Solution to exercise 2.3.1.

\[
\begin{array}{c}
A \vdash A \\
\hline
B \vdash B \\
C \vdash C \\
\hline
\hline
A \vdash A \\
B \vdash C, B \vdash C \\
\hline
A \vdash B \vdash C, A, B \vdash C \\
A \vdash B \vdash C, B, A \vdash C \\
B, A \vdash B \vdash C, A \vdash C \\
\hline
A \vdash A \\
\hline
B, A, A \vdash B \vdash C \vdash C \\
\hline
A \vdash B, A, A, A \vdash B \vdash C \vdash C \\
\hline
A, A \vdash B, A, A \vdash B \vdash C \vdash C \\
\hline
A, A \vdash B, A \vdash B \vdash C \vdash C \\
\hline
A, A \vdash B, A \vdash B \vdash C \vdash A \vdash C \\
\hline
A \vdash B \vdash C \vdash (A \vdash B) \vdash A \vdash C \\
\hline
\vdash (A \vdash B \vdash C) \vdash (A \vdash B) \vdash A \vdash C \\
\hline
\end{array}
\]

\[
\begin{array}{c}
A \vdash A \\
B \vdash B \\
\hline
\hline
A \vdash A \\
\hline
\neg A \vdash \neg B, B \vdash A \\
\hline
\neg A \vdash \neg B, \neg A \vdash \neg B \vdash A \\
\hline
\neg A \vdash \neg B, \neg A \vdash \neg B \vdash A \\
\hline
\neg A \vdash \neg B \vdash (\neg A \vdash B) \vdash A \\
\hline
\vdash (\neg A \vdash \neg B) \vdash (\neg A \vdash B) \vdash A \\
\hline
\end{array}
\]

Other proofs exist too. (A good practice is to check that all the rules are applied correctly above.)