\right]$$
$$\simeq (6.11 \text{ mBAR}) \text{ EXP}\left[\frac{2.50 \times 10^6 \text{ J/kg}}{462 \text{ J/kgK}}\left(\frac{1}{273\text{K}} - \frac{1}{287\text{K}}\right)\right]$$
$$\simeq 14.6 \text{ mBAR}$$

SO THE MIXING RATIO AT SATURATION FOR $T_1 = 14°C$ AND $P_1 = 950$ mBAR IS

$$w_{S1} = \frac{\varepsilon}{P_1} e_{S1} \simeq \frac{0.622}{950 \text{ mBAR}} \cdot 14.6\text{mBAR} \simeq 9.59 \times 10^{-3}$$

$\Rightarrow w_{S1} = 9.59$ g/kg AT INITIAL HEIGHT OF PARCEL

SINCE $w_{S1} > w_1 = 8$g/kg THE PARCEL IS INDEED UNSATURATED.

AS PARCEL RISES, ITS POTENTIAL TEMPERATURE IS CONSTANT UP TO BECOMING SATURATED: $\theta = T_1\left(\frac{P_1}{P_0}\right)^{-x} = T\left(\frac{P}{P_0}\right)^{-x}$

$$\Rightarrow \quad p = P_1 (T/T_1)^{1/x} \qquad (*)$$

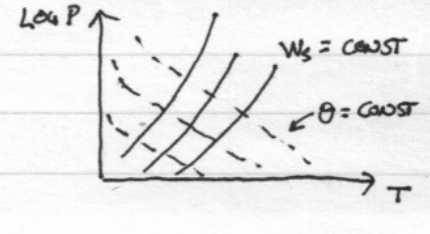
AT SATURATION $w_1 = w_S = \frac{\varepsilon}{p} e_S = \frac{\varepsilon}{p} e_0 \text{ EXP}\left[\frac{l_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$

IN TERMS OF $P \Rightarrow \quad p = \frac{\varepsilon}{w_1} e_S = \frac{\varepsilon}{w_1} e_0 \text{ EXP}\left[\frac{l_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \qquad (**)$

EQUATING $(*)$ + $(**)$ GIVES AN IMPLICIT EQUATION FOR $T = T_S$

$$P_1 (T_S/T_1)^{1/x} = \frac{\varepsilon}{w_1} e_0 \text{ EXP}\left[\frac{l_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T_S}\right)\right] \quad //$$

IN PRACTISE THIS MUST BE SOLVED NUMERICALLY. (FIND $T_S \sim 8°C$ AT $P_S \sim 890$ mBAR) HOWEVER, ATMOSPHERIC SCIENTISTS OFTEN TAKE A SHORT-CUT ~~BY~~ FINDING INTERSECTIONS OF LINES OF CONSTANT θ AND SATURATION MIXING RATIO w_S ON A "SKEW T – LnP" CHART OR A "TEPHIDIAGRAM"



5] (CONT'd)

EXAMPLE ②: A SATURATED AIR PARCEL AT 950 mBAR HAS A TEMPERATURE OF 5°C. FIND ITS POTENTIAL TEMPERATURE AND ITS EQUIVALENT POTENTIAL TEMPERATURE.

SOL^N/

THE POTENTIAL TEMPERATURE IS

$$\theta = T\,(P/P_0)^{-\kappa}$$

$$\simeq (278\text{ K})\left[\frac{950\text{ mBAR}}{1000\text{ mBAR}}\right]^{-2/7}$$

$$\Rightarrow \boxed{\theta \simeq 282\text{ K}}$$

(SO AIR WILL HAVE TEMPERATURE ~9°C IF BROUGHT TO GROUND.)

FOR θ_e, NEED TO FIND W_s

$$W_s \simeq \frac{\varepsilon}{P}e_s \simeq \frac{\varepsilon}{P}e_0 \,\text{EXP}\left[\frac{l_v}{R_v}\left(\frac{1}{T_0}-\frac{1}{T}\right)\right]$$

$$\simeq \frac{0.622}{950\text{ mBAR}}\left[(6.11\text{ mBAR})\,\text{EXP}\left[\frac{2.5\times10^6}{462}\left(\frac{1}{273}-\frac{1}{278}\right)\right]\right]$$

$$\simeq \frac{0.622}{950\text{ mBAR}}\left[8.73\text{ mBAR}\right] \simeq 0.00572$$

(i.e. $W_s \approx 5.72$ g/kg)

$$\text{So}\quad \theta_e = \theta\,\text{EXP}\left[\frac{l_v W_s}{c_p T}\right]$$

$$\simeq (282\text{ K})\,\text{EXP}\left[\frac{2.50\times10^6\text{ J/kg}}{1005\text{ J/kgK}}\cdot\frac{0.00572}{278\text{ K}}\right]$$

$$\simeq 282\text{K}\;\text{EXP}[0.0463]$$

$$\Rightarrow \boxed{\theta_e \simeq 295\text{ K}}$$

(SO IF ALL THE MOISTURE CONDENSES AND RAINS OUT, THE AIR TEMPERATURE WOULD BE 22°C IF BROUGHT TO THE GROUND. THE LATENT HEAT RELEASE AMOUNTS TO A 13°C RISE IN TEMPERATURE.)