Department of Philosophy

Phil 420/Phil 522: Metalogic/Topics in Logic

Katalin Bimbo

[Course description — Winter term (2021)]

Logic has been a center piece of philosophy since the time of Aristotle. In the last 200 years or so, logic underwent a tremendous development, which expanded its scope and applicability. (Nowadays, parts of logic are parts of other disciplines beyond philosophy, such as computer science, mathematics, informatics and linguistics.) A characteristic feature of this progress is an increase in rigor and precision, which is in harmony with the analytic tradition in philosophy.

Two-valued logic with quantification (often abbreviated as Fol) is arguably one of the simplest systems of logic, yet it is rich enough to have interesting properties. (Some of the basics of Fol are taught in the philosophy courses "Symbolic Logic 1" and "Symbolic Logic 2.") The ability to formalize simple English sentences and to check the correctness of inferences is *indispensable in the work of a philosopher*, especially, in exact philosophy. However, a deeper understanding of Fol is needed to appreciate the role of Fol in mathematics and other sciences, and to philosophize about those areas of knowledge.

This course will focus on establishing some properties of Fol that are well-known by their label; we will fill those labels with substance. Instead of a natural deduction or an analytic tableaux formulation of Fol, we will work with an *axiomatic proof system*, which is more amenable to extensions and which permits simpler reasoning about proofs in the system. To prove key theorems such as the *soundness and completeness theorem*, we will work with a precise definition of interpretations for first-order languages. As an easy consequence, we will obtain *compactness*. The theorems named after L. Löwenheim and T. Skolem are some of the most impactful for philosophy; we will prove these too. We might touch upon further methods and theorems such as the *ultrapower construction* (that can bring infinitesimals back to life), the *quantifier elimination* method (that can help to prove certain theories decidable) and *Beth's definability theorem* (that has implications for the philosophy of sciences). We will conclude the course by sampling the ideas that led to the development of formal logic into several *new directions* well beyond Fol.

[The formal prerequisite for the Phil 420 course is Phil 220. The prerequisite may be waived in certain cases upon request. There is no official prerequisite for Phil 522.]

Time: M, W, F 13:00 pm–13:50 pm

Texts: Mendelson, E., *Introduction to Mathematical Logic*, 6th ed., CRC Press, Boca Raton, FL, 2015. Smullyan, R. M., *First-order Logic*, Dover Publications, New York, NY, 1995. (recommended)

For **further information**, please contact the instructor at <bimbo@ualberta.ca>
The (official) **course outline** is available in the e-classroom during the course.