PHYS 310: Thermodynamics and Statistical Mechanics
Sample Final Exam

1. An automobile tire is inflated to a pressure of $270\,\text{kPa}$ at the beginning of a trip. After 3 hours of high-speed driving the pressure is $300\,\text{kPa}$ but the volume remains constant at 0.057 m^3.

   a) What is the change of internal energy of air in the tire between the two pressure measurements?

   b) What is the change of enthalpy?

   [In both a) and b) give your answer in Joules with 2-digit accuracy.]

2. Suppose an ideal diatomic gas operates in a heat engine whose PV cycle, shown to the right, consists of an isochoric process $(A \to B)$, an adiabatic process $(B \to C)$ and an isobaric process $(C \to A)$. Suppose $V_2 = 2V_1$.

   a) Write an expression for P_2 in terms of P_1.

   b) What is the efficiency of this engine?
   [Give your answer as a percent with 3-digit accuracy.]
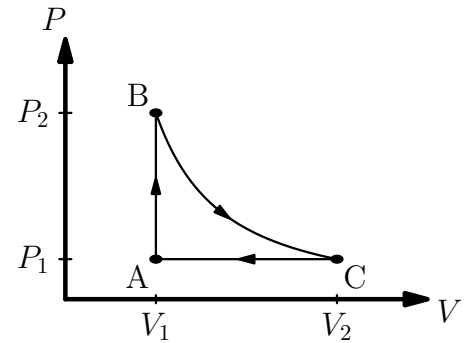
3. This problem demonstrates the importance of the ocean as a heat reservoir.

   a) Consider a vertical column of a dry atmosphere having a horizontal cross-sectional area $A = 1\text{ m}^2$ and assume it is isothermal with temperature $T = 20°\text{C}$ extending from the Earth's surface (where the pressure is $1\,\text{atm}$) to space. Estimate how much energy is required to raise the temperature of this column by $1°\text{C}$ assuming that the volume does not change upon heating. [Give your answer in Joules with 3-digit accuracy.]

   b) Now consider a vertical column of the ocean with the same horizontal area A and isothermal temperature T extending down to some depth D. Give an estimate, as a function of D, for the energy required to raise the temperature of this column by $1°\text{C}$. (For simplicity, suppose the column consists of fresh water and assume D is sufficiently small that you can ignore the thermodynamics of pressure acting upon water.)

   c) Equating the energy found in (b) to your result in (a), find the corresponding depth D of the ocean which contains as much energy as the entire atmosphere above it. [Give your answer in meters with 2-digit accuracy.]

4. A parcel of moist air situated at the base of a cloud has temperature $10°\text{C}$, pressure $900\,\text{mbar}$ and 100% humidity.

   a) What would be the temperature of the air if brought adiabatically directly to the ground where the pressure is $1000\,\text{mbar}$? [Give your answer in $°\text{C}$ with 3-digit accuracy.]

   b) What would be the temperature of the air if it was lifted adiabatically until all the water vapour in it condensed, and then the resulting dry air was brought adiabatically to the ground where the pressure is $1000\,\text{mbar}$? [Give your answer in $°\text{C}$ with 3-digit accuracy.]

5. A simple model of a polymer (a long-chained molecule as in rubber), assumes it forms a one dimensional chain made up of N identical elements, each of length ℓ. The angle between successive elements can be either $0°$ or $180°$, but there is no difference in internal energy between these two possibilities. For the sake of counting, one can think of each element as either pointing to the right ($+$, if oriented at $0°$ from the previous element) or to the left ($-$, if oriented at $180°$ from the previous element). Then one has $N = n_+ + n_-$ for the total number of elements and $L = \ell(n_+ - n_-)$ for the length of the chain to the right of the first element. Assume N, n_+ and n_- are all *very* large.

a) Find the entropy, S, as a function of N, n_+ and k.

b) The relationship between entropy and the tension, F, on the chain is given by the first law of thermodynamics: $0 = TdS + FdL$. Use this to find an explicit expression for the tension as a function of N, k, T, L and ℓ.

c) Using your result in b), show that in the limit, $L \ll N\ell$, the chain satisfies Hookes Law: $F \propto L$. Explicitly find the proportionality constant as it depends upon N, k, T and ℓ.

6. The energy density of photons in a box (a "photon gas") at temperature T is

$$u = \frac{4}{c}\sigma T^4$$

in which c is the speed of light and σ is the Stefan-Boltzmann constant.

a) What is the heat capacity of photons in a box of volume V? [Give your answer in terms of T, V, c and σ.]

b) Assuming this formula is accurate all the way down to absolute zero, what is the entropy of the photon gas, with $S(T)$ defined so that $S(0) = 0$? [Give your answer in terms of T, V, c and σ.]

c) Use $P = -\left(\frac{\partial U}{\partial V}\right)_{S,N}$ to find the pressure of a photon gas as a function of the energy density, u.