PHYS 310: Thermodynamics and Statistical Mechanics

Assignment 1: Due Wednesday, September 17 at 4pm

Submit into box marked PHYS310 opposite CCIS L2-041

In all questions below, give your answer in SI metric units with temperatures in Kelvin, unless otherwise specified. Take the value of absolute zero to be $\sim -273.15°\text{C}$.

1. A tank of volume $0.5\,\text{m}^3$ contains oxygen at a pressure of $P_0 = 1.5 \times 10^6\,\text{Pa}$ and a temperature of 20°C. The molar mass of atomic oxygen is 15.9994 g/mol.

   a) How many moles of oxygen are in the tank?

   b) How many kilograms?

   c) Find the pressure if the temperature is increased to 500°C.

   d) At a fixed temperature of 20°C, how many kilograms can be withdrawn from the tank before the pressure falls by 10% of the original pressure, P_0?

   [In each problem above, give your answers with 3-digit accuracy.]

2. In addition to the van der Waals equation, another formula used to model substances in their liquid and gaseous states is given by the Dieterici equation:

$$P = \frac{RT}{V-b}\ \exp[-a/(RTV)],$$

   in which a and b are constants. By analogy with the calculation for van der Waals equation, find an expression (in terms of a, b and/or R) for the volume, V, at the critical point.

3. The typical density of sea water at the ocean surface is $1025\,\text{kg/m}^3$. For example, this is the density if the temperature is 20°C and the salinity is 38.5 ppt. Around these values, the thermal expansion coefficient is $\alpha_T = 2.1 \times 10^{-4}\,\text{K}^{-1}$ and the haline contraction coefficient is $\alpha_S = 6.7 \times 10^{-4}\,\text{ppt}^{-1}$.

   a) Using the linear approximation for the density of salt water, $\rho = \rho_0[1 - \alpha_T(T - T_0) + \alpha_S(S - S_0)]$, find the density of sea water at a temperature of 25°C and salinity of 35 ppt. [Give your answer in kg/m^3 with 5-digit accuracy.]

   b) Suppose a box with horizontal area A is filled to a depth h_0 with sea water. The temperature of the water is then increased by a small amount ΔT. Find a formula for the increase in depth of water in the box as it depends on ΔT, h and α_T. Show that the change in depth varies approximately linearly with ΔT.

   c) Suppose the top kilometer of the ocean warms by 1°C. Using your result in b) and the value α_T given above, estimate by how much the sea level rises. (For this problem, you can ignore the influence of pressure on the linear approximation to the equation of state of sea water.)
   [Give your answer in cm with 2-digit accuracy.]

4. A hypothetical substance has an expansivity $\beta = bT/v$ and an isothermal compressibility $\kappa = a/v$, in which T is temperature, v is the specific volume, and a and b are constants. Find the corresponding equation of state in the form $f(v, T, P) =$constant, in which P is pressure.