PHYS 310: Thermodynamics and Statistical Mechanics
Final Exam Formula Sheet

Physical Constants

Boltzmann's constant	k	$1.381 \times 10^{-23}\,\mathrm{J/K}$
Avogadro's number	N_A	6.022×10^{23}
Gravity	g	$9.8\,\mathrm{m/s^2}$
Planck's constant	h	$6.626 \times 10^{-34}\,\mathrm{J \cdot s}$
	$\hbar$	$1.055 \times 10^{-34}\,\mathrm{J \cdot s}$
Proton/neutron mass	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$
Bohr magneton (dipole moment)	μ_B	$9.274 \times 10^{-24}\,\mathrm{J/T}$
Universal gas constant	R	$8.315\,\mathrm{J/(mol \cdot K)}$
Gas constant for air	R_a	$287\,\mathrm{J/(kg\ K)}$
Gas constant for H_2O	R_v	$462\,\mathrm{J/(kg\ K)}$
Latent heat of fusion for water	L_f	$3.33 \times 10^5\,\mathrm{J/kg}$
Latent heat of vaporization for water at 100°C	L_v	$2.26 \times 10^6\,\mathrm{J/kg}$
Latent heat of vaporization for water at 0°C	L_v	$2.50 \times 10^6\,\mathrm{J/kg}$
Specific heat of air at constant volume	c_v	$718\,\mathrm{J/(kg\ K)}$
Specific heat of air at constant pressure	c_p	$1005\,\mathrm{J/(kg\ K)}$
Specific heat of water	c_p	$4186\,\mathrm{J/(kg\ K)}$
saturation vapour pressure of H_2O at 0°C	e_0	$611\,\mathrm{Pa}$
molar mass of air	m_a	$28.97\,\mathrm{g/mol}$
molar mass ratio of water to air	ϵ_w	0.622

Unit Conversions

$$
\begin{aligned}
1\,\mathrm{atm} &= 1.013\,\mathrm{bar} = 1.013 \times 10^5\,\mathrm{Pa} \\
0^\circ\mathrm{C} &= 273.15\,\mathrm{K} \\
1\,\mathrm{eV} &= 1.602 \times 10^{-19}\,\mathrm{J}
\end{aligned}
$$

Handy Taylor Series Expansions (for $|\epsilon| \ll 1$)

$$
\begin{aligned}
f(x_0+\epsilon) &\simeq f(x_0) + f'(x_0)\,\epsilon + (1/2) f''(x_0)\,\epsilon^2 \\
(1+\epsilon)^p &\simeq 1 + p\,\epsilon + [p(p-1)/2]\,\epsilon^2 \\
\exp(\epsilon) &\simeq 1 + \epsilon + (1/2)\,\epsilon^2 \\
\ln(1+\epsilon) &\simeq \epsilon - (1/2)\,\epsilon^2
\end{aligned}
$$

Handy Definitions and Formulae

$$
\begin{aligned}
N! &\simeq \sqrt{2\pi N}\; N^N e^{-N}, \text{ for } N \gg 1\\
\ln N! &\simeq N\ln N - N, \text{ for } N \gg 1\\
S &\equiv k\ln\Omega
\end{aligned}
$$

$$
\begin{aligned}
dU &= \delta Q - P\,dV + \mu\,dN \;=\; T\,dS - P\,dV + \mu\,dN\\
dH &= d(U+PV) \;=\; T\,dS + V\,dP + \mu\,dN\\
dF &= d(U-TS) \;=\; -S\,dT - P\,dV + \mu\,dN\\
dG &= d(H-TS) \;=\; -S\,dT + V\,dP + \mu\,dN
\end{aligned}
$$

$$
\begin{aligned}
\tfrac{1}{T} &= \left(\tfrac{\partial S}{\partial U}\right)_{V,N}\\
P &= T\left(\tfrac{\partial S}{\partial V}\right)_{U,N} \;=\; -\left(\tfrac{\partial U}{\partial V}\right)_{S,N}\\
\mu &= -T\left(\tfrac{\partial S}{\partial N}\right)_{V,U}
\end{aligned}
$$

$$U = Nf\left(\tfrac{1}{2}kT\right) \qquad \text{(for } N \text{ molecules with } f \text{ degrees of freedom for manifestation of energy)}$$

$$C_v \equiv \left(\frac{\partial Q}{\partial T}\right)_{V,N} = \left(\tfrac{\partial U}{\partial T}\right)_{V,N}, \quad C_p \equiv \left(\frac{\partial Q}{\partial T}\right)_{P,N} = \left(\tfrac{\partial H}{\partial T}\right)_{P,N}$$

Ideal gas formulae: ($f = 3$ monatomic, $f = 5$ diatomic)

$$
\begin{aligned}
C_v &= (f/2)nR,\; c_v = (f/2)R_g, \qquad C_p \;=\; C_v + nR,\; c_p = c_v + R_g \qquad (R_g = nR/M_g \text{ is gas constant})\\
P &= \tfrac{1}{V}\, nRT = \tfrac{1}{V}\, NkT = \rho R_g T\\
P &\propto V^{-\gamma}, \quad \gamma = c_p/c_v = 1 + 2/f \qquad \text{(for adiabatic gas)}\\
\theta &\equiv T(P/P_0)^{-\kappa}, \quad \kappa = R_g/c_p = 1 - 1/\gamma = 2/(2+f)\\
S &= Nk\left[\ln\left(\tfrac{V}{N}\left(\tfrac{4\pi m U}{3N\hbar^2}\right)^{3/2}\right) + \tfrac{5}{2}\right] \qquad \text{(for monatomic gas)}
\end{aligned}
$$

$$(P + aN^2/V^2)(V - bN) = NkT \qquad \text{(van der Waal's equation)}$$

$$dp/dz = -\rho g \qquad \text{(hydrostatic balance)}$$

$$
\begin{aligned}
dP/dT &= L\,m/(T\Delta V) \qquad \text{(Clausius-Clapeyron equation, } L \text{ is latent heat of vaporization or fusion)}\\
P(T) &\simeq P_0 \exp\left[\tfrac{L_v}{R_v}\left(\tfrac{1}{T_0} - \tfrac{1}{T}\right)\right] \qquad \text{(vapour pressure equation, } T_0 \text{ is boiling temperature at pressure } P_0)\\
w &\equiv m_v/m_a \qquad \text{(mixing ratio: mass of water vapour per mass dry air)}\\
e &\equiv \mathcal{V}p \;\simeq\; \tfrac{w}{\epsilon_w}p \qquad \text{(partial pressure: } \mathcal{V} \text{ is volume mixing ratio)}\\
e_s(T) &\simeq e_0\,\exp\left[\tfrac{L_v}{R_v}\left(\tfrac{1}{T_0} - \tfrac{1}{T}\right)\right] \qquad \text{(partial vapour pressure equation, } e_s \text{ is saturation vapour pressure)}\\
\text{R.H.} &\equiv e/e_s \qquad \text{(relative humidity)}\\
e &= e_s(T_d) \qquad \text{(implicit definition for dew point, } T_d\text{, for given vapour pressure } e)
\end{aligned}
$$

$$
\begin{aligned}
M &= \mu_B(N_\uparrow - N_\downarrow) = -\tfrac{U}{B} \qquad \text{(for two-state paramagnet)}\\
U &= -N\mu_B B\tanh\left(\tfrac{\mu_B B}{kT}\right) \qquad \text{(for two-state paramagnet)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{D}(u) &= \left(\tfrac{m}{2\pi kT}\right)^{3/2} 4\pi u^2 e^{-mu^2/2kT}, \qquad \text{(Maxwell speed distribution)}\\
u_{max} &= \sqrt{2kT/m}, \quad u_{rms} = \sqrt{3kT/m}, \quad \bar{u} = \sqrt{8kT/\pi m}\\
\bar{\ell} &= 1/(4\pi R^2 n) = kT/(4\pi R^2 P) \qquad \text{(quick estimation of mean-free path)}\\
\ell &= \bar{\ell}/\sqrt{2} \qquad \text{(accurate value of mean-free path)}\\
p(x) &= e^{-x/\ell} \qquad \text{(probability of travelling distance } x \text{ with no collision)}\\
\Phi &= \tfrac{1}{4}n\bar{u} \qquad \text{(particle-flux: \# collisions/area/time)}
\end{aligned}
$$