PHYS 310: Thermodynamics and Statistical Mechanics
Assignment 3: Due Wednesday, October 15 at 4pm
Submit into box marked PHYS310 opposite CCIS L2-041

1. When the sun sets, the ground cools rapidly and so cools the air immediately above it. The air well above is unaffected and so remains at the same temperature. As a result, about a hundred meters above the surface the temperature suddenly rises from that of the cooled air below to the unchanged warmer air above. This is called an atmospheric inversion ("inversion" because the temperature locally increases, rather than undergoing its usual decrease with height).

   Suppose the temperature is $T_1 = 10°\mathrm{C}$ from the ground to $h_1 = 100\,\mathrm{m}$, the temperature increases linearly from $T_1 = 10°\mathrm{C}$ to $T_2 = 15°\mathrm{C}$ between $h_1 = 100\,\mathrm{m}$ and $h_2 = 120\,\mathrm{m}$, and from $T_2 = 15°\mathrm{C}$ the temperature decreases at the adiabatic lapse rate above $h_2 = 120\,\mathrm{m}$. Take the pressure at the ground to be $P_0 = 1000\,\mathrm{mbar}$.

   a) Find the potential temperature as a function of height and the variables T_1, T_2, h_1, h_2 and P_0. [Your solution should be a piecewise function of $\theta(z)$.]

   b) Using the values given for these variables, find the buoyancy frequency and buoyancy period at 90 m, 110 m, and 130 m.

2. Suppose n moles of an ideal diatomic gas operates in a heat engine whose PV cycle involves 3 sub-processes:
   (1) $A \to B$ is an isothermal process while the volume doubles;
   (2) $B \to C$ is an isochoric process with decreasing pressure;
   (3) $C \to A$ is adiabatic.
   Denote the volume and temperature at 'A' by V_A and T_A, respectively.

   a) Sketch the process on a PV diagram.

   b) Find the heat change in sub-process $A \to B$ in terms of T_A, n and the universal gas constant, R, and determine if this heat enters or leaves the system.

   c) Find the heat change in sub-process $B \to C$ in terms of T_A and C_v, and determine if this heat enters or leaves the system.

   d) Find the energy efficiency of the cycle, giving a numerical answer as a percentage accurate to one decimal place.

   e) The Clapeyron definition of entropy S (a state variable) is defined implicitly so that $dS = \delta Q/T$, in which dS is the infinitesimal change in entropy in a system resulting from an infinitesimal input of heat, δQ, for the system at temperature T. Find the increase in entropy in the sub-process $A \to B$, the sub-process $B \to C$ and the sub-process $C \to A$, and show that the sum of these three processes is zero.

3. Assume that heat enters an air-conditioned room at a rate proportional to the temperature difference between the hot outside (of temperature T_o) and the room's interior (of temperature T_i). That is, $\Delta Q/\Delta t = K(T_o - T_i)$, for a constant K.

   a) Assuming the air conditioner acts as an ideal Carnot refrigerator, derive an expression in terms of T_o, T_i and K for the power required to drive the air conditioner.

   b) During the afternoon the outdoor temperature increases from 27°C to 30°C. What percentage increase in power is required to drive the air conditioner to maintain the interior temperature at 21°C?