

Solution to exercise 2.3.2.

$$\frac{\frac{\mathcal{A}(y) \vdash \mathcal{A}(y)}{\forall x \mathcal{A}(x) \vdash \mathcal{A}(y)}}{\vdash \forall x \mathcal{A}(x) \supset \mathcal{A}(y)}$$

Notice that x is OK for y in $\mathcal{A}(y)$, hence, y is OK for x in $\mathcal{A}(x)$.

$$\frac{\frac{\frac{\mathcal{A} \vdash \mathcal{A} \quad \mathcal{B}(y) \vdash \mathcal{B}(y)}{\mathcal{A} \supset \mathcal{B}(y), \mathcal{A} \vdash \mathcal{B}(y)}}{\forall x (\mathcal{A} \supset \mathcal{B}(x)), \mathcal{A} \vdash \mathcal{B}(y)}}{\forall x (\mathcal{A} \supset \mathcal{B}(x)), \mathcal{A} \vdash \forall x \mathcal{B}(x)}}{\frac{\forall x (\mathcal{A} \supset \mathcal{B}(x)) \vdash \mathcal{A} \supset \forall x \mathcal{B}(x)}{\vdash \forall x (\mathcal{A} \supset \mathcal{B}(x)) \supset \mathcal{A} \supset \forall x \mathcal{B}(x)}}$$

The side condition of the axiom is used in the application of the $(\vdash \forall)$ rule.