Department of Philosophy

Phil 422/Phil 522: Gödel's incompleteness theorems and computability

Katalin Bimbo

(Course description — Fall term (2026))

Symbolic logic emerged in the 19th century, because of a desire to make reasoning and the language of mathematics more rigorous. Philosophers and mathematicians, for example, De Morgan, Boole, Peirce and Frege played a chief role in the development of modern formal logic. David Hilbert initiated the movement to formalize mathematical theories such as Euclidean geometry. An axiomatic theory — based on 2-valued logic — has to be consistent in order not to be trivial.

Kurt Gödel, while attempting to advance Hilbert's program, proved a theorem, which is nowadays often called Gödel's first incompleteness theorem. Exemplified on **PA** (first-order Peano arithmetic), the theorem claims that there is a sentence of **PA** that is neither provable nor refutable, if **PA** is consistent. The second incompleteness theorem (due, separately, to Gödel and J. von Neumann) states that **PA** cannot prove that **PA** is consistent under the same condition.

The incompleteness theorems trigger a profusion of questions. What is **PA** exactly and how can a claim be proved or refuted in it? Are there other mathematical theories that are susceptible to similar theorems (and if there are, which theories are incomplete)? Does the incompleteness of a theory interfere with mathematical practice that employs that theory? Are there (informally) interesting or natural arithmetical theorems that cannot be proved in **PA**? Do the incompleteness theorems impact Hilbert's program or thinking about the foundations of mathematics in general? Can **PA** be weakened or strengthened to make it into a complete theory? What happens if the underlying logic is changed to a logic that has more than two truth values?

The goal of this course is to address some of the questions above. We will look at precise formulations of arithmetical theories and of the incompleteness theorems. In order to grasp the content of the theorems, we will glance at their proofs in some detail, which will include learning about a handful of notions of computation. In the later part of the course, we will either steer toward more philosophical problems or we will dive into more logical and mathematical questions (or both).

Time: M, W, F 13:00 pm–13:50 pm

Texts: The course will utilize a variety of resources (from papers in journals to chapters in textbooks or monographs), most of which will be available online (through the Library of the University of Alberta).

For *further information*, please contact the instructor at <bimbo@ualberta.ca>.

The (official) **course outline** will be available in the canvas class during the course.