EAS 475/587: Atmospheric and Ocean Dynamics

Assignment 4: Due Thursday, Dec. 7 at 2pm

1. Here we consider the propagation of interfacial inertial, coastal Kelvin and Rossby Waves in the Pacific Ocean. We model the ocean as a 2-layer fluid with total depth $H_T = 4\,\text{km}$ and a pycnocline at $H_1 = 100\,\text{m}$ below the surface separating water with density $\rho_1 = 1020\ \text{kg/m}^3$ above from colder and saltier water with density $\rho_2 = 1026\ \text{kg/m}^3$ below. Because $H_T \gg H_1$, we can treat the abyss as infinitely deep. In each part below, we consider waves propagating near 30°N with wavelength 100 km.
   (a) What is the (internal Rossby) deformation radius?
   (b) What is the phase and group speed of Kelvin waves? How long would a Kelvin wavepacket take to travel 1000 km north-westward along the western coast of Mexico?
   (c) What is the phase and group speed of inertial waves? How long would a inertial wavepacket take to travel 6000 km westward across the Pacific?
   (d) What is the phase and group speed of Rossby waves? How long would a Rossby wavepacket take to travel 6000 km westward across the Pacific?

   In (b)-(d), give your answers for speed in m/s and for travel time in days.)

2. An interfacial inertial wave travels due eastward across the Pacific Ocean at 30°N modelled as in question 1 above with a pycnocline at 100 m depth separating surface waters of density $1020\ \text{kg/m}^3$ from abyssal waters of density $1026\ \text{kg/m}^3$. The waves have zonal wavelength 100 km and the maximum displacement of the interface is $\eta_0 = 1.0\,\text{m}$.
   (a) Find the amplitudes (ie maximum values) of the horizontal velocity components, u and v. (Give your answers in units of m/s.)
   (b) Evaluate the magnitude of the mean zonal energy flux (per unit mass times depth), $\langle F \rangle$.
   (c) compute the power (in Watts) transmitted over a 1km north-south span of the waves.

3. A $W = 19.7\,\text{cm}$ wide tank is filled with uniformly salt-stratified fluid with buoyancy frequency $N = 1.5\,\text{s}^{-1}$. Model sinusoidal hills with width, W, wavelength $\lambda = 13.7\,\text{cm}$ and crest-to-trough hill-height 2.6 cm are towed rightward along the surface at 2 cm/s. As a result, the hills launch down and rightward-propagating internal gravity waves.
   (a) What is the frequency at which the fluid is forced by the towed hills?
   (b) Find the vertical wavelength of the waves and the angle formed between the wave crests and the vertical.
   (c) The mean drag, D, exerted on the hills due to wave generation is the momentum flux times the area. That is $D = \langle F_M \rangle \lambda W$ in which $\langle F_M \rangle \equiv \rho_0 \langle uw \rangle$ is the mean vertical flux of horizontal momentum. Find D, giving your answer in Newtons.

4. The tide flowing over the Kaena Ridge (22°N, 159°W) between Kauai and Oahu in the Hawaiian Islands, launches an inertia gravity wave that emanates northeast and southwest from the Kauai Channel. During the "Hawaii Ocean Mixing Experiment" in 2001-2002, oceanographers attempted to measure the generation and breakdown of these waves in order to assess how energy is transferred from large-scale tides to small-scale mixing. One observation recorded a vertical time-series of horizontal velocity over several weeks (eg see figure below). This showed crests and troughs (maxima and minima of speeds) moving downward from 200 to 700 m depth in 2 hours at nearly constant speed, repeating the process every tidal period of about 12 hours. (Recall that downward vertical phase speed corresponds to upward vertical group speed.) The mean stratification over this depth was $N = 4\,\text{cph}$ (1 cph is a cycle-per-hour $= 2\pi/(60 * 60) = 1.75 \times 10^{-3}\,\text{s}^{-1}$).

   (a) From the wave frequency and knowing N and f_0, compute the angle Θ (in degrees) formed between crests and the vertical. Hence determine the magnitude of the ratio between vertical and horizontal wavenumbers: $|m/k| = \tan\Theta$.

   (b) From the vertical phase speed and your result in (a), find the horizontal wavenumber and horizontal wavelength (in km).

   (c) What is the vertical group speed?

   (d) The maximum horizontal speed of the fluid was measured to be about 0.2 m/s. What was the corresponding maximum vertical displacement?

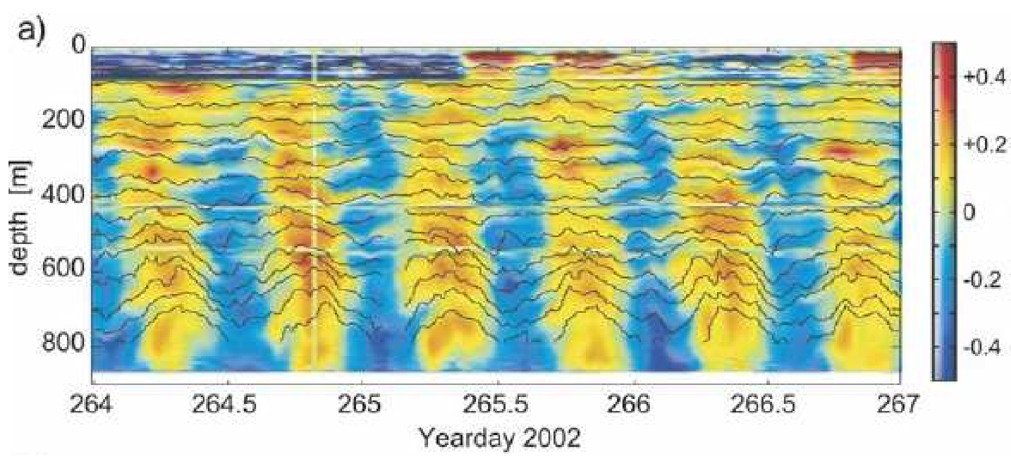

Figure 1: Figure 5a from Rainville and Pinkel, J. Phys. Oceanogr., **36**, 1104-1122 (2006).