ASSIGNMENT 4, SOLUTIONS

1) a) $g' = g\frac{\rho_2-\rho_1}{\rho_2} = (9.8\ m/s^2)\frac{1026-1020}{1026} \simeq 5.7\times10^{-2}\ m/s^2$

$H = H_1 = 100\ m$

$c = \sqrt{g'H_1} = \sqrt{(5.7\times10^{-2}\ m/s^2)(100\ m)} \simeq 2.4\ m/s$

$f_0 = 2\Omega_e \sin(30^\circ) \simeq 7.3\times10^{-5}\ s^{-1}$

$\Rightarrow L_D = \frac{c}{|f_0|} = (2.4\ m/s)/(7.3\times10^{-5}\ s^{-1}) \simeq 3.3\times10^4\ m \simeq \boxed{33\ km}$

b) $\omega = ck$

$\Rightarrow C_p = C_g = c \simeq \boxed{2.4\ m/s}$ FOR KELVIN WAVES

$\Rightarrow t = d/C_g = (1000\times10^3\ m)/(2.4\ m/s) = 4.2\times10^5\ s \simeq \boxed{4.8\ \text{DAYS}}$

c) $k = \frac{2\pi}{100\times10^3\ m} \simeq 6.3\times10^{-5}\ m^{-1}$, $\omega = \sqrt{c^2k^2+f_0^2} = \sqrt{(2.4\times6\times10^{-5})^2+(7.3\times10^{-5})^2} \simeq 1.6\times10^{-4}\ s^{-1}$

$\Rightarrow C_p = \omega/k = \frac{1.6\times10^{-4}}{6.3\times10^{-5}} = \boxed{2.6\ m/s}$

$\Rightarrow C_g = \frac{d\omega}{dk} = \frac{d}{dk}\left((c^2k^2+f_0^2)^{1/2}\right) = \frac{c^2k}{\omega} = \frac{c^2}{C_p} \simeq \frac{(2.4)^2}{2.6} \simeq \boxed{2.3\ m/s}$

$\Rightarrow t = d/C_g = (6000\times10^3\ m)/(2.3\ m/s) \simeq 2.7\times10^6\ s \simeq \boxed{31\ \text{DAYS}}$

d) $k = -6.3\times10^{-5}\ m^{-1}$, $\omega = \frac{-\beta k}{k^2+1/L_D^2}$ WITH $\beta = \frac{2\Omega_e}{R_e}\cos(30^\circ) \simeq 2.1\times10^{-11}\ \frac{1}{ms}$

$\Rightarrow \omega \simeq (2.1\times10^{-11})(6.3\times10^{-5})/\left[(6.3\times10^{-5})^2 + 1/(3.3\times10^4)^2\right] \simeq 2.7\times10^{-7}\ s^{-1}$

$\Rightarrow |C_p| = \omega/|k| = (2.7\times10^{-7}\ s^{-1})/(6.3\times10^{-5}\ m^{-1}) \simeq \boxed{4.3\times10^{-3}\ m/s}$

$|C_g| = \left|\frac{d\omega}{dk}\right| = \left|\frac{\beta}{[k^2+1/L_D^2]^2}(k^2 - 1/L_D^2)\right| = \left|-C_p\left(\frac{k^2L_D^2-1}{k^2L_D^2+1}\right)\right| = 4.3\times10^{-3}\left(\frac{(2.1)^2-1}{(2.1)^2+1}\right) \simeq \boxed{2.7\times10^{-3}\ m/s}$

$t = d/|C_g| = (6000\times10^3\ m)/(2.7\times10^{-3}\ m/s) \simeq 2.2\times10^9\ s = \boxed{2.6\times10^4\ \text{DAYS}}$

(ABOUT 71 YEARS)

2a) FROM (1) WE FOUND:

$$g' = 5.7\times10^{-2}\,m/s^2, \quad c \simeq 2.4\,m/s$$
$$f_0 = 7.3\times10^{-5}\,s^{-1}, \quad L_D \simeq 3.3\times10^4\,m$$
$$\lambda = 100\,km \Rightarrow |k| = \frac{2\pi}{\lambda} \simeq 6.3\times10^{-5}\,m^{-1}$$
AND $\omega \simeq 1.6\times10^{-4}\,s^{-1}$

FOR (1-LAYER) SHALLOW WATER WAVES FOUND
$$|A_u| = \frac{\omega}{kH}\eta_0, \quad |A_v| = \frac{f_0}{kH}\eta_0$$
SO, FOR THIS 1½ LAYER FLUID, HAVE
$$|A_u| = \frac{\omega}{kH_1}\eta_0 \simeq \{(1.6\times10^{-4}\,s^{-1})/[(6.3\times10^{-5}\,m^{-1})(100\,m)]\}(1\,m)$$
$$\boxed{\simeq 0.025\ m/s}$$
$$|A_v| = \frac{f_0}{kH_1}\eta_0 \simeq \{(7.3\times10^{-5}\,s^{-1})/[(6.3\times10^{-5}\,m^{-1})(100\,m)]\}(1\,m)$$
$$\boxed{\simeq 0.012\ m/s}$$

b) FOR (1-LAYER) SHALLOW WATER WAVES FOUND MEAN ZONAL ENERGY FLUX TO BE
$$\langle \mathcal{F}_{E,x}\rangle = c^2\langle u\eta\rangle = \frac{1}{2}c^2\frac{\omega}{kH}\eta_0^2$$

HERE WE HAVE $\langle \mathcal{F}_{E,x}\rangle = \frac{1}{2}c^2\frac{\omega}{kH_1}\eta_0^2$
$$\simeq \frac{1}{2}(2.4\,m/s)^2(0.025\,m/s)(1\,m)$$
$$\boxed{\simeq 0.072\ m^4/s^3}$$

[ALTERNATELY, FIND ENERGY $\langle E\rangle = \langle \frac{1}{2}H_1(u^2+v^2) + \frac{1}{2}g\eta^2\rangle = \left[\frac{1}{4}H_1\left(\frac{\omega^2+f_0^2}{(kH_1)^2}\right) + \frac{1}{4}g\right]\eta_0^2$
THEN FIND $\langle \mathcal{F}_{E,x}\rangle = C_{gx}\langle E\rangle$]

c) $P = \rho_0\langle \mathcal{F}_{E,x}\rangle W \simeq (1020\,kg/m^3)(0.072\,m^4/s^3)(1000\,m)$
$$\simeq 7.3\times10^4\ kg\,m^2/s^3$$
$$\Rightarrow \boxed{P = 7.3\times10^4\ W} \qquad \left(1\,W = 1\,J/s = 1\left(kg\frac{m^2}{s^2}\right)/s\right)$$

3) a) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{13.7\,cm} \simeq 0.46\ cm^{-1}$

$\Rightarrow \omega = Uk = (2\,cm/s)(0.46\,cm^{-1}) \simeq \boxed{0.92\ s^{-1}}$

b) WITH NO ROTATION $\omega^2 = N^2 \cos^2 \Theta$

$\Rightarrow \Theta = \cos^{-1}(\omega/N) = \cos^{-1}(0.92/1.5) = \boxed{52^\circ}$

$\Rightarrow |m| = k \tan\Theta = 0.46 \tan(52^\circ) \simeq 0.60\ cm^{-1}$

[ALTERNATELY $\omega^2 = N^2k^2/(k^2+m^2) \Rightarrow m^2 = \frac{N^2k^2}{\omega^2} - k^2 \Rightarrow |m| = k\sqrt{\frac{N^2}{\omega^2} - 1} = 0.60\,cm^{-1}$]

So $\lambda_z = \frac{2\pi}{|m|} = \boxed{11\ cm}$

c) $A_u = \omega \frac{m}{k} \eta_0$, $A_w = \omega \eta_0$, $\eta_0 = \frac{1}{2}(2.6\,cm) = 1.3\,cm$ (HALF CREST-TROUGH)

$\Rightarrow \langle uw \rangle = \frac{1}{2}\omega^2 \frac{m}{k} \eta_0^2 \simeq \frac{1}{2}(0.92\,s^{-1})^2 \left(\frac{0.60\,cm^{-1}}{0.46\,cm^{-1}}\right)(1.3\,cm)^2$

$\simeq 0.93\ cm^2/s^2$

$\Rightarrow \langle F_M \rangle \equiv \rho_0 \langle uw \rangle \simeq (1\ g/cm^3)(0.93\,cm^2/s^2) \simeq 9.3 \times 10^{-1}\ g/(cm\,s^2)$

$\Rightarrow D = \langle F_M \rangle \lambda W = (9.3 \times 10^{-1} \frac{g}{cm\,s^2})(13.7\,cm \times 19.7\,cm) = 2.5 \times 10^2\ g\,cm/s^2$

$\Rightarrow \boxed{D = 2.5 \times 10^{-3}\ N}$

4) a) $\omega = \frac{2\pi}{(12\ \text{HOURS})\times(3600\ \text{s/HR})} \simeq 1.5\times10^{-4}\ s^{-1}$

$f_0 = 2\Omega_e \sin 22^\circ \simeq 5.5\times10^{-5}\ s^{-1}$

$N \simeq 4\ \text{cph} \times 1.75\times10^{-3} = 7.0\times10^{-3}\ s^{-1}$

So $\tan\Theta = \pm\left(\frac{N^2-\omega^2}{\omega^2-f_0^2}\right)^{1/2} \simeq \left(\frac{(7.0\times10^{-3})^2-(1.5\times10^{-4})^2}{(1.5\times10^{-4})^2-(5.5\times10^{-5})^2}\right)^{1/2} \simeq \boxed{50 = \left|\frac{m}{k}\right|}$

$\Rightarrow \boxed{\Theta \simeq 89^\circ}$

$\alpha = \left|\frac{m}{k}\right| = 50$

b) $C_{pz} = \frac{\omega}{|K|^2 m} \Rightarrow k^2 = \frac{\omega m}{C_{pz}} - m^2 = \frac{\omega\alpha}{C_{pz}}k - \alpha^2 k^2$

$\Rightarrow (1+\alpha^2)k^2 = \frac{\omega\alpha}{C_{pz}}k$

$\Rightarrow k = \left(\frac{\omega\alpha}{C_{pz}}\right)/(1+\alpha^2) \overset{\alpha\gg1}{\simeq} \frac{\omega}{\alpha C_{pz}}$

GIVEN $|C_{pz}| \simeq (700\text{m}-200\text{m})/(2\text{HRS}) \simeq 500\text{m}/(2\times3600\text{s}) \simeq 6.9\times10^{-2}\ \text{m/s}$

$\Rightarrow k \simeq (1.5\times10^{-4}s^{-1})/(50\times6.9\times10^{-2}\text{m/s}) \simeq \boxed{4.3\times10^{-5}\ m^{-1}}$

$\Rightarrow \lambda_x = 2\pi/k \simeq 2\pi/(4\times10^{-5}m^{-1}) \simeq 1.5\times10^5 m = \boxed{1.5\times10^2\ km}$

c) $|C_{gz}| = \frac{1}{\omega}\frac{N^2k^2}{m^3} = \frac{1}{k}\frac{1}{\omega}N^2\left(\frac{m}{k}\right)^{-3}$

$\simeq (4.3\times10^{-5}m^{-1})^{-1}(1.5\times10^{-4}s^{-1})^{-1}(7.0\times10^{-3}s^{-1})^2(50)^{-3}$

$\simeq \boxed{6.1\times10^{-2}\ m/s}$

d) SINCE $\omega > f_0$, MAXIMUM SPEED IS $\|A_u\| \sim \omega\left|\frac{m}{k}\right|\eta_0$

SO MAXIMUM VERTICAL DISPLACEMENT IS

$\eta_0 = \|A_u\|/(\omega\cdot\left|\frac{m}{k}\right|) \simeq (0.2m/s)/(1.5\times10^{-4}s^{-1}\cdot 50)$

$\boxed{\simeq 27m}$

[NOTE: THIS IS A BIG VALUE FOR TYPICAL MOTION IN OCEANS, BUT WAS CONFIRMED IN SEPARATE MEASUREMENTS].