PHYS 310: Thermodynamics and Statistical Mechanics
Assignment 4: Due Wednesday, November 19 at 4pm
Submit into box marked PHYS310 opposite CCIS L2-041

In all questions below, give your answer in SI metric units unless otherwise specified. Use $0°C = 273.15\,K$.

1. Another unusual property of water is that its solid form is less dense than liquid water: near freezing, the density of liquid water if $1000\,kg/m^3$ and the density of ice is $917\,kg/m^3$.
   a) Use the Clausius-Clapeyron relation $(dP/dT = LM/(T\Delta V))$ to explain why in a P-T diagram for H_2O the slope of the phase boundary between ice and water is negative.
   b) Assume the densities of ice and liquid water change negligibly under pressure and that the latent heat of fusion, $L_f = 3.33 \times 10^5$ J/kg, is constant. How much pressure would you have to put on an ice cube to make it melt at $-1°C$. (Give your answer in bars accurate to 3 digits.)
   c) Approximately how deep under a glacier would you have to be before the weight of ice above gives the pressure you found in part (b)? (Give your answer in km, accurate to one decimal place.) (This mechanism provides lubrication which accelerates the advance of a thick glacier flowing down valleys.)

2. The approximate formula for the saturation vapour pressure is
$$e_s(T) = e_0 \exp\left[\frac{L_v}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right],$$
   in which $T_0 = 273.15\,K$, $e_0 = 6.11\,mbar$, $L_v = 2.50 \times 10^6$ J/kg and $R_v = 462$ J/(kg K).
   Air at temperature $T = 5°C$ has a relative humidity of 85%. Find the partial vapour pressure (in mbar) and the dew point T_d (accurate to 1 decimal place in °C).

3. A parcel of moist air at the ground (where $p = p_0 = 1\,bar$) is at its dew point temperature of 5°C. The parcel is carried upward cooling until it loses all of its contained water vapour as a consequence of condensation. Thereafter the dry air descends adiabatically back to the ground. What is the temperature of this dry parcel of air (accurate to one decimal place in °C)?

4. a) Rewrite the Maxwell-Boltzmann speed distribution $\mathcal{N}(u)$ in terms of the non-dimensional velocity $\tilde{u} \equiv u/u_{\rm m}$, in which $u_{\rm m} = \sqrt{2kT/m}$ is the most probable speed. And so write an expression for the probability of the nondimensional speed lying between $\tilde{u}$ and $\tilde{u} + d\tilde{u}$.
   b) A particle moving faster than the escape speed of $U_e = 11$ km/s has enough kinetic energy to leave the gravitational potential of the Earth. Consider a nitrogen molecule in the "thermosphere" where the temperature is 1000 K. What is the escape speed relative to $u_{\rm m}$ for a nitrogen molecule.
   c) Use tabulated values of integrals of the probability distribution, $D(v) \equiv \frac{4}{\sqrt{\pi}} v^2 e^{-v^2}$, to find the probability that a nitrogen molecule in a gas at 1000 K is moving faster than 11 km/s.
   d) The escape speed from the moon's surface is $U_m = 2.4$ km/s. Compute the probability that a nitrogen molecule in an atmosphere at 1000 K will have the moon's escape velocity. Based on this calculation, speculate why the moon has no atmosphere.

Definite integrals of Distribution function D(v) = [4/sqrt(pi)] v^2 exp(-v^2)											
Integrals of D(v)		Complementary Integrals of D(v): 1 - int_0^V D(v) dv									
V	int_0^V D(v) dv	V	int_V^inf D(v) dv	V	int_V^inf D(v) dv	V	int_V^inf D(v) dv	V	int_V^inf D(v) dv		
0.0	0.00000E+00	1.1	4.89922488E-01	4.6	3.43133092E-09	8.1	2.95224130E-28	11.6	4.78467688E-58		
0.1	7.47756E-04	1.2	4.10499239E-01	4.7	1.38198731E-09	8.2	5.85466711E-29	11.7	4.69512521E-59		
0.2	5.87563E-03	1.3	3.36662438E-01	4.8	5.45364696E-10	8.3	1.13790199E-29	11.8	4.51569716E-60		
0.3	1.92485E-02	1.4	2.70233272E-01	4.9	2.10871436E-10	8.4	2.16751057E-30	11.9	4.25682718E-61		
0.4	4.37758E-02	1.5	2.12290288E-01	5.0	7.98917987E-11	8.5	4.04643598E-31	12.0	3.93306667E-62		
0.5	8.11086E-02	1.6	1.63218111E-01	5.1	2.96582822E-11	8.6	7.40355888E-32	12.1	3.56173140E-63		
0.6	1.31510E-01	1.7	1.22818311E-01	5.2	1.07883369E-11	8.7	1.32759646E-32	12.2	3.16137538E-64		
0.7	1.93909E-01	1.8	9.04546010E-02	5.3	3.84532295E-12	8.8	2.33319248E-33	12.3	2.75027725E-65		
0.8	2.66112E-01	1.9	6.52065781E-02	5.4	1.34303028E-12	8.9	4.01878879E-34	12.4	2.34510801E-66		
0.9	3.45136E-01	2.0	4.60117061E-02	5.5	4.59640275E-13	9.0	6.78423820E-35	12.5	1.95990776E-67		
1.0	4.27593E-01	2.1	3.17823320E-02	5.6	1.54146631E-13	9.1	1.12245622E-35	12.6	1.60544434E-68		
1.1	5.10078E-01	2.2	2.14915877E-02	5.7	5.06568162E-14	9.2	1.82012512E-36	12.7	1.28896800E-69		
1.2	5.89501E-01	2.3	1.42279164E-02	5.8	1.63129927E-14	9.3	2.89266165E-37	12.8	1.01432399E-70		
1.3	6.63338E-01	2.4	9.22207087E-03	5.9	5.14784088E-15	9.4	4.50567248E-38	12.9	7.82346848E-72		
1.4	7.29767E-01	2.5	5.85266269E-03	6.0	1.59190066E-15	9.5	6.87841195E-39	13.0	5.91439673E-73		
1.5	7.87710E-01	2.6	3.63696461E-03	6.1	4.82402646E-16	9.6	1.02916329E-39	13.1	4.38238402E-74		
1.6	8.36782E-01	2.7	2.21312964E-03	6.2	1.43254940E-16	9.7	1.50920721E-40	13.2	3.18272890E-75		
1.7	8.77182E-01	2.8	1.31879547E-03	6.3	4.16887562E-17	9.8	2.16911632E-41	13.3	2.26557514E-76		
1.8	9.09545E-01	2.9	7.69609484E-04	6.4	1.18888742E-17	9.9	3.05553368E-42	13.4	1.58069203E-77		
1.9	9.34793E-01	3.0	4.39849664E-04	6.5	3.32259894E-18	10.0	4.21854249E-43	13.5	1.08095192E-78		
2.0	9.53988E-01	3.1	2.46204791E-04	6.6	9.09983077E-19	10.1	5.70833867E-44	13.6	7.24529644E-80		
2.1	9.68218E-01	3.2	1.34978199E-04	6.7	2.44235961E-19	10.2	7.57059114E-45	13.7	4.75989032E-81		
2.2	9.78508E-01	3.3	7.24805066E-05	6.8	6.42405699E-20	10.3	9.84064113E-46	13.8	3.06499005E-82		
2.3	9.85772E-01	3.4	3.81227262E-05	6.9	1.65590389E-20	10.4	1.25369328E-46	13.9	1.93442925E-83		
2.4	9.90778E-01	3.5	1.96410928E-05	7.0	4.18301299E-21	10.5	1.56543215E-47	14.0	1.19665403E-84		
2.5	9.94147E-01	3.6	9.91241202E-06	7.1	1.03555683E-21	10.6	1.91581233E-48	14.1	7.25565551E-86		
2.6	9.96363E-01	3.7	4.90046537E-06	7.2	2.51241571E-22	10.7	2.29799005E-49	14.2	4.31198549E-87		
2.7	9.97787E-01	3.8	2.37329193E-06	7.3	5.97370839E-23	10.8	2.70159643E-50	14.3	2.51171764E-88		
2.8	9.98681E-01	3.9	1.12598285E-06	7.4	1.39198171E-23	10.9	3.11293905E-51	14.4	1.43402763E-89		
2.9	9.99230E-01	4.0	5.23346670E-07	7.5	3.17879111E-24	11.0	3.51559808E-52	14.5	8.02486416E-91		
3.0	9.99560E-01	4.1	2.38305437E-07	7.6	7.11428554E-25	11.1	3.89140928E-53	14.6	4.40161045E-92		
3.1	9.99754E-01	4.2	1.06310072E-07	7.7	1.56042627E-25	11.2	4.22176741E-54	14.7	2.36635304E-93		
3.2	9.99865E-01	4.3	4.64643136E-08	7.8	3.35428245E-26	11.3	4.48912807E-55	14.8	1.24692910E-94		
3.3	9.99928E-01	4.4	1.98965369E-08	7.9	7.06646335E-27	11.4	4.67854239E-56	14.9	6.44018782E-96		
3.4	9.99962E-01	4.5	8.34749310E-09	8.0	1.45899004E-27	11.5	4.77903918E-57	15.0	3.26024409E-97		