

Solution to exercise 2.3.1.

$$\begin{array}{c}
 \frac{\frac{\frac{A \vdash A}{A \supset B \supset C, A, B \vdash C}}{A \supset B \supset C, B, A \vdash C}}{B, A \supset B \supset C, A \vdash C}}{A \supset B, A, A, A \supset B \supset C \vdash C}}{A, A \supset B, A, A \supset B \supset C \vdash C}}{A, A, A \supset B, A \supset B \supset C \vdash C}}{A, A \supset B, A \supset B \supset C \vdash C}}{A \supset B, A \supset B \supset C \vdash A \supset C}}{A \supset B \supset C \vdash (A \supset B) \supset A \supset C}}{\vdash (A \supset B \supset C) \supset (A \supset B) \supset A \supset C}
 \end{array}$$

$$\begin{array}{c}
 \frac{\frac{\frac{A \vdash A}{\vdash A, \neg A}}{A \vdash A}}{\vdash A, \neg A}}{\neg A \supset B, \neg A \supset \neg B \vdash A, A}}{\neg A \supset B, \neg A \supset \neg B \vdash A}}{\neg A \supset \neg B \vdash (\neg A \supset B) \supset A}}{\vdash (\neg A \supset \neg B) \supset (\neg A \supset B) \supset A}
 \end{array}$$

Other proofs exist too. (A good practice is to check that all the rules are applied correctly above.)