Mathematics is often grouped together with the sciences. However, mathematics seems to study something strikingly different from the bits and pieces of “nature,” that are the objects of investigation in the empirical sciences. Philosophical questions arise immediately: Do numbers, triangles, manifolds and functions exist? What sort of existence do they have? How can we acquire beliefs and knowledge about these objects? What does make mathematics “unreasonably effective” in its applications?

The history of philosophy and of mathematics lists plenty of thinkers who gave various answers to such questions. In this course, we will touch upon three main approaches from the early part of the 20th century that aimed at explaining the status of mathematical objects and mathematical knowledge: 
logicism, intuitionism and formalism.

Further ways to tackle some of the same issues go under the labels platonism, realism, constructivism, nominalism, structuralism and fictionalism. These latter approaches will also be mentioned — some of them at greater length than others.

Reasoning with symbols, often in a completely abstract setting, plays a central role in mathematics, especially, since the late 19th century. The loss of immediacy, which was provided by geometric and physical insights in earlier times, necessitated the introduction of more rigorous methods of proof. This course will explore some of the well-known (as well as some less well-known) connections between mathematics and various systems of logic. We will also glance at other questions such as the role of notation, of pictures, of computers and of experiments in mathematical practice.

The course — inevitably — will include some examples from mathematics. They will be either quite simple (i.e., high-school-level illustrations), or they will be explained in the textbook or in the lectures.

[There is no formal prerequisite for the course, and it is not assumed that you have taken a university-level course in mathematics. Nevertheless, interest in or knowledge of mathematics will surely be helpful or even advantageous.]