PHYS 310: Thermodynamics and Statistical Mechanics

Assignment 5: Due Wednesday, December 3 at 4pm

Submit into box marked PHYS310 opposite CCIS L2-041

In all questions below, give your answer in SI metric units unless otherwise specified. The following universal constants may come in handy:

Avogadro's number	N_A	6.02×10^{23}
Boltzmann's constant	k	1.38×10^{-23} J/K
Planck's constant	h	6.63×10^{-34} J · s
proton, neutron mass	m_p,m_n	1.67×10^{-27} kg
molar mass of argon	M_{Ar}	39.9 g/mol
energy converstion	1 eV	1.60×10^{-19} J

1. a) A rectangular box contains 100 molecules of a gas. What is the probability that all the molecules are situated on the left half of the box?

   b) What is the probability that the 100 molecules are all situated in the leftmost 99% of the box?

   c) If the box contains 10,000 molecules, what is the probability they are all situated in the leftmost 99% of the box.

2. Without the assumption that the number of energy units, q, is much larger than the number of oscillators, N, in an Einstein solid, the multiplicity is given approximately by

$$\Omega \sim \left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N.$$

   a) Using this formula and writing the internal energy as $U = q\epsilon$ for constant ϵ, find an expression for the temperature of the Einstein solid. Simplify your result as much as possible.

   b) From your result in a), find an expression for the heat capacity, C.

3. Use the Sackur-Tetrode equation in the form

$$S = Nk\left\{\ln\left[\frac{V}{N}\left(\frac{4\pi mE}{3Nh^2}\right)^{3/2}\right] + \frac{5}{2}\right\}$$

   to calculate the entropy of a mole of argon gas at room temperature (293 K) and at an atmospheric pressure of 1 bar.

4. The ground state of a hydrogen atom has energy −13.6 eV. The first excited state, which occurs in 4 different configurations (one of the s- or 3 p-orbitals) has energy −3.4 eV. For this problem, you can ignore the higher excited states.

   a) Estimate the probability that a hydrogen atom at 293K is in one of its first excited states (given relative to the probability of being in the ground state).

   b) Repeat the calculation for a hydrogen atom in the atmosphere of a star whose surface temperature is 9500 K.