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Fangwei Xu

Date: . . . .

"Progress is the activity of today and the assurance of tomorrow."

- Ralph Waldo Emerson

University of Alberta

ASSESSMENT OF CONTROL SYSTEM PERFORMANCE: THE EFFECTS OF DISTURBANCES

by

Fangwei Xu

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of **Doctor of Philosophy**

in

Process Control

Department of Chemical and Materials Engineering

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University of Alberta

Faculty of Graduate Studies and Research

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **Assessment of Control System Performance: The Effects of Disturbances** submitted by Fangwei Xu in partial fulfillment of the requirements for the degree of **Doctor of Philosophy**.

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To my mother, my father and my wife

Abstract

Over the last few decades, controller performance assessment has become one of the most active research areas in process control community. Though most algorithms are based on minimum variance control (MVC) benchmark, other methods, with consideration of time varying dynamics, disturbances, model plant mismatch etc., are gaining ground as more realistic benchmarks for advanced control monitoring. This thesis focuses on the controller performance assessment under disturbance effects. Linear matrix inequalities (LMIs) and covariance analysis methods are used as main mathematical tools for solving problems.

First, the controller performance of a class of linear processes is investigated under linear time invariant (LTI) control subject to linear time varying (LTV) disturbances, abbreviated as the LTVD problem. The structured closed-loop response is introduced to formulate the performance limit problem and performance assessment problem. The problems are solved for both SISO and MIMO processes by using LMI techniques. The regular, weighted and generalized LTVD benchmarks are derived respectively with distinct objective functions which result in different control performance in dealing with different disturbances.

A more general framework based on the structured closed-loop response is proposed for performance assessment subject to a pre-specified variance/covariance upper bound constraint. Its feasibility equivalence is derived with covariance control methods, giving rise to a full or reduced order solutions accordingly. An optional optimization strategy is presented for a practical solution by minimizing the gap between the resultant structured closed-loop response and the existing one in the sense of H_∞ norm.

A higher level performance assessment for model predictive control (MPC) applications is studied with the consideration of disturbance effects. Both variability and constraint are taken into account for economic benefit potential. They are utilized as two tuning

knobs to improve economic performance. The variance performance is shown to be readily transformed to the economic performance. A systematic approach is given to evaluate the performance of existing MPC applications, which includes variance and economic performance assessment, sensitivity analysis and tuning guidelines.

Finally, a practical framework for industrial implementation is suggested. The software package developed in this thesis is plant-oriented with standard DCS interfaces and is readily applied to process industries.

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Glossary

Notation

η_T	economic performance index with MVC as the benchmark
η_{wot}	economic performance index without tuning
ΔJ_B	optimal benefit potential by reducing variability and relaxing constraint simultaneously
ΔJ_C	optimal benefit potential by relaxing constraint
ΔJ_E	existing benefit potential
ΔJ_I	ideal benefit potential
ΔJ_{MVC}	benefit potential that is achieved by MVC
ΔJ_T	theoretical benefit potential
ΔJ_V	optimal benefit potential by reducing variability
a_{ki}	quadratic objective coefficient for i-th CV at time stamp k
a_{kj}	quadratic objective coefficient for j-th MV at time stamp k
a_t, a_k	zero mean white noise sequence with variance σ^2
b_{ki}	linear objective coefficient for i-th CV at time stamp k
b_{kj}	linear objective coefficient for j-th MV at time stamp k
d	order of interactor matrix (MIMO), process time delay (SISO)
m	number of MV
p	number of CV
r_{yi}	percentage of variability change of i-th CV
s_{uj}	percentage of constraint limit change of j-th MV
s_{yi}	percentage of constraint limit change of i-th CV
u_{kj0}	sampled data value of j-th MV at time stamp k
y_{ki0}	sampled data value of i-th CV at time stamp k
y_t, y_k	process output at time t or k
$y_t^{(i)}$	process output at time t subject to the i-th disturbance
$\tilde{y}_t$	interactor-filtered process output
$\hat{y}_t$, $\hat{y}_k$	estimated process output
D	interactor matrix
$E(.)$	expectation (mean) operator
G_{cl}	closed-loop response model
$\hat{G}_{cl}$	estimated closed-loop response model
G_R	desired structured closed-loop response
K_{ij}	the steady state gain value
L_R	desired structured closed-loop response
N_L	sampled data length
N_i	i-th disturbance dynamics affecting the process

R_C	target benefit potential ratio by tuning constraint
R_V	target benefit potential ratio by tuning variability
R_{yi}	specified variability reduction percentage of i-th CV
S_{uj}	specified constraint percentage change of j-th MV
S_{yi}	specified constraint percentage change of i-th CV
T	process transfer function or matrix
$\tilde{T}$	time delay free process transfer function or matrix
U_{holkj}	half of the constraint range of j-th MV at time stamp k
U_{qorj0}	quarter of existing operating range of j-th MV
U_{stdj}	standard deviation of j-th MV
U_{Hkj}	high limit of j-th MV at time stamp k
U_{Lkj}	low limit of j-th MV at time stamp k
Y_{holki}	half of the constraint range of i-th CV at time stamp k
Y_{stdi0}	sampled standard deviation of i-th CV
Y_{Hki}	high limit of i-th CV at time stamp k
Y_{Lki}	low limit of i-th CV at time stamp k

Acronyms

APC	Advanced process control
ARMA	Auto-regressive moving average
CLP	Closed-loop potential
CV	Controlled variable
DCS	Distributed control system
DMC	Dynamic matrix control
DV	Disturbance variable
MA	Moving average
FCOR	Filtering and correlation analysis
GCC	Generalized covariance constrained
GOHTU	Gas oil hydrotreating unit
IMC	Internal model control
LMIPA	APC economic performance assessment and tuning guidelines
LP	Linear programming
LQG	Linear quadratic gaussian
LMI	Linear matrix inequality
LTI	Linear time invariant
LTV	Linear time-varying
LTVD	Linear time-varying disturbances
MIMO	Multi-input multi-output
MPC	Model predictive control
MV	Manipulated variable
MVPA	Multivariate controller performance assessment
NLP	Nonlinear programming
NMIR	Normalized multivariate impulse response
OPC	OLE for process control
PATS	Performance analysis technology and solutions
PID	Proportional-integral-derivative
QP	Quadratic programming
RMPCT	Robust model predictive control technology, Honeywell product
RTW	Real-time workshop
RTWT	Real-time windows target
SDP	Semi-definite programming
SIMO	Single-input multi-output
SISO	Single-input single-output
SRU	Sulphur recovery unit
SVD	Singular value decomposition

1 Introduction

1.1 An overview of controller performance monitoring

Since Harris' seminal paper (Harris, 1989), controller performance assessment has attracted significant interests from both academia and industries. The lower bound of output variance is feedback control invariant and can be estimated directly from closed-loop routine operating data with an *a priori* known process time delay. A significant distinction from previous approaches is that it studies directly the existing control system which is subject to a variety of unknown practical disturbances on the run-time stage rather than on the design stage. It takes advantage of closed-loop routine operating data with no detailed information of the controller and the process. The performance measure can give a straight insight into the potential of improving the performance of an existing control system. Continuous performance monitoring allows timely detection of performance degradation in control loops and routine maintenance of such loops at optimal settings can result in huge monetary savings for a typical chemical complex. Survey papers and books in this area have been published by Qin (1998), Harris *et al.* (1999), Harris and Seppala (2001), Huang and Shah (1999), Horch (2000), Hoo *et al.* (2003), and Jelali (2006).

For a given stable single-input single-output (SISO) process with time delay d, the closed-loop relationship between the unmeasured disturbances and the process output can be expressed as an infinite-order moving average (MA) process,

$$y_t = (f_0 + f_1 q^{-1} + \cdots + f_{d-1} q^{d-1} + f_d q^{-d} + \cdots) a_t \tag{1.1}$$

where a_t is white noise with variance σ_a^2. The first d terms define the minimum variance which is feedback control invariant (Harris, 1989),

$$\sigma_{mv}^2 = (f_0^2 + f_1^2 + \cdots + f_{d-1}^2)\sigma_a^2 \tag{1.2}$$

For a process with actual output variance σ_y^2, a normalized performance index (Desborough and Harris, 1992; Harris, 2004; Quinn *et al.*, 2005) is defined as

$$\eta_{Harris} = 1 - \frac{\sigma_{mv}^2}{\sigma_y^2} \tag{1.3}$$

and the least squares regression approach is suitable to estimate the minimum variance as well as the performance index. Many researchers adopted the close-loop potential (CLP) factor as the performance index (Kozub, 1996; Huang and Shah, 1999):

$$\eta_{clp} = \frac{\sigma_{mv}^2}{\sigma_y^2} \tag{1.4}$$

which can be easily estimated using an efficient stable filtering and correlation analysis (FCOR) algorithm (Huang and Shah, 1999). This minimum variance approach was also extended to the performance assessment of feedback/feedforward control loops (Stanfelj *et al.*, 1993; Desborough and Harris, 1993), cascade control loops (Ko and Edgar, 2000) and non-minimum phase systems (Tyler and Morari, 1995). Besides the minimum variance control (MVC) benchmark, some other optional benchmarks have also been studied with the replacement of σ_{mv}^2 by some optimal criterion value σ_{opt}^2 (Eriksson and Isaksson, 1994; Hugo, 2001) or some user specified response value σ_{user}^2 (Huang and Shah, 1999; Li *et al.*, 2003), i.e.,

$$\eta_{opt} = \frac{\sigma_{opt}^2}{\sigma_y^2} \quad or \quad \eta_{user} = \frac{\sigma_{user}^2}{\sigma_y^2} \tag{1.5}$$

For a multi-input multi-output (MIMO) process, the multivariate controller performance can also be evaluated by the MVC benchmark (Harris *et al.*, 1996; Huang *et al.*, 1997*a*; Ko and Edgar, 2001*b*; Shah *et al.*, 2001). The difficulty in multivariate controller performance assessment is the factorization of the time delay matrix, which is known as the interactor matrix. The interactor matrix is an equivalent form of the time delay in multivariate systems. It is shown that a unitary interactor matrix is an optimal factorization of time delays for multivariate systems in terms of minimum variance control and controller performance assessment (Huang and Shah, 1999). A key to performance assessment of multivariate processes using MVC as the benchmark, is to estimate the benchmark performance from routine operating data with an *a priori* knowledge of time delays or interactor matrices. The expression for the feedback control invariant term (minimum variance) is then derived by using the interactor matrix. The multivariate performance indices can be estimated by multivariate FCOR algorithm (Huang and Shah, 1999). Likewise, the MVC benchmark was extended to evaluate the performance of feedforward plus feedback controller of MIMO systems (Huang *et al.*, 2000) and MIMO non-minimum phase systems (Huang and Shah, 1999) as well. In addition, the LQG (Linear Quadratic Gaussian) benchmark (Huang and Shah, 1999; Shah *et al.*, 2001) takes both the control effort and the output performance into account. Its tradeoff curve can be obtained in terms of the $\mathcal{H}_2$ norm of the appropriate transfer function matrices.

With more and more model predictive control (MPC) applications in the process industry, the problem of how to evaluate their performance has drawn great attention. Ko and Edgar (2001*b*) presented a benchmark based on the finite horizon minimum variance controller by using closed-loop data and the knowledge of the order of the delay matrix. With the knowledge of the process and disturbance models, the lower bounds on constrained performance of a finite horizon minimum variance controller can be obtained (Ko and Edgar, 2001*a*). The MVC benchmark has also been used to investigate the performance of industrial MPC applications with consideration of constraints, optimization and interaction (Gao *et al.*, 2003). The design versus achieved benchmark (Patwardhan *et al.*, 2002) uses a criterion that commensurates with the actual design objective value of the controller and then compares the achieved performance. This idea is analogous to the method of Kammer *et al.* (1996), which was based on the frequency domain comparison of the achieved and design objective functions for LQG. For a model predictive controller, if the design objective function is denoted by $\hat{J}_k$ and the optimal control moves by Δu_k^*, the optimal value of the design objective function is given by

$$\hat{J}_k^* = \hat{J}_k(\Delta u_k^*)$$

The performance index using design versus achieved as a benchmark is defined as

$$\eta(k) = \frac{\hat{J}_k^*}{J_k} \tag{1.6}$$

where J_k is the actual achieved objective function value. This performance index actually reflects whether or not the achieved performance meets the design requirements. The historical benchmark (Huang and Shah, 1999) is based on the comparison of the achieved objective functions in the good region with the current performance. Schäfer and Cinar

(2004) tried to integrate the historical benchmark and design versus achieved benchmark for monitoring and diagnosis.

It should be noted that no single performance index is sufficient for effective performance monitoring (Kozub, 1996). For the MPC performance assessment, one should not just rely on any one specific index and it would be more appropriate to check all relevant indices that reflect performance measures from different aspects (Shah *et al.*, 2001).

1.2 A class of linear time-varying processes

For the linear time varying (LTV) processes, little work has been done on the controller performance assessment since it is not a straightforward extension of its linear time invariant (LTI) counterpart (Li *et al.*, 1997; Li and Evans, 1997; Huang, 2002). For a linear process T controlled by Q shown in Figure 1.1 , it is driven by a sequence of zero mean white noise a_t and subject to several different disturbance dynamics, $N_i, i = 1, 2, \cdots$, in different time durations respectively. This is often observed in chemical processes due

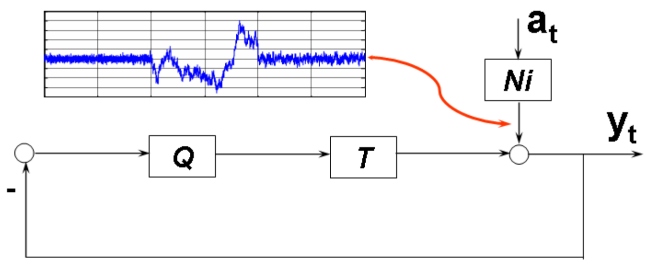

Figure 1.1: Illustration of a class of time-varying processes

to the temporary interference of operations by operators or a temporary grade change of raw materials. The change of disturbances is often reflected in the disturbance model parameters, which can be significantly different from its previous ones. Since LTI controllers are dominantly applied in industrial practice, they are often de-tuned in order to cope with such kind of time varying disturbances. Then, what is the best that an LTI controller can do in the presence of LTV disturbances? What is the achievable performance of an LTI controller that can be estimated from closed-loop routine operating data?

Huang (1999) and Olaleye *et al.* (2002, 2004*a*, 2004*b*) studied this kind of processes and proposed the Type-C benchmark to evaluate the controller performance. Both the process and controller are considered linear time invariant, and the disturbance models are piecewise constant in the parameters, where one of them may be the most representative and one is a major but transient upset. The Type-C benchmark (Huang, 1999) is characterized by a controller that minimizes the variance of the most representative section of the disturbances subject to some controller performance requirement in regulating the major transient disturbance. This is beneficial to improve the controller performance to the best in regulating the most representative disturbance while the performance of regulating the major transient upset is not sacrificed.

1.3 Output covariance upper bound problem

Most of current controller performance assessment algorithms are based on the MVC benchmark. It does provide an absolute theoretical lower bound of output variance against which real controllers can be compared (Harris, 1989). This variance lower bound is feedback control invariant due to the process time delay and could be achieved under MVC strategy. Unfortunately, MVC is seldom implemented in practice because of its lack of robustness to model uncertainty and use of excessive input actions (Astrom and Wittenmark, 1997). Therefore, the closed-loop response of a general regulatory feedback control loop can be divided into two parts, the first part is feedback control invariant and the second part is dependent on the feedback controller. This can be seen clearly from the closed-loop impulse response. For a linear process with time delay d under different feedback control strategies, its closed-loop impulse responses to the disturbance could appear as shown in Figure 1.2 and the magnitude of minimum variance is dependent on the process time delay d and output disturbance. The minimum variance results from the impulse response of the first part and becomes the lower bound of output variance. For

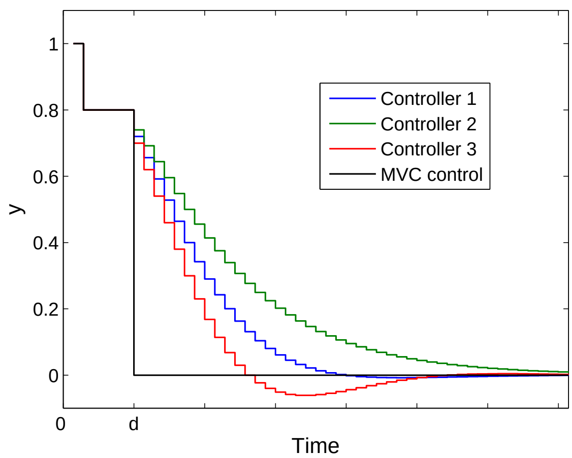

Figure 1.2: The closed-loop impulse responses under different feedback controllers

the same output disturbance, the additional output variance beyond the minimum variance comes from the impulse response of the second part which is affected directly by the feedback controller. This second part could vary for different feedback controllers which give rise to different output variances. It can be used to check whether the closed-loop response meets the specified desired specification, such as impulse response (Tyler and Morari, 1996), time constant or settling time (Kozub, 1996), closed-loop pole assignment (Horch and Isaksson, 1999), and so on, which is classified as the user specified benchmark (Huang and Shah, 1999). Since the output variance typically represents product quality consistency, if the output variance satisfies an upper bound constraint, the product quality should thus be guaranteed. In this case, what should the second part look like? This is referred to as the output variance upper bound problem and the resultant closed-loop response is defined as the structured closed-loop response (Xu and Huang, 2006). For MIMO systems, the output covariance is the direct extension and the corresponding problem is called the output covariance upper bound problem.

1.4 Performance monitoring of MPC applications

MPC applications have been widely used in the process industry. How to evaluate the performance of an MPC application and its cascaded regulatory control loops has become one of greatest interests in the process control community. The past research focused mainly on the comparison of the dynamic performance, such as the MVC benchmark, the design versus achieved benchmark (Patwardhan *et al.*, 2002) and the historical benchmark (Huang and Shah, 1999). The economic performance assessment of MPC applications has rarely been covered except for a few (Zhou and Forbes, 2003). However, most MPC applications are not in full capacity in practice due to lack of maintenance and too conservative constraint settings (Singh and Seto, 2002). Only 40% of the benefit is usually captured and the desired benefit is often not guaranteed. Therefore, a systematic and stardardized approach is demanded to monitor MPC applications and facilitate the task of MPC maintenance.

As a matter of fact, the latest generation of MPC technology itself includes a steady state economic optimization and a dynamic optimization, where the steady state optimization aims at searching for the optimal operating condition that can be driven by the dynamic optimization (Qin and Badgwell, 2003). Thus the steady state optimization is directly related to the economic benefit and the economic objective function becomes explicit. On the other hand, the back off approach is often utilized in the optimal process design stage with the consideration of possible disturbances and uncertainty parameters (Loeblein and Perkins, 1998). For the on-line MPC applications, this back off often implies benefit loss due to disturbances and conservative operations. The optimal back off should be the minimum move of the operating point away from the nominal one, which is usually located on the constraint limit, such that the feasibility and operability are not violated. Therefore, the optimal economic benefit could be achieved by pushing the quality variables toward their constraint limits. In addition, if the back off could be reduced or the constraint limits could be relaxed, further economic benefit could be obtained. Even though the disturbances are never avoidable, it is possible to reduce the variability by tuning the cascaded regulatory control loops. This variability analysis would bridge the MVC benchmark and the economic benefit, which result in variance and economic performance respectively. With these ideas in mind, variability and constraint would be the two keys on the performance monitoring of MPC applications.

1.5 Linear matrix inequalities

The linear matrix inequality (LMI) technique is employed to solve most of the problems proposed in this thesis. We only give a brief discussion here.

A general form of LMI is given as

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i \succ 0 \tag{1.7}$$

where $x \in \mathbf{R}^m$ is the unknown decision variables and $F_i \in \mathbf{R}^{n \times n}$ for $i = 0, 1, \cdots, m$ are known symmetric matrices. "$\succ$" means that $F(x)$ is a positive definite matrix, i.e.,

$$z^T F(x) z > 0, \forall z \neq 0, z \in \mathbf{R}^n \tag{1.8}$$

The LMI (1.7) is equivalent to n polynomial inequalities, which means its principal minors are positive. An important property of LMIs is that the set $\{x | F(x) \succ 0\}$ is convex, that is, the LMI (1.7) forms a convex constraint on x. The convexity of LMIs plays a crucial role in optimization since a convex function has a global optimum over a convex set. The *Schur complement lemma* is often used to convert a class of convex nonlinear inequalities into the equivalent LMI.

Lemma 1.5.1 (*Schur complement lemma*) *The following two statements are equivalent:*

- $R(x) \succ 0$, $Q(x) - S(x)R(x)^{-1}S(x)^T \succ 0$.

- $\begin{pmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{pmatrix} \succ 0.$

A lot of control problems can be transformed into LMIs, such as linear constraints, stability of linear systems, the quadratic objective function (Boyd *et al.*, 1994; VanAntwerp and Braatz, 2000). The advantage of LMI technique is that LMIs can be solved numerically very efficiently using ellipsoid algorithm or interior-point methods, which converge in polynomial time. Some software toolboxes are actually available, such as Matlab LMI toolbox (Gahinet *et al.*, 1995), SeDuMi (Sturm, 1998-2001) and SDPT3 (Tütüncü *et al.*, 2001), just to name a few.

1.6 Outline of the thesis

With the variety of methods available, this thesis aims at addressing some of the relevant issues that have not been resolved in the literature. Whereas some of the results are extensions and generalizations of the available results, some new concepts are introduced.

This thesis can be broadly divided into the following three parts:

1. Performance assessment of control loops with LTV disturbances (Chapter 2 and 3);
2. Covariance analysis approach for controller performance assessment (Chapter 4)
3. Performance assessment of MPC applications (Chapter 5 and 6)

A block diagram of the thesis is shown in Figure 1.3. The controller performance in regulating the process subject to disturbances is studied by developing different approaches in different chapters. Each individual chapter of this thesis is self-contained: starting with introduction, problem formulation or derivation, and then presenting the proposed solution or algorithms, followed by case studies and conclusions.

Chapter 2 is concerned with performance assessment of univariate control loops subject to linear time varying disturbance dynamics. The problem is motivated by the observation that most industrial controllers are linear time invariant (LTI) but the process, particularly the disturbance dynamics, is time varying. The time varying behavior of disturbance dynamics is modelled by piecewise constant parameters of linear disturbance models, namely linear time varying (LTV) dynamics. Thus, during a period of process operation, the process may be affected by several disturbances in terms of different disturbance dynamics or models. This problem has been previously solved using the standard Type-C benchmark, by minimizing the variance of a most representative disturbance while satisfying a structured regulatory performance requirement for one of other disturbances, typically the transient but most significant disturbance. This leaves performance in regulating the remaining disturbances unspecified. In this chapter, this problem is

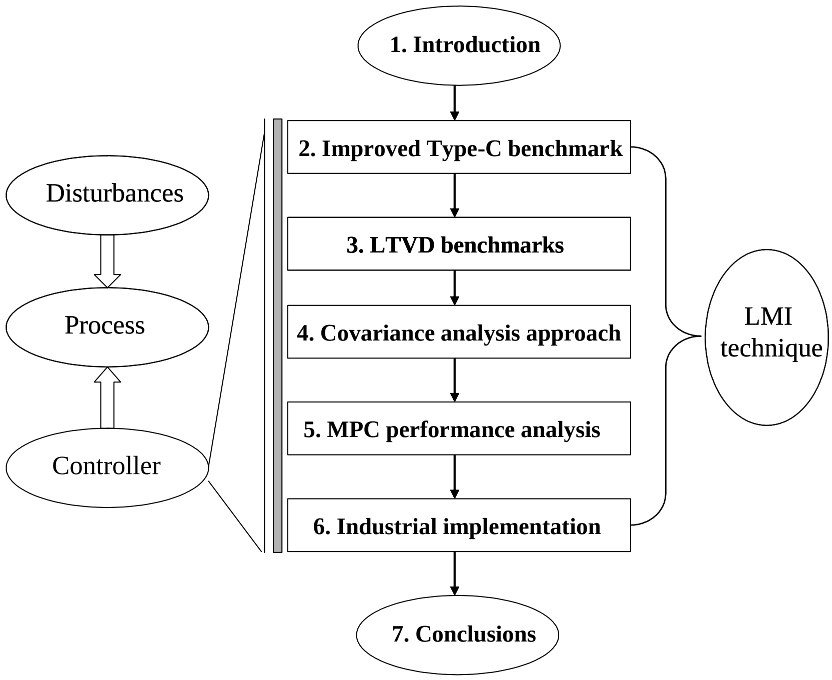

Figure 1.3: Block diagram of thesis

formulated as minimization of the sum of the weighted variances of all but one major disturbance that is considered under the structured regulatory performance requirement. It is solved from the following two perspectives: 1) Models of LTV disturbances are given, the limit of the achievable structured closed-loop performance of any LTI controller for the LTV disturbances is calculated, and the optimal LTI control law is derived if the process model is also known; 2) no complete models about the process or the disturbances are available except for the time delay of the process, an algorithm is developed to assess the performance of the existing LTI controller in the presence of LTV disturbances. This leads to an improved Type-C benchmark and a better trade-off can be obtained in regulating different disturbances in the sense of output variance.

In Chapter 3, the standard and improved Type-C benchmarks are extended into MIMO systems. Similar result is obtained as that of SISO systems. In this chapter, the standard Type-C benchmark is redefined as the regular linear time varying disturbances (LTVD) benchmark and the improved Type-C benchmark as the weighted LTVD benchmark. The regular LTVD benchmark aims at minimizing the total output variance subject to a most representative disturbance and the weighted LTVD benchmark is to minimize the sum of weighted total output variances subject to different disturbances. Another LTVD benchmark, the generalized LTVD benchmark, is also proposed to minimize the maximum total variance subject to different disturbances. These three LTVD benchmarks are compared and the result shows that the weighted and generalized LTVD benchmarks can always lead to better trade-offs on the total output variances under different disturbances than the regular one.

In Chapter 4, a covariance analysis approach is proposed to monitor the control loop performance. Owing to the process time delays, the closed-loop response can be divided into feedback control invariant part and feedback controller dependent part. If the latter part is replaced by a user specified response trajectory, the resultant closed-loop response is referred to as structured closed-loop response. The user specified structured closed-loop response has been used as an achievable control against which one can assess performance of control loops. In the control performance monitoring literature, the user specified response is often given as a first-order transfer function with some specified performance requirement, such as time constant. In this chapter, this problem is solved from a systematic approach, i.e., in viewpoint of a variance/covariance upper bound on the outputs. With available closed-loop routine operating data and process time delay/interactor matrix, the desired structured closed-loop response can be obtained directly via an estimated closed-loop time series model. A significant feature is that the output variance/covariance upper bound constraint can be explicitly specified according to the product specifications and is always satisfied when the problem is feasible. This desired structured closed-loop response can thus be served as a benchmark against which the existing controller performance can be compared. Two approaches, linearizing change of variables and Frank and Wolfe algorithm, are shown to be suitable for solving this problem, which result in full order and reduced order solutions respectively.

In Chapter 5, the performance of existing MPC applications is to be evaluted on the basis of the routine operating data. Even the multivariate controller performance assessment (MVPA) has been developed for several years, its application in advanced model predictive control (MPC) has been limited mainly due to issues associated with comparability of variance control objective and that of MPC applications. MPC has been proven as one of the most effective advanced process control (APC) strategies to deal with multivariable constrained control problems with an ultimate objective towards economic optimization. Any attempt to evaluate MPC performance should therefore consider constraints and economic performance. This chapter shows that the variance performance assessment may be transferred to performance assessment of MPC applications. A systematic approach is put forward to evaluate the economic performance of MPC applications, including economic performance assessment, sensitivity analysis and variance/constraint tuning guidelines. The MPC economic performance can be evaluated by solving benefit potentials through either variability reduction of quality output variables or tuning of constraints. The result shows the possible economic improvement potentials of existing MPC applications with/without variability/constraint tuning efforts. The sensitivity analysis gives rise to the impact of variability/constraint changes of different variables on the economic benefit potential. The result can be employed to identify the importance of different variables to the economic benefit potential and used as a reference in choosing tuning variables on either variability or constraint. The variability/constraint tuning guidelines tell which variables should be selected and how much should be tuned on either variability or constraint in order to achieve the desired target benefit potential.

In Chapter 6, an industrial APC performance monitoring framework is introduced with the implementation background. This framework is based on the developed software package which is named as Performance Analysis Technology and Solutions (PATS). The algorithms of two main components are summarized and adapted for suitability and convenience of industrial application. At the same time, a plant-oriented solution for APC performance monitoring is proposed within this framework, integrated by the components for real-time data collection, multivariate controller performance assessment (MVPA), APC economic performance assessment and tuning guidelines (LMIPA). It is illustrated by performance analysis of an industrial MPC application, which is used to control the reactor section of a gas oil hydrotreating unit (GOHTU).

This thesis has been written in the format in accordance with the rules and regulations of the Faculty of Graduate Studies and Research, University of Alberta. Some chapters have been published in journals and conference proceedings. In order to link the different chapters, there is some overlap and redundancy of material. This has been done to ensure completeness and cohesiveness of the thesis material and help the reader understand the material easily.

2

Performance Monitoring of SISO Systems with LTV Disturbances *

2.1 Introduction

The change of process disturbance dynamics in chemical processes is often observed, due to, for example, the temporary interference of operations by operators or a temporary grade change of raw materials. The change of disturbances is reflected in process model parameters, in particular, the disturbance model parameters, which can be significantly different from its previous ones. Even though the optimal strategy to handle this kind of time varying disturbances is to use parameter varying control or switching control (Sun and Ge, 2005), the controllers in industrial practice are, however, dominantly linear time invariant (LTI) controllers. To avoid the problem associated with such time varying disturbances, the LTI controller is often de-tuned or sometimes even suspended (if the abnormal disturbance is significant). This practice can compromise performance in regulating normal disturbances.

Motivated by these observations, one would naturally consider an optimal LTI control strategy to regulate time varying disturbances; in particular, we consider linear time varying (LTV) disturbances that can be modeled by a series of piecewise constant disturbance

*This chapter has been published in Xu, F. and Huang, B., Performance monitoring of SISO control loops subject to LTV disturbance dynamics: an improved LTI benchmark, Journal of Process Control, 16, 567-579, 2006 models. One would ask, what is the best that an LTI controller can do in the presence of LTV disturbances? This problem is formulated as the assessment of the limit of achievable performance. This limit of performance serves as a useful guideline for the design of an LTI controller and answers questions, such as, whether a switching or an adaptive controller is necessary. Another equally important problem is the performance assessment problem. In this case, a controller has been implemented in the process. One does not have complete knowledge about the process and disturbance models except for the process time delay as required by most conventional control performance assessment algorithms. One would like to estimate the achievable performance of the LTI controllers from the closed-loop routine operating data. Its importance has been well addressed by many authors (Huang and Shah, 1999; Desborough and Harris, 1992; Harris, 1989).

Based on the above rationale, Huang (1999) and Olaleye *et al*. (2002, 2004*a*, 2004*b*) proposed three optional benchmarks. They assumed that the process and controller are both linear time invariant, and the disturbance models are piecewise constant in the parameters subject to abrupt changes and thus a disturbance trajectory may be separated into several stationary sections, where one of them may be the most representative and one is a major but transient upset. The Type-C benchmark (Huang, 1999) is characterized by a controller that minimizes the output variance of the most representative section of the disturbances subject to some predefined regulatory performance on one of the remaining disturbances. That is to say: the abnormal disturbance or major upset, which is typically transient, should have been settled down along some user specified reference trajectory or funnel. This is beneficial because it ensures that a specified performance of the controlled variable is achieved in regulating transient but major upset (Qin and Badgwell, 1996). The desired closed-loop response to this section of the disturbance could be first order or higher order with a predefined regulatory requirement and it often leaves some degree of freedom that could be used to search for an LTI controller that minimizes the output variance of the most representative section of the disturbances (Huang, 1999). In general, there is a control performance requirement such as the settling time or time constant to regulate the abnormal disturbances of the process in industrial practice; for example, the closed-loop time constant is often tuned to be equal to the open-loop time constant.

In (Huang, 1999) and (Olaleye, 2002; Olaleye *et al.*, 2004*b*), the desired regulatory response to the major disturbance was assumed to take the form of a first order transfer function with only one free parameter to be determined by optimization. The Type-C benchmark was obtained via an *ad hoc* method or unconstrained nonlinear programming (NLP) technique, e.g., *Nelder-Simplex* method. But this leaves the performance of remaining disturbances unspecified in the previous work. The contributions of this chapter are in four aspects: 1) The problem is reformulated as a minimization of the sum of the weighted variances of all but one major disturbance that is subject to a structured regulatory performance requirement. 2) It is shown that when the decision variables are only present as the numerator coefficients in the transfer function of structured closed-loop response, the optimization problem can always be converted into linear matrix inequalities (LMIs) and thus can be solved efficiently. 3) The user specified structured closed-loop response is extended to more general first order and second order dynamics. 4) The performance limit problem together with the optimal LTI controller law is solved in addition to performance assessment problem. In consequence, a better trade-off on output variances in regulating different disturbances can be achieved in a more practical manner.

The remainder of this chapter is organized as follows. The SISO controller performance assessment is revisited in Section 2.2. In Section 2.3 an improved Type-C benchmark problem is formulated, together with the statement of performance limit and performance assessment problems. Section 2.4 describes the computation strategy for the improved Type-C benchmark problem via the LMI technique. The algorithms for performance limit problem and performance assessment problem are given in Section 2.5. Simulation and industrial examples are provided in Section 2.6, followed by concluding remarks in Section 2.7.

2.2 Revisit of SISO controller performance assessment

Harris (1989) has shown that minimum variance control can be used as the benchmark to measure an existing controller's performance by using closed-loop routine operating data alone. For a given time invariant process with time delay d, a moving average model can be used to fit the closed-loop output data y_t :

$$y_t = (\underbrace{f_0 + f_1 q^{-1} + ... + f_{d-1} q^{-(d-1)}}_{F(q^{-1})} + q^{-d} R_{cl}(q^{-1}))a_t \tag{2.1}$$

where a_t is a white noise sequence. Then the first part is feedback control invariant due to the process time delay d. Under minimum variance control, $R_{cl}(q^{-1})$ of the second part vanishes. Therefore, the lower bound of process output variance can be used as a benchmark and estimated from its closed-loop response:

$$y_t|_{mv} = (f_0 + f_1 q^{-1} + ... + f_{d-1} q^{-(d-1)})a_t = F(q^{-1})a_t \tag{2.2}$$

For a linear time varying process, Huang (1999) has proposed three optional benchmarks to do controller performance assessment, which were studied further by Olaleye *et al.* (2002, 2004*a*, 2004*b*) . They are referred to as Type-A, Type-B and Type-C benchmarks respectively. The Type-A benchmark is suitable to evaluate the performance of time varying controllers. It is assumed that both the process and disturbance are time varying, the closed-loop response under time varying minimum variance control can be written as

$$y_t|_{mv} = (f_0(t) + f_1(t)q^{-1} + ... + f_{d-1}(t)q^{-(d-1)})a_t = F(q^{-1}, t)a_t \tag{2.3}$$

which can be established on the basis of a moving window. The Type-B and Type-C benchmarks, however, only consider abrupt changes of disturbance dynamics, i.e., the parameters of the disturbance model are piecewise time invariant, and the process remains time invariant. It is further assumed that one of the disturbances is most representative affecting the process and the corresponding process output data is denoted as i-th section, the others are denoted as k-th sections respectively, where $k = 1, 2, \cdots$ and $k \neq i$. The Type-B benchmark uses minimum variance control of the most representative section of data as a benchmark to evaluate the controller performance over the entire time period. The benchmarking closed-loop response of the i-th section is exactly the same as (2.2), i.e.,

$$y_t^{(i)}|_{Type-B} = y_t^{(i)}|_{mv} = (f_0^{(i)} + f_1^{(i)}q^{-1} + ... + f_{d-1}^{(i)}q^{-(d-1)})a_t = F_i(q^{-1})a_t \tag{2.4}$$

By applying this benchmark to the k-th section, its closed-loop response is derived as

$$y_t^{(k)}|_{Type-B} = \frac{N_k(q^{-1})}{N_i(q^{-1})}F_i(q^{-1})a_t \tag{2.5}$$

where $N_i(q^{-1})$ and $N_k(q^{-1})$ are referred to the i-th and k-th disturbance models respectively. Note that minimum variance control of one portion of data may not be suitable to other sections of data when disturbance dynamics changes significantly (Huang, 1999). If this happens, the Type-C benchmark is suggested and the j-th disturbance is considered as the major transient disturbance. The Type-C benchmark is defined to minimize the output variance of the most representative section of the disturbances subject to some predefined specifications in regulating other sections of disturbances. For the j-th section of data, its benchmarking closed-loop response is formulated as

$$y_t^{(j)}|_{Type-C} = (F_j(q^{-1}) + q^{-d}G_R(q^{-1}))a_t \tag{2.6}$$

where $G_R(q^{-1})$ satisfies some predefined specifications but leaves some unknown parameters to be determined. For the i-th section of data,

$$y_t^{(i)}|_{Type-C} = \frac{N_i(q^{-1})}{N_j(q^{-1})}(F_j(q^{-1}) + q^{-d}G_R(q^{-1}))a_t \tag{2.7}$$

By minimizing its output variance, the unknown parameters of $G_R(q^{-1})$ can be solved and the Type-C benchmark is established. For the remaining sections, e.g., k-th section of data, its benchmarking closed-loop response can be simply calculated from

$$y_t^{(k)}|_{Type-C} = \frac{N_k(q^{-1})}{N_j(q^{-1})}(F_j(q^{-1}) + q^{-d}G_R(q^{-1}))a_t \tag{2.8}$$

The challenge in solving Type-C benchmark lies in the fact that an optimal LTI control should be solved for LTV disturbances, and most problems remain open in the literature.

2.3 Formulation of the improved Type-C benchmark

2.3.1 Preliminary

The derivation in this section has the following assumptions: (1) The plant is linear time invariant with minimum phase. (2) The disturbance model is piecewise constant linear time varying. (3) The controller is time invariant.

Consider a SISO process $q^{-d}\tilde{T}(q^{-1})$ shown in Figure 2.1 subject to several different disturbance dynamics, $N_k(q^{-1}), k = 1, 2, \cdots$, in different enough long time durations respectively, a_t is a sequence of white noise with zero mean, d is the process time delay, and $\tilde{T}(q^{-1})$ is the time delay free part of the process transfer function with minimum

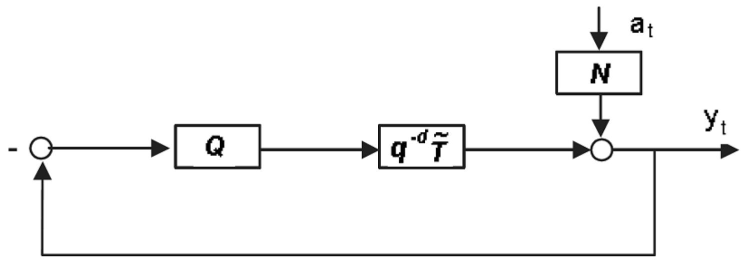

Figure 2.1: General feedback control framework

phase. The process is controlled by an LTI controller $Q(q^{-1})$. The change of disturbance dynamics takes place at time $t = \theta$. Without loss of generality, it is assumed that the i-th disturbance dynamics is the most representative of the disturbances that are affecting the process while the j-th section of the disturbances corresponds to the significant but transient upset affecting the process. For the Type-C benchmark, it is required that the closed-loop response to the j-th section of the disturbances settles down along some user specified trajectory. Thus, the closed-loop response to the j-th section of the disturbances can be written in a general form (Huang, 1999):

$$y_t^{(j)} = (\underbrace{f_0^{(j)} + f_1^{(j)} q^{-1} + ... + f_{d-1}^{(j)} q^{-(d-1)}}_{F_j(q^{-1})} + q^{-d} G_R(q^{-1})) a_t \tag{2.9}$$

where $G_R(q^{-1})$ is a stable and proper transfer function that is specified by the user and is feedback control dependent. The performance specification, such as settling time specification alone, can typically leave some free parameters that can be used to find a controller that optimizes the regulatory performance of the i-th section of the disturbances. A simple choice of $G_R(q^{-1})$, for example, is suggested as

$$G_R(q^{-1}) = \frac{\alpha}{1 - \lambda q^{-1}} \qquad \lambda = exp(-\frac{\Delta T}{\tau}) \tag{2.10}$$

where α is the free parameter to be determined and λ is specified according to the desired closed-loop time constant τ with sampling period ΔT (Kozub, 1996). In this chapter, we define the response of equation (2.9) as a structured closed-loop response, meaning that $G_R(q^{-1})$ is chosen by the user according to specified dynamic structure, first order in this instance. This leaves α as a free parameter. Another choice of $G_R(q^{-1})$ can take, for example, the form of a lead-lag system. Here we consider more general expressions for $G_R(q^{-1})$, i.e., the first order and second order transfer functions. The rationale is that most process dynamic behavior may be approximated by either first order or second order models. In particular an under-damped dynamic response must be represented by at least second order dynamics, justifying the necessity of exploring the second or high order structure. We will consider the following two more common forms of $G_R(q^{-1})$ in the sequel.

$$G_R(q^{-1}) = \frac{\alpha + \beta q^{-1}}{1 - \lambda q^{-1}} \tag{2.11}$$

$$G_R(q^{-1}) = \frac{\alpha_0 + \alpha_1 q^{-1} + \alpha_2 q^{-2}}{1 + \lambda_1 q^{-1} + \lambda_2 q^{-2}} \tag{2.12}$$

where

$$\lambda_1 = \begin{cases} -e^{-\frac{\xi \Delta T}{\tau}} (e^{\frac{\sqrt{\xi^2-1}\Delta T}{\tau}} + e^{\frac{-\sqrt{\xi^2-1}\Delta T}{\tau}}) & \text{when} \quad \xi \geq 1, \\ -2e^{-\frac{\xi \Delta T}{\tau}} \cos(\frac{\sqrt{1-\xi^2}\Delta T}{\tau}) & \text{when} \quad 0 < \xi < 1. \end{cases}$$

$$\lambda_2 = e^{-\frac{2\xi \Delta T}{\tau}}$$

and ξ is the damping coefficient, τ is the natural period or the inverse natural frequency (Ogunnaike and Ray, 1994). For the second order system, ξ and τ can be determined according to the desired characteristic of the under-damped response, such as rise time, overshoot, decay ratio, settling time, and so on.

According to Huang (1999), for a minimum phase process, the output of the major disturbance, $y_t^{(j)}$, in equation (2.9) is an achievable process response, and the output of the most representative disturbance, $y_t^{(i)}$, can be formulated as

$$y_t^{(i)} = \frac{N_i(q^{-1})}{N_j(q^{-1})}(F_j(q^{-1}) + q^{-d}G_R(q^{-1}))a_t \tag{2.13}$$

The corresponding controller $Q^*(q^{-1})$ can be solved as

$$Q^*(q^{-1}) = \frac{R_j(q^{-1}) - G_R(q^{-1})}{\tilde{T}(q^{-1})(F_j(q^{-1}) + q^{-d}G_R(q^{-1}))} \tag{2.14}$$

where $F_j(q^{-1})$ and $R_j(q^{-1})$ can be obtained from the Diophantine equation, $N_j(q^{-1}) = F_j(q^{-1}) + q^{-d}R_j(q^{-1})$. It can be shown this controller is stable and proper as far as $G_R(q^{-1})$ has stable first order dynamics and $\tilde{T}(q^{-1})$ is minimum phase (Olaleye *et al.*, 2004*b*).

For other disturbances (other than i-th and j-th) affecting the process in different time durations, their closed-loop responses can always be expressed as the same form as equation (2.13) with $i = k, k \neq j$.

The optimization problem of the Type-C benchmark (Olaleye *et al.*, 2004*b*; Huang, 1999) only considers two different disturbance dynamics, one is the most representative and the other one is a major but transient upset. Their outputs are referred to as $y_t^{(i)}$ and $y_t^{(j)}$, respectively. It is obvious that the best result is to gain a trade-off between these two different disturbances. If there exist more than two different disturbances, the suitability of the Type-C benchmark is not guaranteed.

The improved Type-C benchmark proposed in this chapter considers all disturbances simultaneously. It is also assumed that there exists one major but transient upset disturbance which should be regulated to satisfy a user specified trajectory, i.e., a structured closed-loop response. This requirement forms the constraint for the benchmark optimization problem. The objective function is chosen to minimize the sum of the weighted output variances of all remaining disturbances, which is referred to as total variance. The importance in regulating different disturbances can be obtained through a trade-off by assigning different weighting values to the different disturbances in the objective function. Although this problem is not a multivariable control problem but an LTV control problem, it will be shown that it is solvable via an optimal multivariable control formulation.

The improved Type-C benchmark problem can be summarized as follows: the process with time delay d is assumed to be subject to piecewise constant disturbance dynamics.

It is further assumed that the j-th section of the disturbances is a major but transient upset, and it should be regulated to satisfy some user specified performance requirement, through the specification of $G_R(q^{-1})$. In general, not all parameters are determined by this performance specification. The optimal values of these free parameters are then obtained by minimizing the total variance, i.e., the sum of the weighted output variances of all sections of disturbances other than the j-th section of the disturbances. If there are totally n sections of different disturbances affecting the process, this problem can be formulated as

$$G_R(q^{-1}) = \min_{\mathbf{x}} \left\{ \sum_{i=1, i \neq j}^{n} \rho_i^2 \cdot Var\left\{ y_t^{(i)} \right\} \right\} \tag{2.15}$$

where the decision variable **x** includes all unknown free parameters of $G_R(q^{-1})$, ρ_i^2 represents the weighting coefficient for the i-th section of the disturbances, and $y_t^{(i)}$ is given in equation (2.13). It is obvious that if there is only one weighting coefficient that is assigned a nonzero value, then the improved Type-C benchmark optimization problem reduces to the standard Type-C benchmark (Huang, 1999). Therefore, the standard Type-C benchmark is a special case of the improved Type-C benchmark.

Remark 2.3.1 *How does the proposed benchmark compare with conventional ones including moving window type of algorithms? What will happen if the proposed algorithm is not used in the presence of time varying disturbance? Let's consider several distinct disturbances affecting a process. By applying the conventional performance benchmarking, we would compare the existing control performance with a benchmark control that can minimize output variance of all disturbances with different dynamics. This benchmark control must therefore be time varying and is not consistent with most practical controls that are time invariant, and will tend to underestimate existing control performance. Then how to interpret the performance if two indices are different for two different disturbance sections, say the first section is large and the second is small? Our answer is: if these two indices are calculated from the proposed algorithm, then one can make a recommendation that the second section has the possibility to reduce its output variance to the percentage as indicated by the index by simply tuning time invariant control. However, if they are calculated from the conventional algorithm, then one can only recommend that the second section has the possibility to reduce its output variance to the percentage as indicated by the index if the existing time invariant control is replaced by time varying control.*

2.3.2 Performance limit problem

The output expression of $y_t^{(i)}$ has been derived subject to the user specified structured closed-loop response in the j-th section of the disturbances, as is shown in equation (2.13). In the case that the disturbance models are all known, the unknown free parameters of $G_R(q^{-1})$ can be calculated via the following improved Type-C benchmark optimization problem:

$$\begin{aligned} G_R(q^{-1}) &= \min_{\mathbf{x}} \left\{ \sum_{i=1, i\neq j}^{n} \rho_i^2 \cdot Var\left\{ \frac{N_i(q^{-1})}{N_j(q^{-1})}(F_j(q^{-1}) + q^{-d}G_R(q^{-1}))a_t \right\} \right\} \\ &= \min_{\mathbf{x}} \sum_{i=1, i\neq j}^{n} \rho_i^2 \cdot \| \frac{N_i(q^{-1})}{N_j(q^{-1})}(F_j(q^{-1}) + q^{-d}G_R(q^{-1})) \|_2^2 \end{aligned} \tag{2.16}$$

where $N_j(q^{-1}) = F_j(q^{-1}) + q^{-d}R_j(q^{-1})$, $\mathbf{x}$ is the decision variable which includes all the unknown free parameters of $G_R(q^{-1})$, and the corresponding optimal LTI control law can be calculated from equation (2.14).

2.3.3 Performance assessment problem

In industrial implementation, control loop performance assessment has to be done using routine operating data. By time series analysis, the closed-loop transfer function from the white noise to the process output can be estimated directly from routine operating data. Once the benchmark control response is known, the control loop performance assessment problem is readily solved by comparing the benchmark control response with the existing process output and identifying the opportunity to improve.

For a SISO process that is affected by different disturbance dynamics, the process outputs of the i-th and j-th sections can be expressed as

$$y_t^{(i)} = G_{cl}^{(i)}(q^{-1})a_t \quad \text{and} \quad y_t^{(j)} = G_{cl}^{(j)}(q^{-1})a_t$$

where $G_{cl}^{(i)}$ and $G_{cl}^{(j)}$ can be directly estimated from routine operating data. The output of the i-th section can thus be formulated as (Huang, 1999; Olaleye *et al.*, 2004*b*)

$$y_t^{(i)} = \frac{G_{cl}^{(i)}(q^{-1})}{G_{cl}^{(j)}(q^{-1})}(\hat{F}_j(q^{-1}) + q^{-d}G_R(q^{-1}))a_t \tag{2.17}$$

where $\hat{F}_j(q^{-1})$ can be obtained from Diophantine equation of $G_{cl}^{(j)}$ and $G_R(q^{-1})$ is specified by the user.

For the performance assessment problem based on the improved Type-C benchmark, the purpose is to minimize the sum of the weighted variances of all sections of the outputs with respect to different disturbances except for the j-th output, but subject to the performance specification on the j-th disturbance.

As we have shown, for the systems without knowing the process and disturbance models, the closed-loop transfer function from the white noise to the output can be directly estimated from routine operating data by time series analysis. The ratio of $N_i(q^{-1})$ and $N_j(q^{-1})$ is the same as that of $G_{cl}^{(i)}(q^{-1})$ and $G_{cl}^{(j)}(q^{-1})$, the closed-loop transfer functions from the white noise to the output. In this case, the unknown free parameters of $G_R(q^{-1})$ can be calculated via the following improved Type-C benchmark optimization problem:

$$\begin{aligned} G_R(q^{-1}) &= \min_{\mathbf{x}} \left\{ \sum_{i=1, i\neq j}^{n} \rho_i^2 \cdot Var \left\{ \frac{G_{cl}^{(i)}(q^{-1})}{G_{cl}^{(j)}(q^{-1})} (\hat{F}_j(q^{-1} + q^{-d} G_R(q^{-1})) a_t \right\} \right\} \\ &= \min_{\mathbf{x}} \sum_{i=1, i\neq j}^{n} \rho_i^2 \cdot \| \frac{G_{cl}^{(i)}(q^{-1})}{G_{cl}^{(j)}(q^{-1})} (\hat{F}_j(q^{-1} + q^{-d} G_R(q^{-1})) \|_2^2 \end{aligned} \tag{2.18}$$

where $\hat{F}_j(q^{-1})$ can be obtained from Diophantine equation of $G_{cl}^{(j)}(q^{-1})$.

2.4 Computation of the improved Type-C benchmark

2.4.1 SIMO formulation of SISO LTV problem

To calculate the improved Type-C benchmark, we need to minimize the sum of the weighted variances of the process output in response to different disturbance dynamics subject to the specified performance for the j-th disturbance dynamics. Although this is a SISO LTI control design problem for LTV disturbances, we may solve the problem by stacking different closed-loop models together to form a single-input multi-output (SIMO) problem. Each element of this SIMO system consists of the same process model but different disturbance models. By Parseval's theorem (Ljung, 1999), each output variance can be related to the $\mathcal{H}_2$ norm of its corresponding system, and the variance of the SIMO system is the sum of variance of each subsystem. The above heuristic argument can be rigorously shown by the following theorem,

Theorem 2.4.1 *For a process under a single LTI control subject to n piecewise constant disturbance dynamics occurring at n different time periods with closed-loop models $G_i \in \mathcal{RH}_2$, $i = 1, \cdots, n$, if the stacked system $\begin{bmatrix} G_1 & G_2 & \cdots & G_n \end{bmatrix}^T \in \mathcal{RH}_2$, then the sum of* *the output variances over n different time periods is the same as the variance of the stacked system.*

$$\sum_{i=1}^{n} \|G_i\|_2^2 = \left\| \begin{array}{c} G_1 \\ \vdots \\ G_n \end{array} \right\|_2^2$$

Proof: Assume that, without loss of generality, the closed-loop system is excited by the white noise a_t with zero mean and unit variance. The outputs of different time periods with respect to different piecewise constant disturbance dynamics can be written as

$$y_t^{(i)} = G_i(q^{-1})a_t, \quad i = 1, 2, \cdots, n$$

In the case of $E(a_t)^2 = \sigma^2 \neq 1$, $G_i(q^{-1})$ can be normalized by multiplying a factor of $1/\sigma$ so that the white noise has unit variance. Using Parseval's Theorem (Ljung, 1999; Söderström and Stoica, 1989), the sum of variances over n different time periods can be formulated as

$$\begin{aligned}
\sum_{i=1}^{n} Var(y_t^{(i)}) &= \sum_{i=1}^{n} E([G_i(q^{-1})a_t][G_i(q^{-1})a_t]^T) \\
&= \sum_{i=1}^{n} \frac{1}{2\pi} \int_{-\pi}^{\pi} |G_i(e^{j\omega})|^2 \Phi_{a_t}(\omega) d\omega \\
&= \sum_{i=1}^{n} \frac{1}{2\pi} \int_{-\pi}^{\pi} |G_i(e^{j\omega})|^2 d\omega \\
&= \sum_{i=1}^{n} \|G_i\|_2^2
\end{aligned}$$

where Φ represents spectrum of signal (Ljung, 1999) and $\Phi_{a_t}(\omega)$ is constant 1.

For the stacked SIMO system, the output can be written as

$$Y_t = \begin{bmatrix} y_t^{(1)} \\ \vdots \\ y_t^{(n)} \end{bmatrix} = \begin{bmatrix} G_1(q^{-1}) \\ \vdots \\ G_n(q^{-1}) \end{bmatrix} a_t = G(q^{-1})a_t$$

Then its output variance yields

$$
\begin{aligned}
trace[Cov(Y_t)] &= trace\left\{E([G(q^{-1})a_t][G(q^{-1})a_t]^T)\right\} \\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi} trace\left\{G(e^{j\omega})\Phi_{a_t}(\omega)G^*(e^{-j\omega})\right\} d\omega \\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi} trace\left\{G(e^{j\omega})G^*(e^{-j\omega})\right\} d\omega \\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi} \sum_{i=1}^{n} |G_i(e^{j\omega})|^2 d\omega \\
&= \sum_{i=1}^{n} \|G_i\|_2^2
\end{aligned}
$$

This gives $\sum_{i=1}^{n} Var(y_t^{(i)}) = trace[Cov(Y_t)]$. Since

$$
\frac{1}{2\pi}\int_{-\pi}^{\pi} trace\left\{G(e^{j\omega})G^*(e^{-j\omega})\right\} d\omega = \|G\|_2^2
$$

The above derivation also implies

$$
\sum_{i=1}^{n} \|G_i\|_2^2 = ||G||^2 = \left\| \begin{array}{c} G_1 \\ \vdots \\ G_n \end{array} \right\|_2^2
$$

□

Therefore, the objective function of the optimization problem of the improved Type-C benchmark can be formulated as the $\mathcal{H}_2$ norm of a stacked system with SIMO structure. We can deal with this stacked system in a similar manner as a single system. For example, if two SISO systems, G_1 and G_2, with their state space realizations as $\left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right]$ and $\left[\begin{array}{c|c} A_2 & B_2 \\ \hline C_2 & D_2 \end{array}\right]$ respectively, then the stacked system G becomes (Chen and Francis, 1995)

$$
G = \left[\begin{array}{c|c} \mathrm{A} & \mathrm{B} \\ \hline \mathrm{C} & \mathrm{D} \end{array}\right] = \left[\begin{array}{cc|c} A_1 & 0 & B_1 \\ 0 & A_2 & B_2 \\ \hline C_1 & 0 & D_1 \\ 0 & C_2 & D_2 \end{array}\right] \tag{2.19}
$$

In this way, we can convert several SISO systems into a single stacked SIMO system in order to solve the optimization problem of the improved Type-C benchmark. In the case when the weighted coefficients are not the same, they could be integrated into their corresponding subsystems respectively before a SIMO system is stacked. Then the total variance of a SISO LTV system is equal to the variance of the corresponding single stacked SIMO system. This will transform the objective function of total variance into the variance of the single stacked SIMO system, and therefore make it convenient to calculate the objective function in the optimization.

2.4.2 Optimization formulation

As far as the improved Type-C benchmark is concerned, the optimization problem can always be converted into an $\mathcal{H}_2$ norm optimization problem of a system or a stacked system. The derived discrete-time transfer function or transfer function matrix, such as equation (2.16) and equation (2.18), can further be converted into the state space form as

$$\begin{aligned} \xi_{k+1} &= A\xi_k + Ba_k \\ \Psi_k &= C\xi_k + Da_k \end{aligned} \tag{2.20}$$

where A is asymptotically stable (its eigenvalues are all located within the unit circle). Then its $\mathcal{H}_2$ norm can be formulated as (Zhou *et al.*, 1996)

$$\|G\|_2^2 = trace\left\{D^T D + B^T W_o B\right\} = trace\left\{DD^T + CW_c C^T\right\} \tag{2.21}$$

where W_c and W_o are referred to as the controllability and observability gramians which are positive definite and satisfy

$$\begin{aligned} AW_cA^T - W_c + BB^T &= \mathbf{0} \\ A^T W_o A - W_o + C^T C &= \mathbf{0} \end{aligned} \tag{2.22}$$

Therefore the optimization problem can be finally converted into one of the following two forms,

$$\min_{\mathbf{x}} \left\{trace(DD^T + CW_cC^T)\right\} \tag{2.23}$$

subject to

$$\begin{aligned} &AW_cA^T - W_c + BB^T = \mathbf{0} \\ &W_c \succ \mathbf{0} \end{aligned}$$

where A is asymptotically stable and (A,B) is controllable. Or dually,

$$\min_{\mathbf{x}} \left\{trace(D^T D + B^T W_o B)\right\} \tag{2.24}$$

subject to

$$\begin{aligned} &A^T W_o A - W_o + C^T C = \mathbf{0} \\ &W_o \succ \mathbf{0} \end{aligned}$$

where A is asymptotically stable and (C,A) is observable.

2.4.3 Solution via LMI

The standard Type-C benchmark problem was studied by Huang (1999) and Olaleye *et al.* (2002,2004), they assumed that the desired regulatory response to the major disturbance takes the form of a first order transfer function with only one unknown free parameter to be determined by optimization. The Type-C benchmark was obtained via an *ad hoc* method or unconstrained nonlinear programming (NLP) technique, e.g., *Nelder-Simplex* method. In the following we will show that the optimization problem of the improved Type-C benchmark can be converted into LMIs under certain conditions, i.e., when the decision variables are only present as the numerator coefficients in the transfer function of the structured closed-loop response, and thus can be solved in a more efficient manner.

For the following transfer function (d_0 is constant),

$$G(z) = d_0 + \frac{\beta_1 z^{n-1} + \beta_2 z^{n-2} + \cdots + \beta_{n-1} z + \beta_n}{z^n + \alpha_1 z^{n-1} + \cdots + \alpha_{n-1} z + \alpha_n} \tag{2.25}$$

its observable canonical form of the state space realization is given as the following:

$$G(z) = \left[\begin{array}{c|c} \mathrm{A} & \mathrm{B} \\ \hline \mathrm{C} & \mathrm{D} \end{array}\right] = \left[\begin{array}{ccccc|c} -\alpha_1 & 1 & 0 & \cdots & 0 & \beta_1 \\ -\alpha_2 & 0 & 1 & \cdots & 0 & \beta_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ -\alpha_{n-1} & 0 & 0 & \cdots & 1 & \beta_{n-1} \\ -\alpha_n & 0 & 0 & \cdots & 0 & \beta_n \\ \hline 1 & 0 & 0 & \cdots & 0 & d_0 \end{array}\right] \tag{2.26}$$

For a SISO system with its objective transfer function in the form of equation (2.25), if the unknown free parameters present only in the coefficients of the numerator in the proper transfer function of $G_R(q^{-1})$, they will also appear only in the coefficients of the numerator in the corresponding closed-loop transfer function, such as equation (2.13) and equation (2.17). In addition, d_0, the first coefficient of the closed-loop transfer function from the white noise to the process output must be non-zero due to the time delay in the open-loop plant. Therefore, in its observable canonical form of state space realization, A, C and D are all constant matrices or scalar without containing any unknown free parameters. The elements of B, however, are linear combinations of the free parameters. If we formulate the optimization problem as (2.23), when (A, B) is controllable and all the eigenvalues of A are located within the unit circle, it is readily converted into LMIs and can be solved efficiently.

Similarly, for the improved Type-C benchmark optimization problem, each SISO system in the objective function can be readily converted into its observable canonical form of the state space realization. These SISO systems can then be stacked into a SIMO system, as is shown in equation (2.19). Therefore, if the unknown free parameters all present in the elements of B as their linear combinations for each SISO system, the above conclusion is applicable to the stacked SIMO system. That means, in the state space realization of the stacked SIMO system, A, C and D are all constant matrices without including any unknown free parameters and the elements of B are linear combinations of the unknown free parameters. Consequently the corresponding optimization problem for the stacked SIMO system can also be transformed into the formulation as (2.23) and ready to be solved.

To summarize, if the unknown free parameters only present as the coefficients of the numerator in the proper transfer function $G_R(q^{-1})$, the optimization problem of the improved Type-C benchmark can be transformed into the form as (2.23). When the stacked SIMO system is asymptotically stable and controllable, the optimization problem (2.23) can be finally solved by a semi-definite programming (SDP) problem (Boyd *et al.*, 1994). It is given by the following theorem,

Theorem 2.4.2 *For a discrete time LTI system (A, B, C, D) with linear combinations of unknown free parameters in B, its $\mathcal{H}_2$ norm optimization problem (2.23) can be converted into the following SDP problem as*

$$\min_{\mathbf{x}} \{trace(\Phi)\} \tag{2.27}$$

subject to

$$\begin{aligned} &\begin{pmatrix} A\Sigma A^T - \Sigma & B \\ B^T & -I \end{pmatrix} \preceq \boldsymbol{0} \\ &DD^T + C\Sigma C^T - \Phi \preceq \boldsymbol{0} \\ &-\Sigma \prec \boldsymbol{0} \end{aligned}$$

where $\mathbf{x}$ is the decision variable which includes all the unknown free parameters.

Proof: Refer to Sato and Liu (1999) or Wang *et al.* (2000).

□

In the following, we will show that the elements of B can be expressed as the linear combinations of the unknown free parameters by an example. If $G_R(q^{-1})$ takes the form of equation (2.10), the closed-loop transfer functions for the two sections in the objective function are derived as

$$G_1(z) = 1 + \frac{0.27z^4 - 0.27\lambda z^3 + (\alpha - 0.6)z + 0.6\lambda - \alpha}{z^5 - (0.67 + \lambda)z^4 + 0.67\lambda z^3}$$

and

$$G_3(z) = 1 + \frac{0.47z^4 - 0.47\lambda z^3 + (\alpha - 0.6)z + 0.6\lambda - \alpha}{z^5 - (0.87 + \lambda)z^4 + 0.87\lambda z^3}$$

Their observable canonical forms are given respectively by

$$G_1(z) = \left[\begin{array}{c|c} A_1 & B_1 \\ \hline C_1 & D_1 \end{array}\right] = \left[\begin{array}{ccccc|c} 0.67+\lambda & 1 & 0 & 0 & 0 & 0.27 \\ -0.67\lambda & 0 & 1 & 0 & 0 & -0.27\lambda \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \alpha - 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0.6\lambda - \alpha \\ \hline 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

and

$$G_3(z) = \left[\begin{array}{c|c} A_3 & B_3 \\ \hline C_3 & D_3 \end{array}\right] = \left[\begin{array}{ccccc|c} 0.87+\lambda & 1 & 0 & 0 & 0 & 0.47 \\ -0.87\lambda & 0 & 1 & 0 & 0 & -0.47\lambda \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \alpha - 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0.6\lambda - \alpha \\ \hline 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Let the two weighting coefficients both take the unit value, i.e., $\rho_1 = \rho_3 = 1$, and the stacked system can be expressed as the structure of equation (2.19). When λ is assigned a value, the stacked A, C and D are all constant matrices. The free parameter α only exists in the stacked matrix B. It can be further formulated as

$$B = \begin{bmatrix} B_1 \\ B_3 \end{bmatrix} = S_1 S S_2 + S_3$$

where

$$S_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}^T, \quad S = \alpha, \quad S_2 = 1$$
$$S_3 = \begin{bmatrix} 0.27 & -0.27\lambda & 0 & -0.6 & -0.6\lambda & 0.47 & -0.47\lambda & 0 & -0.6 & 0.6\lambda \end{bmatrix}^T$$

Since λ is specified a value *a priori*, the only free parameter α appears only in S. This is a suitable formulation that can be used directly by the Matlab LMI toolbox (Gahinet *et al.*, 1995).

Likewise, if the state space realization takes the controllable canonical form, then the corresponding optimization problem (2.24) can also be converted into an SDP problem. This is shown in the following theorem.

Corollary 2.4.1 *For a discrete time LTI system* (A, B, C, D) *with linear combinations of unknown free parameters in C, its* $\mathcal{H}_2$ *norm optimization problem (2.24) can be converted into the following SDP problem as*

$$\min_{\mathbf{x}} \{trace(\Phi)\} \tag{2.28}$$

subject to

$$\begin{aligned} &\begin{pmatrix} A^T\Sigma A - \Sigma & C^T \\ C & -I \end{pmatrix} \preceq \boldsymbol{0} \\ &D^T D + B^T \Sigma B - \Phi \preceq \boldsymbol{0} \\ &-\Sigma \prec \boldsymbol{0} \end{aligned}$$

*where **x** is the decision variable which includes all the unknown free parameters.*

2.5 Algorithms for performance limit and performance assessment problems

2.5.1 Algorithm for performance limit problem

The procedure for calculating the performance limit based on the improved Type-C benchmark can be summarized in the following algorithm, where σ^2_{opt} represents the achievable variance under the optimal feedback control.

Algorithm 2.5.1 *Given a process model with stable inverse, and the disturbance models of n different dynamics, the performance limit of the LTI control subject to piecewise constant disturbance dynamics can be calculated through the following steps:*

(1) *Denote the major but transient upset disturbance model as $N_j(q^{-1})$ and the other disturbance models as $N_i(q^{-1})$, $i = 1, 2, \cdots$. Determine the process time delay according to the process model. Specify the predefined response $G_R(q^{-1})$, such as a first order transfer function in equation (2.11) or a second order transfer function in equation (2.12). $F_j(q^{-1})$ can be obtained from Diophantine equation as $N_j(q^{-1}) = F_j(q^{-1}) + q^{-d}R_j(q^{-1})$.*

(2) *According to equation (2.16), minimize the $\mathcal{H}_2$ norm and obtain the optimal values of the unknown free parameters of $G_R(q^{-1})$. The $\mathcal{H}_2$ norm minimization of equation (2.16) can be formulated as $\mathcal{H}_2$ norm minimization of a multivariable system which has been shown in the previous section.*

(3) *With known $G_R(q^{-1})$, calculate the $\mathcal{H}_2$ norm of $\frac{N_i(q^{-1})}{N_j(q^{-1})}(F_j(q^{-1}) + q^{-d}G_R(q^{-1}))$ and denote it as $\sigma^2_{opt}(y_i)$, which is the performance limit in regulating the i-th section of the disturbances, for $i = 1, 2, \cdots, n$.*

(4) *With known $G_R(q^{-1})$ and the process model $T(q^{-1})$, the corresponding optimal controller can be obtained from equation (2.14).*

2.5.2 Algorithm for performance assessment problem

The controller performance index is defined as the following,

$$\hat{\eta} = \frac{\hat{\sigma}^2_{opt}}{\hat{\sigma}^2_{act}} \tag{2.29}$$

where $\hat{\sigma}^2_{opt}$ represents the estimated achievable variance at the optimum, and $\hat{\sigma}^2_{act}$ stands for the estimated actual variance. The procedure for performance assessment based on the improved Type-C benchmark can be summarized in the following algorithm.

Algorithm 2.5.2 *If a controller has already been implemented in the control loop, its performance assessment problem can be done using routine operating data via the following steps:*

(1) *Separate data into appropriate sections according to the disturbance dynamics. For off-line assessment problem, this can be done through the stationarity test of time series data (Shiavi, 1999). For on-line assessment, this can be done through detection of abrupt change algorithms as discussed in (Olaleye* et al.*, 2004*b*). Denote the major but transient upset section as the j-th section, which is desired to be controlled within certain time frame. Denote any one of the other sections as the i-th section, $i = 1, 2, \cdots, n, i \neq j$, and assign the weighting values for different sections. Specify the predefined response $G_R(q^{-1})$, such as a first order transfer function in equation (2.11) or a second order in equation (2.12).*

(2) *By time series analysis, estimate the closed-loop transfer functions for the i-th and j-th sections respectively, denote them as $G^{(i)}_{cl}(q^{-1})$ and $G^{(j)}_{cl}(q^{-1})$. Calculate $\hat{F}_j(q^{-1})$ from the following Diophantine equation,*

$$G^{(j)}_{cl}(q^{-1}) = \hat{F}_j(q^{-1}) + q^{-d}R_j(q^{-1})$$

(3) *According to equation (2.18), minimize the $\mathcal{H}_2$ norm and obtain the optimal values of the unknown free parameters of $G_R(q^{-1})$. With known $G_R(q^{-1})$, calculate the $\mathcal{H}_2$ norm of the transfer function in equation (2.17) and denote it as $\hat{\sigma}^2_{opt}(y_i)$ for the i-th section, $i = 1, 2, \cdots, n, i \neq j$. Calculate the $\mathcal{H}_2$ norm for the j-th data section from $\hat{F}_j(q^{-1}) + q^{-d}G_R(q^{-1})$, and denote it as $\hat{\sigma}^2_{opt}(y_j)$.*

(4) *Estimate the actual variances for the different sections from their corresponding routine operating data, denote them as $\hat{\sigma}^2_{act}(y_i)$, $i = 1, 2, \cdots, n, i \neq j$, and $\hat{\sigma}^2_{act}(y_j)$, respectively.*

(5) *Estimate the performance index* $\hat{\eta}(y_i)$ *from the ratio of* $\hat{\sigma}^2_{opt}(y_i)$ *and* $\hat{\sigma}^2_{act}(y_i)$, $i = 1, 2, \cdots, n, i \neq j$. *Estimate the performance index* $\hat{\eta}(y_j)$ *from the ratio of* $\hat{\sigma}^2_{opt}(y_j)$ *and* $\hat{\sigma}^2_{act}(y_j)$.

2.6 Simulation and industrial examples

2.6.1 Simulation example

A jacketed reactor (Cooper *et al.*, 2004) is considered to demonstrate the proposed algorithms, where the sampling time is chosen as 10 second. The reactor is a continuously stirred vessel in which an exothermic reaction occurs. The reactor exit stream temperature is controlled by manipulating a valve to adjust the cooling liquid flow rate. The disturbance variable of interest for this process is the temperature of cooling liquid entering the jacket.

The process transfer function is given by

$$q^{-d}\tilde{T} = q^{-6}\frac{-0.02633 - 0.009711q^{-1}}{1 - 0.8999q^{-1}}$$

The transfer functions of three disturbance models with respect to three different sections are assumed to be

$$\begin{aligned} N_1(q^{-1}) &= \frac{0.07899}{1 - 0.9169q^{-1}} & & 1 \leqslant t < 2001 \\ N_2(q^{-1}) &= \frac{0.06729}{1 - q^{-1}} & & 2001 \leqslant t < 4001 \\ N_3(q^{-1}) &= \frac{0.06185}{1 - 0.9535q^{-1}} & & 4001 \leqslant t \leqslant 6000 \end{aligned}$$

The second section is assumed as a major but transient upset section of the disturbances. It is required to make it satisfy a user specified structured closed-loop response, which may take any one of the following two forms,

$$G_R(q^{-1}) = \frac{\alpha + \beta q^{-1}}{1 - \lambda q^{-1}} \qquad or \qquad G_R(q^{-1}) = \frac{\alpha_0 + \alpha_1 q^{-1} + \alpha_2 q^{-2}}{1 + \lambda_1 q^{-1} + \lambda_2 q^{-2}}$$

where $\alpha, \beta, \alpha_0, \alpha_1$ and α_2 are the decision variables, and $\lambda, \lambda_1, \lambda_2$ can be specified a value or a bounded region. The true $F_2(q^{-1})$ can be obtained from Diophantine equation of $N_2(q^{-1})$, which is given below,

$$F_2(q^{-1}) = 0.0673 + 0.0673q^{-1} + 0.0673q^-2 + 0.0673q^{-3} + 0.0673q^{-4} + 0.0673q^{-5}$$

The first section is assumed to be the most representative one so that we can also evaluate the performance based on the standard Type-C benchmark in order to do comparisons with the improved Type-C benchmark. In addition, the true variances of the closed-loop transfer functions under current feedback control for three different disturbances can be calculated directly from their true models respectively. Meanwhile, the actual variances of the closed-loop systems for three different disturbances can be calculated theoretically.

2.5.1.1 Performance limit calculation

Case 1 Assume that the user specified structured closed-loop response of the second disturbance takes the first order transfer function, where the time constant τ is given as 40, 95, 180 and 270, respectively for the sake of comparison. The free parameters α and β should be determined such that the sum of the weighted variances of the first and third sections ($\rho_1 = \rho_3 = 1$) is minimized.

According to Algorithm 2.5.1 for the performance limit problem, the results are given in Table 2.1. It can be seen that with the increasing value of time constant, τ, which is

Table 2.1: Performance limit results with first order $G_R(q^{-1})$

τ	α^*	β^*	$\sigma^2_{opt}(y_1)$	$\sigma^2_{opt}(y_2)$	$\sigma^2_{opt}(y_3)$
40	0.0142	0.0089	0.0300	0.0284	0.0205
95	0.0161	0.0066	0.0289	0.0298	0.0202
180	0.0178	0.0047	0.0279	0.0319	0.0199
270	0.0188	0.0035	0.0274	0.0341	0.0196

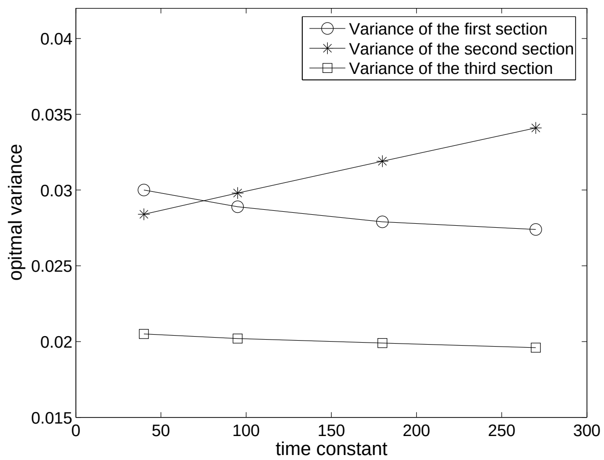

Figure 2.2: The trend of optimal variance vs user chosen τ value

specified for the desired structured closed-loop response of the second section, the optimal variances of the first and third sections are all decreasing gradually while the corresponding

Table 2.2: Optimal feedback controllers with respect to different τ values

$\tau(s)$	$Q^*(q^{-1})$
40	$\frac{-29.99+53.59q^{-1}-28.95q^{-2}+4.51q^{-3}}{1-0.41q^{-1}-0.29q^{-2}-0.79q^{-6}+0.41q^{-7}+0.13q^{-8}-0.05q^{-9}}$
95	$\frac{-28.89+54.84q^{-1}-29.7q^{-2}+3.38q^{-3}}{1-0.53q^{-1}-0.33q^{-2}-0.76q^{-6}+0.48q^{-7}+0.18q^{-8}-0.04q^{-9}}$
180	$\frac{-27.93+53.67q^{-1}-28.34q^{-2}+2.39q^{-3}}{1-0.58q^{-1}-0.35q^{-2}-0.74q^{-6}+0.48q^{-7}+0.21q^{-8}-0.03q^{-9}}$
270	$\frac{-27.35+52.58q^{-1}-27.15q^{-2}+1.79q^{-3}}{1-0.59q^{-1}-0.36q^{-2}-0.72q^{-6}+0.47q^{-7}+0.22q^{-8}-0.02q^{-9}}$

variance of the second section at the optimum is increasing (Figure 2.2). It is predictable that the variance of the second section will become larger with the relaxation of its specified value of time constant τ, while the variance of the first and third section is decreasing owing to the relaxation of the performance in the second section. In addition, the corresponding optimal feedback controllers can be derived according to equation (2.14). The obtained optimal feedback controllers are listed in Table 2.2.

Case 2 Assume that the user specified structured closed-loop response of the second section takes the second order transfer function, where $\lambda = [\lambda_1, \lambda_2]$ is determined by ξ and τ. Now τ is fixed and equals to 100, and ξ is given as 0.2155, 0.3441, 0.5912, 0.8261 and 1.0, respectively. The free parameters in α ($\alpha = [\alpha_0, \alpha_1, \alpha_2]$) should be determined such that the sum of the weighted variances of the first and third sections ($\rho_1 = \rho_3 = 1$) is minimized.

With given values of ξ and τ, $\lambda = [\lambda_1, \lambda_2]$ can be obtained according to equation (2.12). The results are given in Table 2.3. It can be seen that with the increasing value of the

Table 2.3: Performance limit results with second order $G_R(q^{-1})$

ξ	α^*	$\sigma^2_{opt}(y_1)$	$\sigma^2_{opt}(y_2)$	$\sigma^2_{opt}(y_3)$
0.2155	[0.0142, -0.0220, 0.0089]	0.0300	0.0287	0.0205
0.3441	[0.0152, -0.0214, 0.0078]	0.0294	0.0294	0.0204
0.5912	[0.0166, -0.0202, 0.0062]	0.0286	0.0307	0.0201
0.8261	[0.0175, -0.0192, 0.0052]	0.0281	0.0319	0.0199
1.0	[0.0180, -0.0185, 0.0046]	0.0278	0.0327	0.0198

damping coefficient ξ, the variances of the first and the third sections are both decreasing while the variance of the second section is increasing (Figure 2.3). It shows once again that the decreasing of the variances in the first and third sections is obtained at the cost of the variance inflation in the second section. Since the user specified response $G_R(q^{-1})$ presents

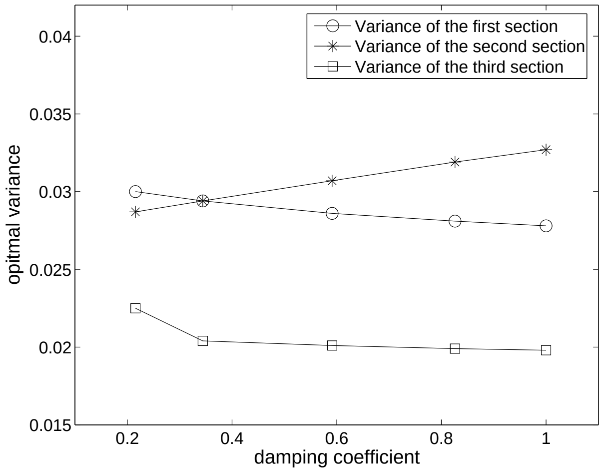

Figure 2.3: The trend of optimal variance vs user chosen damping coefficient

Table 2.4: Optimal feedback controllers with respect to different ξ values

ξ	$Q^*(q^{-1})$
0.2155	$\frac{-29.97+80.52q^{-1}-67.1q^{-2}+12q^{-3}+4.52q^{-4}}{1-1.58q^{-1}+0.24q^{-2}+0.35q^{-3}-0.79q^{-6}+1.12q^{-7}+0.02q^{-8}-0.32q^{-9}-0.05q^{-10}}$
0.3441	$\frac{-29.4+78.88q^{-1}-66.17q^{-2}+12.71q^{-3}+3.95q^{-4}}{1-1.56q^{-1}+0.22q^{-2}+0.34q^{-3}-0.77q^{-6}+1.1q^{-7}+0.01q^{-8}-0.3q^{-9}-0.43q^{-10}}$
0.5912	$\frac{-28.62+76.34q^{-1}-64.35q^{-2}+13.43q^{-3}+3.16q^{-4}}{1-1.51q^{-1}+0.2q^{-2}+0.33q^{-3}-0.75q^{-6}+1.05q^{-7}-0.004q^{-8}-0.28q^{-9}-0.03q^{-10}}$
0.8261	$\frac{-28.11+74.41q^{-1}-62.63q^{-2}+13.66q^{-3}+2.64q^{-4}}{1-1.47q^{-1}+0.17q^{-2}+0.31q^{-3}-0.74q^{-6}+1.02q^{-7}-0.008q^{-8}-0.26q^{-9}-0.03q^{-10}}$
1.0000	$\frac{-27.82+123.5q^{-1}-216.5q^{-2}+184.6q^{-3}-72.62q^{-4}+6.95q^{-5}+1.91q^{-6}}{1-3.25q^{-1}+3.58q^{-2}-1.15q^{-3}-0.42q^{-4}+0.25q^{-5}-0.73q^{-6}+2.32q^{-7}-2.41q^{-8}}$ $\overline{+0.59q^{-9}+0.41q^{-10}-0.15q^{-11}-0.02^{-12}}$

under-damped dynamic characteristics, the variance of the second section is increasing rapidly with the reduced damping coefficient while the variance decreasing in the first and third sections is rather slow. Similarly, the obtained optimal feedback controllers are listed in Table 2.4.

2.5.1.2 Performance assessment calculation

As is shown in the previous two cases, the performance limit problem is solved when the process and disturbance models are all known and the corresponding optimal controller can also be obtained thereafter based on the improved Type-C benchmark. On the other hand, for a given process under regulatory control, the performance of the existing controller can be evaluated based on the improved Type-C benchmark. In the following two cases, a well-designed controller has been implemented to control this process, which is given by

$$Q(q^{-1}) = \frac{-2.7 + 2.419q^{-1}}{1 - q^{-1}}$$

Our task is to evaluate its performance using routine operating data. Performance assessment problem will be solved for the structured closed-loop responses with first order and second order transfer functions respectively.

Case 3 For the same specification as Case 1 in Section 2.5.1.1, study the corresponding performance assessment problem.

The performance assessment problem is different from the performance limit problem in that we have no knowledge about the process model other than the time delay. Time series modeling has to be done first before performance estimation. According to Algorithm 2.5.2 for the performance assessment problem based on the improved Type-C benchmark, we obtain the results shown in Table 2.5. Compared with the results of the performance limit problem in Table 2.1, the overall estimated results resemble their theoretical ones well with certain error due to the estimation (Figure 2.4) and the error can be reduced by increasing the sampling size of the routine operating data. For all three sections, the trends of variance change with respect to increasing of τ value are almost the same as their corresponding ones of the performance limit problem. In addition, performance indices of three sections are also shown in Figure 2.5. We can see that with larger time constant specified for the structured closed-loop response of the second section, a better trade-off can be gained among three sections with closer performance indices in this case.

Table 2.5: Performance assessment results with first order $G_R(q^{-1})$

τ	$\hat{\alpha}^*$	$\hat{\beta}^*$	$\hat{\sigma}^2_{opt}(y_1)$	$\hat{\sigma}^2_{opt}(y_2)$	$\hat{\sigma}^2_{opt}(y_3)$	$\hat{\eta}(y_1)$	$\hat{\eta}(y_2)$	$\hat{\eta}(y_3)$
40	0.0101	0.0068	0.0346	0.0266	0.0217	0.7822	0.6962	0.7708
95	0.0113	0.0047	0.0339	0.0272	0.0215	0.7679	0.7131	0.7619
180	0.0122	0.0028	0.0333	0.0280	0.0214	0.7536	0.7343	0.7594
270	0.0127	0.0013	0.0329	0.0286	0.0215	0.7451	0.7501	0.7626

Case 4 For the same specification as Case 2 in Section 2.5.1.1, study the corresponding performance assessment problem.

According to Algorithm 2.5.2 for the performance assessment problem based on the improved Type-C benchmark, we get the results shown in Table 2.6 and Figure 2.6. Compared with the corresponding results in Table 2.3 and Figure 2.3, similar conclusions can be drawn. The performance indices of all three sections are also given in Figure 2.7. In this case, a better trade-off is gained among three sections with closer performance indices

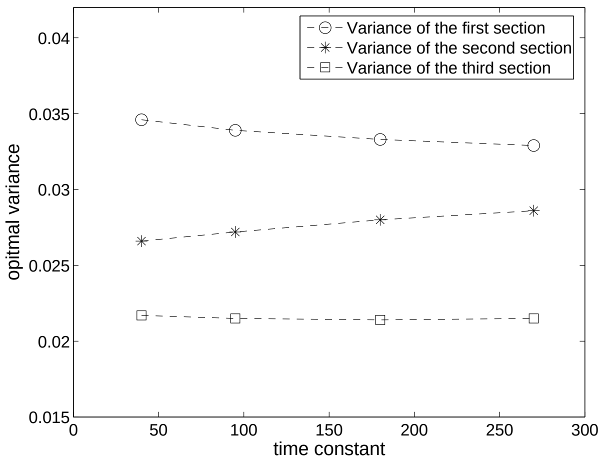

Figure 2.4: The trend of optimal variance vs user chosen τ value

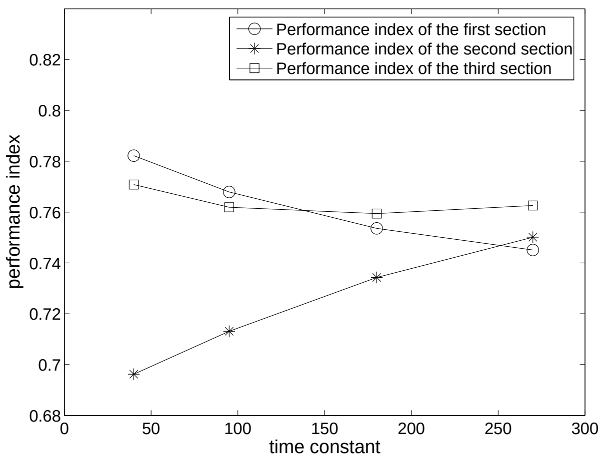

Figure 2.5: The trend of performance index vs user chosen τ value

when $\xi = 1.0$.

2.6.2 Industrial example

The improved Type-C benchmark is used to evaluate the control loop performance in a Sulphur Recovery Unit (SRU), which was studied by Olaleye *et al.* (2004*b*) for the standard Type-C benchmark problem. In this example, a proportional-integral-derivative (PID) controller is applied to control the difference, $2SO_2 - H_2S$, by manipulating the flowrate of the trim air. This data set inludes 740 data points with three data sections (Figure 4.6) and the time delay is 2 sampling units.

According to Algorithm 2.5.2 for the performance assessment problem, the improved Type-C benchmark is applied by minimizing the sum of variances of the first and third

Table 2.6: Performance assessment results with second order $G_R(q^{-1})$

ξ	$\hat{\alpha}^*$	$\hat{\sigma}^2_{opt}(y_1)$	$\hat{\sigma}^2_{opt}(y_2)$	$\hat{\sigma}^2_{opt}(y_3)$	$\hat{\eta}(y_1)$	$\hat{\eta}(y_2)$	$\hat{\eta}(y_3)$
0.2155	[0.0108, -0.0171, 0.0071]	0.0345	0.0269	0.0215	0.7800	0.7047	0.7628
0.3441	[0.0117, -0.0171, 0.0067]	0.0341	0.0272	0.0214	0.7722	0.7137	0.7585
0.5912	[0.0121, -0.0155, 0.0053]	0.0337	0.0277	0.0213	0.7618	0.7267	0.7565
0.8261	[0.0125, -0.0145, 0.0043]	0.0334	0.0281	0.0214	0.7551	0.7359	0.7575
1.0	[0.0126, -0.0136, 0.0036]	0.0332	0.0283	0.0214	0.7513	0.7414	0.7590

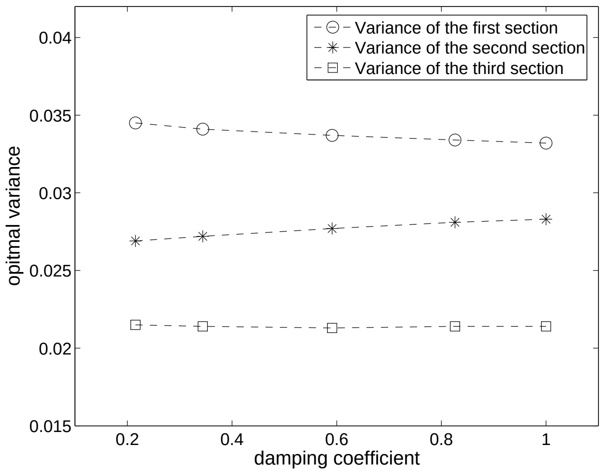

Figure 2.6: The trend of optimal variance vs user chosen damping coefficient

sections subject to some user specified performance requirement in regulating the major disturbance occurring in the second section of data set. In this example, the time constant of the desired close-loop system is taken as 1.0 min when regulating the major disturbance occurring at the second section. ARMA (auto-regressive moving average) models of second order are used to fit the time series of all three data sections. When the first order transfer function of time constant 1 min is taken as the user specified structured closed-loop response for the second section, we have the result shown in Table 2.7. Compared with the result in (Olaleye *et al.*, 2004*b*), we can see that the optimal variances of the first and third sections are all decreased to some degree while there is an increase of variance on the second section satisfying the time constant requirement. This is in part due to the improved benchmark with the objective to minimize the weighted sum of the output variances of the first and third sections, and also due to the selection of a lead-lag first order system with one more degree of freedom for the user specified closed-loop response on the second section. Therefore, a better trade-off is gained while meeting the time constant constraint.

When the second order transfer function is applied for the structured closed-loop response on the second section, the result is given in Table 2.8. The natural period and

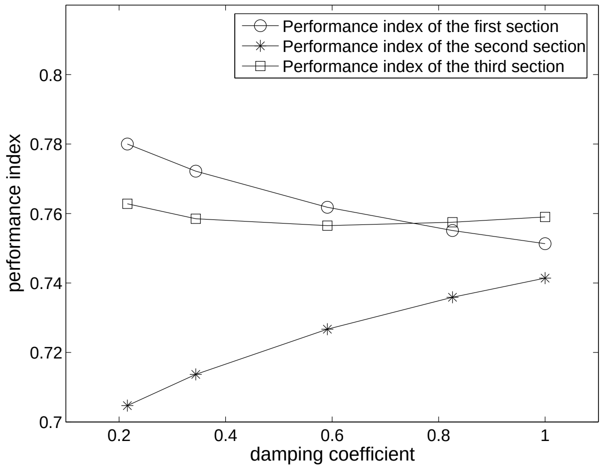

Figure 2.7: The trend of performance index vs user chosen damping coefficient

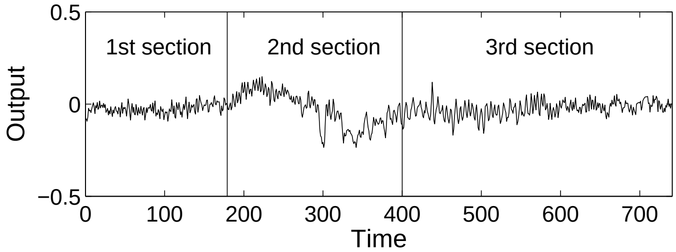

Figure 2.8: Time series plot of process output

damping coefficient are selected as $\tau = 0.5705$ and $\xi = 0.7071$ such that the structured closed-loop response on the second section has the same responding speed as the open-loop system without any oscillation. Since there are three decision variables in this case, it can be seen that the performance on the three sections can be further balanced. Especially when the structured closed-loop response is required to have some kind of oscillations, the second order transfer function is no doubt a better choice to gain not only a good trade-off among different disturbances but also a good satisfaction for the user specified requirements.

In this example, the performance index of the second section is much smaller than the first and third sections in both situations. The results are consistent with what we observed from the actual output variances. Since the performance of the second section is poor while the first and third sections have good performance, it is a reminder for the user to take actions to tune the existing LTI controller in order to improve the performance of the second section and lower down the performance of other two sections such that a better trade-off could be obtained.

It is noted that, the values of the performance indices may be greater than 1, which

Table 2.7: Performance assessment results with first order $G_R(q^{-1})$

$\tau = 1.0$	$\lambda = 0.3679$	$\hat{\alpha}^* = 0.6341$	$\hat{\beta}^* = 0.0801$
Data section	$\hat{\sigma}^2_{opt}$	$\hat{\sigma}^2_{act}$	$\hat{\eta}$
1st	1.4389	1.3160	1.0934
2nd	3.1660	11.7948	0.2684
3rd	1.9255	2.1273	0.9052

Table 2.8: Performance assessment results with second order $G_R(q^{-1})$

$\tau = 0.5705$	$\xi = 0.7071$	$\hat{\alpha}^* =$	$[0.3264, 0.1801, 0.1255]$
Data section	$\hat{\sigma}^2_{opt}$	$\hat{\sigma}^2_{act}$	$\hat{\eta}$
1st	1.4524	1.3160	1.1037
2nd	3.4931	11.7948	0.2962
3rd	1.9108	2.1273	0.8982

indicates that the existing control is better than the benchmark control with respective to a specific section of the disturbances. Note that the optimal (benchmark) control is LTI and is optimal for the specified objective function. For the section with the major disturbance, if the performance index is larger than 1, it can be regarded as an indication that shows the user specified response is actually relaxed with respect to its current response in the sense of variance. In addition, the trade-off can also be further balanced by adjusting different weighting values.

2.7 Conclusions

This chapter has considered performance analysis problems for a SISO process that is subject to time varying disturbance dynamics. In particular, we consider the disturbance dynamics that can be represented as piecewise constant models. An improved LTI benchmark is proposed to calculate the performance limit and do the performance assessment. It is shown that the problems can be formulated as an optimal SIMO control problem and solved via LMI technique. The solutions are given under both performance limit and performance assessment framework. The simulation and industrial examples demonstrate the suitability of the improved Type-C benchmark which always leads to a better trade-off in regulating different disturbances in the sense of process output variance.

3

Performance Assessment of MIMO Systems with LTV Disturbances

3.1 Introduction

In this chapter, we extend the linear time varying disturbance assessment (LVTD) problem from SISO into MIMO processes. Following a similar structure as the last chapter, we first solve the regular LTVD problem; two other problems are also solved for MIMO systems, which are referred to as the weighted LTVD problem and generalized LTVD problem, respectively. For the weighted LTVD problem, we formulate the problem as the minimization of the sum of the weighted total variances (trace of covariance matrix) of all but one major disturbance that is considered under the structured regulatory performance requirement. For the generalized LTVD problem, the objective is to minimize the maximum total variance of all but one major disturbance. The performance limit problem is solved in addition to performance assessment problem.

The remainder of this chapter is organized as follows. Multivariate controller performance assessment is revisited in Section 3.2. The LTVD problems for MIMO processes are derived in Section 3.3, and they are referred to as the regular, weighted and generalized LTVD benchmarks, respectively. The solutions to these LTVD benchmark problems are given in Section 3.4. Simulation examples are provided in Section 3.5, followed by concluding remarks in Section 3.6.

3.2 Revisit of multivariate controller performance assessment

Several authors have proposed solutions for the performance evaluation of multivariate controllers based on the MVC benchmark (Harris *et al.*, 1996; Huang *et al.*, 1997*a*; Ko and Edgar, 2001*b*). The difficulty in multivariate controller performance assessment is the factorization of the time delay matrix, which is known as the interactor matrix. In the following, the interactor matrix is introduced first, and then the FCOR algorithm for multivariate controller performance assessment is summarized thereafter.

3.2.1 Interactor matrix

If a multivariate process is represented by

$$y_t = T(q^{-1})u_t + N(q^{-1})a_t \tag{3.1}$$

where $T(q^{-1})$ is a $p \times m$ proper, rational polynomial transfer function matrix, $N(q^{-1})$ is a disturbance model, y_t, u_t and a_t are output, input and white noise vectors of appropriate dimensions. Then there exists a unique, non-singular, $p \times p$ lower left triangular polynomial matrix D, such that $|D| = q^r$ and

$$\lim_{q^{-1}\to 0} DT = \lim_{q^{-1}\to 0} \tilde{T} = K_o \tag{3.2}$$

where K_o is a full rank (full column rank or full row rank) constant matrix, the integer r is defined as the number of infinite zeros of T, and $\tilde{T}$ is the time delay free transfer function (factor) matrix of T which contains only finite zeros. The matrix D is defined as the interactor matrix and can be written as

$$D = D_0 q^d + D_1 q^{d-1} + D_2 q^{d-2} + \cdots + D_{d-1} q \tag{3.3}$$

where d is denoted as the order of the interactor matrix and is unique for a given transfer function matrix, and D_is (for $i = 0, 1, \cdots, d-1$) are coefficient matrices (Huang *et al.*, 1997*b*).

Instead of taking the lower triangular form, if the interactor matrix satisfies

$$D^T(q^{-1})D(q) = I \tag{3.4}$$

then this interactor matrix is defined as the unitary interactor matrix. The existence of the unitary interactor matrix has been established by (Peng and Kinnaert, 1992). Huang and Shah (1999) have found that a unitary interactor matrix is an optimal factorization of time delays for multivariate systems in terms of minimum variance control and controller performance assessment. This unitary interactor is an all-pass term, and factorization of such a unitary interactor matrix does not change the spectral property of the underlying system. It has been shown that multiplying a unitary interactor matrix to the output variables does not change the quadratic measure or the variance of the corresponding output variables, i.e.,

$$E[\tilde{y}_t{}^T \tilde{y}_t] = E[y_t^T y_t] \tag{3.5}$$

where $\tilde{y}_t = q^{-d} D y_t$, which can be called as the interactor filtered output. Therefore, in terms of the quadratic measure of the variance, the overall performance assessment of y_t is the same as that of interactor filtered output $\tilde{y}_t$.

The interactor matrix is an equivalent form of the time delay in multivariate systems. It needs process model (or at least the first few Markov Matrices) to capture the delay terms. The interactor matrix can be factored out by using QR factorization (Rogozinski *et al.*, 1987; Peng and Kinnaert, 1992), or SVD (Singular Value Decomposition) (Huang *et al.*, 1997*b*), or direct definition (Lu, 2005).

3.2.2 FCOR algorithm

A key to performance assessment of multivariate processes using MVC as a benchmark, is to estimate the benchmark variance from routine operating data with *a priori* knowledge of interactor matrices. This feedback control invariant term (minimum variance) can be estimated from routine operating data by multivariate time series analysis (Huang *et al.*, 1997*b*). The overall and individual performance indices can be calculated by multivariate FCOR (filtering and correlation analysis) algorithm (Huang and Shah, 1999).

With unitary interactor matrix D, the process output in (3.1) can be expressed as

$$y_t = T(q^{-1})u_t + N(q^{-1})a_t = D^{-1}\tilde{T}u_t + Na_t \tag{3.6}$$

Since $q^{-d}DN = F + q^{-d}R$, the interactor filtered output can be written as

$$\tilde{y}_t = q^{-d}Dy_t = q^{-d}\tilde{T}u_t + q^{-d}DNa_t = Fa_t + q^{-d}(\tilde{T}u_t + Ra_t) \tag{3.7}$$

and then the closed-loop response of the filtered output under minimum variance control can be obtained as

$$\tilde{y}_t|_{mv} = Fa_t = (F_0 + F_1 q^{-1} + \cdots + F_{d-1}q^{-d+1})a_t \tag{3.8}$$

and $y_t|_{mv}$ can be solved as

$$y_t|_{mv} = q^d D^{-1}(F_0 + F_1 q^{-1} + \cdots + F_{d-1}q^{-d+1})a_t \tag{3.9}$$

For the unitary interactor matrix, we have $D^{-1}(q) = D^T(q^{-1})$,i.e.,

$$D^{-1} = (D_0 q^d + \cdots + D_{d-1}q)^{-1} = D_0^T q^{-d} + \cdots + D_{d-1}^T q^{-1} \tag{3.10}$$

Therefore

$$\begin{aligned} y_t|_{mv} &= (D_0^T + \cdots + D_{d-1}^T q^{d-1})(F_0 + F_1 q^{-1} + \cdots + F_{d-1}q^{-d+1})a_t \\ &\triangleq (E_0 + E_1 q^{-1} + \cdots + E_{d-1}q^{-d+1})a_t \end{aligned} \tag{3.11}$$

where

$$(E_0, E_1, \cdots, E_{d-1}) \triangleq (D_0^T, D_1^T, \cdots, D_{d-1}^T) \begin{pmatrix} F_0 & F_1 & \cdots & F_{d-1} \\ F_1 & F_2 & \cdots & \\ \vdots & \vdots & & \\ \vdots & F_{d-1} & & \\ F_{d-1} & & & \end{pmatrix} \tag{3.12}$$

If $E(a_t a_t^T) = \Sigma_a$, then we have

$$E(y_t y_t^T)|_{mv} = trace(E_0 \Sigma_a E_0^T + E_1 \Sigma_a E_1^T + \cdots + E_{d-1}\Sigma_a E_{d-1}^T) \tag{3.13}$$

The overall and individual performance indices can thus be calculated respectively as the following,

$$\eta_o = \frac{trace(E(y_t y_t^T)|_{mv})}{trace(E(y_t y_t^T))} = \frac{trace(E_0 \Sigma_a E_0^T + E_1 \Sigma_a E_1^T + \cdots + E_{d-1}\Sigma_a E_{d-1}^T)}{trace(E(y_t y_t^T))} \tag{3.14}$$

and

$$\eta_i = \frac{diag(E(y_t y_t^T)|_{mv})}{diag(E(y_t y_t^T))} = \frac{diag(E_0 \Sigma_a E_0^T + E_1 \Sigma_a E_1^T + \cdots + E_{d-1}\Sigma_a E_{d-1}^T)}{diag(E(y_t y_t^T))} \tag{3.15}$$

An important assumption of these existing results with minimum variance control as the benchmark is that the disturbance dynamics is linear time invariant. However, practical experiences indicate that the disturbance dynamics are often time varying. In the following sections, we will consider a class of time varying disturbance dynamics and then present our solutions.

3.3 The LTVD benchmarks for MIMO processes

To start, we reiterate the following assumptions: (1) The plant is linear time invariant with no unstable zeros. (2) The disturbance model is piecewise linear time varying. (3) The controller is time invariant. The derivations in this chapter are based on the internal model control (IMC) framework that is mathematically equivalent to the conventional feedback control.

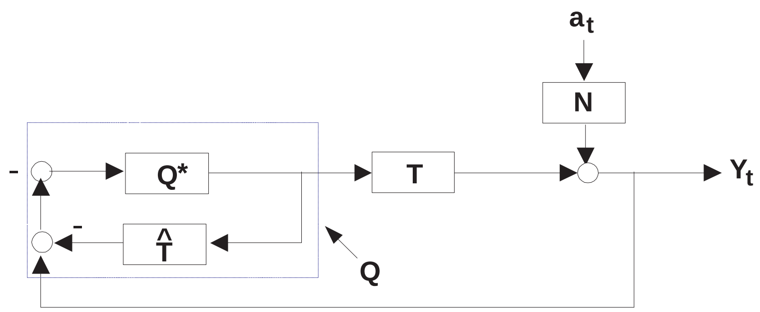

Figure 3.1: Control loop configuration under IMC framework

Consider a MIMO process $T(q^{-1})$ shown in Figure 3.1 subject to piecewise constant disturbance dynamics, $N_i(q^{-1})$ and $N_j(q^{-1})$, in different time durations respectively. a_t is white noise sequences with zero mean. $T(q^{-1})$ is assumed minimum phase which can be factorized as two different parts, $T(q^{-1}) = D^{-1}\tilde{T}(q^{-1})$, where $\tilde{T}(q^{-1})$ is the time delay free part of the process model and D is the unitary interactor matrix, which is the generalized time delay for MIMO processes (Huang and Shah, 1999). $\hat{T}(q^{-1})$ is the internal process model. The process is controlled by a controller $Q^*(q^{-1})$ specified under the IMC framework (Morari and Zafiriou, 1989), which can be converted into a conventional feedback controller $Q(q^{-1})$ as

$$Q(q^{-1}) = Q^*(q^{-1})(I - T(q^{-1})Q^*(q^{-1}))$$

We have assumed that the disturbance dynamics is piecewise linear time varying. This type of time varying disturbances are reflected as different output trajectories in different time sections, each section having a linear time invariant disturbance model. It is assumed that the i-th disturbance dynamics is the most representative of the disturbances that are affecting the process while the j-th section of the disturbances corresponds to the significant but transient upset within the process. It is required that the closed-loop response to the j-th section of disturbance is to be settled down along some user specified trajectory.

The process is also affected by some other disturbances in addition to the two specified above. Since different disturbances affect the process at different time periods, we denote the closed-loop response to the k-th disturbance as the k-th section response or simply call it as the k-th section for simplicity, where $k = 1, 2, \cdots, n$. For the regular LTVD benchmark, $i \neq j, j \neq k, i \neq k$, and the k-th section is only used to verify the suitability of the obtained benchmark. For the weighted LTVD benchmark and the generalized LTVD benchmark, we only differentiate the j-th section from other sections, i.e., k will include the value of i. For simplicity the operator q^{-1} is omitted in the following derivations. In addition, we call the trace of the covariance matrix as total variance, which is equal to the square of $\mathcal{H}_2$ norm of the transfer matrix when the input is white noise with identity covariance matrix.

3.3.1 Performance limit problem

For this problem, the true models, T, N and Q^* are all known. The output can be expressed as the following:

$$\begin{aligned} y_t &= (I - D^{-1}\tilde{T}Q^*)Na_t \\ &= q^d D^{-1}(q^{-d}DN - q^{-d}\tilde{T}Q^*N)a_t \\ &= q^d D^{-1}(F + q^{-d}R - q^{-d}\tilde{T}Q^*N)a_t \\ &= q^d D^{-1}(F + q^{-d}(R - \tilde{T}Q^*N))a_t \end{aligned}$$

where d is denoted as the order of the unitary interactor matrix D (Huang and Shah, 1999). The last term of the right side of the above equation, $D^{-1}(R - \tilde{T}Q^*N)a_t$, depends on the controller Q^* and thus can be shaped by it (Huang and Shah, 1999). The first term, $q^d D^{-1}F$, does not depend on the control and is also known as feedback control invariant, a term representing minimum variance control output.

For the j-th data section, it is required to be settled down according to some user specified response G_R, thus

$$y_t^{(j)} = q^d D^{-1}(F_j + q^{-d}G_R)a_t = (I - D^{-1}\tilde{T}Q^*)N_j a_t \tag{3.16}$$

A simple and practical choice of G_R elements can take any one of the following two forms,

$$\frac{\alpha_{ij}}{1 - \lambda_{ij}q^{-1}} \qquad \text{or} \qquad \frac{\alpha_{ij} + \beta_{ij}q^{-1}}{1 - \lambda_{ij}q^{-1}} \tag{3.17}$$

where α_{ij} and β_{ij} are unknown free parameters to be determined and λ_{ij} represents the response dynamics that can be specified by the user.

If the user specified closed-loop response for the j-th section is satisfied under the controller Q^*, then the output of any one of the other sections, i.e., k-th section, can be derived as follows,

$$\begin{aligned} y_t^{(k)} &= (I - D^{-1}\tilde{T}Q^*)N_k a_t \\ &= q^d D^{-1}(F_j + q^{-d}G_R)N_j^{-1}N_k a_t \end{aligned} \tag{3.18}$$

where

$$q^{-d}DN_j = F_j + q^{-d}R_j \tag{3.19}$$

Then the output of the i-th section is given by

$$y_t^{(i)} = q^d D^{-1}(F_j + q^{-d}G_R)N_j^{-1}N_i a_t \tag{3.20}$$

In this situation, the optimal controller Q^* can be obtained as

$$Q^* = \tilde{T}^{-1}(R_j - G_R)N_j^{-1} \tag{3.21}$$

It is ready to obtain the corresponding optimal controller Q under conventional feedback framework (see Figure 3.1) as

$$\begin{aligned} Q &= Q^*(I - D^{-1}\tilde{T}Q^*)^{-1} \\ &= \tilde{T}^{-1}(R_j - G_R)(F_j + q^{-d}G_R)^{-1}q^{-d}D \end{aligned} \tag{3.22}$$

The objective of the regular LTVD benchmark is to determine the unknown free parameters of the transfer function matrix G_R such that the total variance of the output due to the i-th disturbance is minimized, i.e., the $\mathcal{H}_2$ norm of the i-th closed-loop transfer function matrix as in equation (3.20) is minimized. For the weighted LTVD benchmark, the cost function is the sum of the weighted total variances of different sections of the outputs except for that of the j-th output, i.e., the sum of the weighted $\mathcal{H}_2$ norms of transfer function matrices of all the n sections other than that of the j-th section, as shown in equation (3.18). For the generalized LTVD benchmark, the objective is to minimize the maximum $\mathcal{H}_2$ norm of the transfer function matrices of all the other $n-1$ sections without including the j-th section.

3.3.2 Performance assessment problem

For the performance assessment problem, we have neither the model of the process nor the models of the disturbances. What we have is only the unitary interactor matrix D, which can be obtained from the first few Markov matrices of the process under study (Huang and Shah, 1999; Huang *et al.*, 1997*b*). By time series analysis, we can obtain the closed-loop transfer function matrices $G_{cl}^{(j)}$ and $G_{cl}^{(k)}$ under feedback control, i.e.,

$$\begin{aligned} y_t^{(k)} &= G_{cl}^{(k)}a_t = (I + D^{-1}\tilde{T}Q)^{-1}N_k a_t \\ y_t^{(j)} &= G_{cl}^{(j)}a_t = (I + D^{-1}\tilde{T}Q)^{-1}N_j a_t \end{aligned}$$

Therefore, the following equation holds,

$$G_{cl}^{(j)-1} G_{cl}^{(k)} = N_j^{-1} N_k$$

If a filter, $q^{-d}D$, is applied to the process output, it separates the filtered output into the feedback control independent part and the feedback control dependent part via Diophantine equation. For the interactor filtered output of the j-th data section, its closed-loop response has to meet some user specified requirement as G_R; therefore we have

$$q^{-d} D y_t^{(j)} = (F_j + q^{-d} G_R) a_t = q^{-d} D (I + D^{-1} \tilde{T} Q)^{-1} N_j a_t$$

For the interactor filtered output of the k-th data section, we have

$$\begin{aligned} q^{-d} D y_t^{(k)} &= q^{-d} D (I + D^{-1} \tilde{T} Q)^{-1} N_k a_t \\ &= (F_j + q^{-d} G_R) N_j^{-1} N_k a_t \\ &= (F_j + q^{-d} G_R) G_{cl}^{(j)-1} G_{cl}^{(k)} a_t \end{aligned}$$

Therefore, the output of the i-th, j-th and k-th sections can be written respectively as

$$y_t^{(i)} = q^d D^{-1} (F_j + q^{-d} G_R) G_{cl}^{(j)-1} G_{cl}^{(i)} a_t \tag{3.23}$$

$$y_t^{(j)} = q^d D^{-1} (F_j + q^{-d} G_R) a_t \tag{3.24}$$

$$y_t^{(k)} = q^d D^{-1} (F_j + q^{-d} G_R) G_{cl}^{(j)-1} G_{cl}^{(k)} a_t \tag{3.25}$$

and F_j can be obtained from the Diophantine equation as

$$q^{-d} D G_{cl}^{(j)} = F_j + q^{-d} R_j \tag{3.26}$$

In order to achieve the user specified response for the j-th section shown in equation (3.24), the regular LTVD benchmark aims at searching through the unknown free parameters of G_R such that the $\mathcal{H}_2$ norm of the transfer function matrix of the i-th section as in equation (3.23) is minimized. Meanwhile, the objective of the weighted LTVD benchmark is to minimize the sum of the weighted $\mathcal{H}_2$ norms of the transfer function matrices of all sections other than that of the j-th section, and the aim of the generalized LTVD benchmark is to minimize the maximum $\mathcal{H}_2$ norm of the transfer function matrices of all sections except for that of the j-th section, as shown in equation (3.25).

3.4 Solutions to the LTVD benchmarks

For the regular LTVD benchmark problem, all the unknown free parameters of G_R, which are the decision variables in **x**, can be obtained by solving an $\mathcal{H}_2$ optimization problem. It can be expressed as

$$G_R = \min_{\mathbf{x}} \quad \|q^d D^{-1}(F_j + q^{-d} G_R) N_j^{-1} N_i\|_2^2 \tag{3.27}$$

for the performance limit problem and

$$G_R = \min_{\mathbf{x}} \quad \|q^d D^{-1}(F_j + q^{-d} G_R) G_{cl}^{(j)-1} G_{cl}^{(i)}\|_2^2 \tag{3.28}$$

for the performance assessment problem.

For the weighted LTVD benchmark, if there are totally n different piecewise constant disturbances, it can be formulated by

$$G_R = \min_{\mathbf{x}} \sum_{k=1, k\neq j}^{n} \rho_k^2 \cdot \|q^d D^{-1}(F_j + q^{-d} G_R) N_j^{-1} N_k\|_2^2 \tag{3.29}$$

for the performance limit problem and

$$G_R = \min_{\mathbf{x}} \sum_{k=1, k\neq j}^{n} \rho_k^2 \cdot \|q^d D^{-1}(F_j + q^{-d} G_R) G_{cl}^{(j)-1} G_{cl}^{(k)}\|_2^2 \tag{3.30}$$

for the performance assessment problem, where ρ_k^2 represents the weighting coefficient with respect to the k-th disturbance.

For the generalized LTVD benchmark, it can be formulated as

$$G_R = \min_{\mathbf{x}} \max_{k=1,\cdots,n, k\neq j} \|q^d D^{-1}(F_j + q^{-d} G_R) N_j^{-1} N_k\|_2^2 \tag{3.31}$$

for the performance limit problem and

$$G_R = \min_{\mathbf{x}} \max_{k=1,\cdots,n, k\neq j} \|q^d D^{-1}(F_j + q^{-d} G_R) G_{cl}^{(j)-1} G_{cl}^{(k)}\|_2^2 \tag{3.32}$$

for the performance assessment problem.

The regular LTVD benchmark problem can be directly solved by optimizing the $\mathcal{H}_2$ norm of a system. For the weighted LTVD benchmark, however, the objective is the sum of the weighted $\mathcal{H}_2$ norm of the transfer function matrices of different sections. Each section has its own transfer function matrix which can be imagined as a system. In the objective function, we have $n - 1$ different systems which are related to the corresponding $n-1$ sections respectively. Those different systems can be integrated with the corresponding weighting values into themselves and can be arranged into a new expanded multi-input multi-output (MIMO) structure with inputs unchanged and outputs expanded. As a consequence, the weighted LTVD benchmark can be dealt with as the $\mathcal{H}_2$ norm optimization problem of an expanded system. If $G_k = q^d D^{-1}(F_j + q^{-d}G_R)N_j^{-1}N_k$ or $G_k = q^d D^{-1}(F_j + q^{-d}G_R)G_{cl}^{(j)-1}G_{cl}^{(k)}$, then we have

$$\sum_{k=1,k\neq i}^{n} \|\rho_k G_k\|_2^2 = \left\| \begin{array}{c} \rho_1 G_1 \\ \vdots \\ \rho_n G_n \end{array} \right\|_2^2$$

For the generalized LTVD benchmark, it is actually a min-max $\mathcal{H}_2$ norm optimization problem.

Since any system has its state space respresentation

$$\begin{aligned} \xi_{k+1} &= A\xi_k + Ba_k \\ \Psi_k &= C\xi_k + Da_k \end{aligned} \tag{3.33}$$

the above optimization problems from equation (3.27) to equation (3.32) can all be converted and solved via the following optimization problem (Zhou *et al.*, 1996),

$$\min_{\mathbf{x}} \left\{ trace(DD^T + CW_cC^T) \right\} \tag{3.34}$$

subject to

$$\begin{aligned} &AW_cA^T - W_c + BB^T = \mathbf{0} \\ &W_c \succ \mathbf{0} \end{aligned}$$

where A is stable and (A,B) is controllable. Or dually,

$$\min_{\mathbf{x}} \left\{ trace(D^TD + B^TW_oB) \right\} \tag{3.35}$$

subject to

$$\begin{aligned} &A^TW_oA - W_o + C^TC = \mathbf{0} \\ &W_o \succ \mathbf{0} \end{aligned}$$

where A is stable and (A,C) is observable.

This kind of optimization problem can be solved by nonlinear programming (NLP) techniques. Under certain conditions, it can be further converted into linear matrix inequalities (LMIs) and thus can be solved more efficiently (Xu and Huang, 2006), as described in the previous chapter.

3.5 Simulation examples

The following MIMO system is considered to investigate the different cases of the LTVD benchmark problems.

The process transfer matrix and the controller are given respectively by

$$T = \begin{bmatrix} \frac{q^{-1}}{1-0.4q^{-1}} & \frac{q^{-2}}{1-0.1q^{-1}} \\ \frac{0.3q^{-1}}{1-0.1q^{-1}} & \frac{q^{-2}}{1-0.8q^{-1}} \end{bmatrix}, \qquad Q = \begin{bmatrix} \frac{0.5-0.2q^{-1}}{1-0.5q^{-1}} & 0 \\ 0 & \frac{0.25-0.2q^{-1}}{1-0.25q^{-2}} \end{bmatrix}$$

The process is assumed to be affected by three different piecewise constant disturbance dynamics with the models provided below with respect to these disturbances.

$$N_1 = \begin{bmatrix} \frac{1}{1-0.5q^{-1}} & \frac{-0.6q^{-1}}{1-0.5q^{-1}} \\ \frac{0.5q^{-1}}{1-0.5q^{-1}} & \frac{1}{1-0.5q^{-1}} \end{bmatrix} \qquad 1 \leqslant t < 2001$$

$$N_2 = \begin{bmatrix} \frac{1}{1-0.9q^{-1}} & \frac{-0.6q^{-1}}{1-0.9q^{-1}} \\ \frac{0.5q^{-1}}{1-0.9q^{-1}} & \frac{1}{1-0.9q^{-1}} \end{bmatrix} \qquad 2001 \leqslant t < 3001$$

$$N_3 = \begin{bmatrix} \frac{1}{1-0.7q^{-1}} & \frac{-0.6q^{-1}}{1-0.7q^{-1}} \\ \frac{0.5q^{-1}}{1-0.7q^{-1}} & \frac{1}{1-0.7q^{-1}} \end{bmatrix} \qquad 3001 \leqslant t \leqslant 4500$$

The second disturbance has the slowest dynamics and is considered here as the major disturbance. Control of the second disturbance needs to satisfy the user specified closed-loop response, which takes a lead-lag transfer function matrix as

$$G_R = \begin{bmatrix} \frac{1}{1-\lambda_1 q^{-1}} & 0 \\ 0 & \frac{1}{1-\lambda_2 q^{-1}} \end{bmatrix} \begin{bmatrix} \alpha_{11}+\beta_{11}q^{-1} & \alpha_{12}+\beta_{12}q^{-1} \\ \alpha_{21}+\beta_{21}q^{-1} & \alpha_{22}+\beta_{22}q^{-1} \end{bmatrix} = \begin{bmatrix} \frac{\alpha_{11}+\beta_{11}q^{-1}}{1-\lambda_1 q^{-1}} & \frac{\alpha_{12}+\beta_{12}q^{-1}}{1-\lambda_1 q^{-1}} \\ \frac{\alpha_{21}+\beta_{21}q^{-1}}{1-\lambda_2 q^{-1}} & \frac{\alpha_{22}+\beta_{22}q^{-1}}{1-\lambda_2 q^{-1}} \end{bmatrix}$$

where $\alpha = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}$ and $\beta = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}$ are decision variables, and $\lambda = [\lambda_1 \quad \lambda_2]$ can be specified by given values or bounded regions within which λ_1 and λ_2 are allowed to be changing (Huang and Shah, 1998).

From the process transfer function matrix, a unitary interactor matrix D can be factored out as:

$$D = \begin{bmatrix} -0.9578q & -0.2873q \\ -0.2873q^2 & 0.9578q^2 \end{bmatrix}$$

We can factorize F_2 and R_2 from the Diophantine equation $q^{-d}DN_2 = F_2 + q^{-2}R_2$ as:

$$F_2 = \begin{bmatrix} -0.9578q^{-1} & -0.2873q^{-1} \\ -0.2873+0.2203q^{-1} & 0.9578+1.034q^{-1} \end{bmatrix}, \qquad R_2 = \begin{bmatrix} \frac{-1.006}{1-0.9q^{-1}} & \frac{0.3161}{1-0.9q^{-1}} \\ \frac{0.1983}{1-0.9q^{-1}} & \frac{0.931}{1-0.9q^{-1}} \end{bmatrix}$$

Since in a SISO system, σ^2 is referred to as the variance of a signal time series, for convenience, here we also use the same notation for the total variance in the MIMO system, i.e., the trace of the output covariance matrix. For the performance assessment problems, $\hat{\sigma}^2$ represents the estimated total variance and $\hat{\eta}$ is the corresponding performance index. The performance index is defined as the ratio of optimal total variance and the present total variance. In the following simulation examples, we choose $\lambda_1 = \lambda_2$ in order to observe the changes of optimal total variances with different λ values.

3.5.1 Performance limit calculation

If the process and disturbance models are known, the optimal output total variances can be calculated with different user chosen λ values, based on the regular, weighted and generalized LTVD benchmarks respectively. The results are given in Table 3.1, Table 3.2 and Table 3.3 respectively. The optimal total variances are also shown in Figure 3.2, Figure 3.3 and Figure 3.4 respectively with solid lines.

Table 3.1: Performance limit results based on the regular LTVD benchmark

$\begin{bmatrix}\lambda_1 & \lambda_2\end{bmatrix}$	$\begin{bmatrix}0.3 & 0.3\end{bmatrix}$	$\begin{bmatrix}0.5 & 0.5\end{bmatrix}$	$\begin{bmatrix}0.7 & 0.7\end{bmatrix}$
α^*	$\begin{bmatrix}-0.3007 & -0.0727 \\ 0.0535 & 0.4753\end{bmatrix}$	$\begin{bmatrix}-0.3239 & -0.0934 \\ 0.0384 & 0.5120\end{bmatrix}$	$\begin{bmatrix}-0.3542 & -0.0787 \\ -0.0210 & 0.5595\end{bmatrix}$
β^*	$\begin{bmatrix}-0.1530 & -0.0251 \\ 0.0303 & 0.2419\end{bmatrix}$	$\begin{bmatrix}-0.1244 & -0.0001 \\ 0.0290 & 0.1967\end{bmatrix}$	$\begin{bmatrix}-0.0818 & -0.0426 \\ 0.0095 & 0.1291\end{bmatrix}$
$\sigma^2_{opt}(y_1)$	2.7938	2.7223	2.6298
$\sigma^2_{opt}(y_2)$	3.6753	3.8844	4.3277
$\sigma^2_{opt}(y_3)$	3.0154	3.0028	2.9846

Table 3.2: Performance limit results based on the weighted LTVD benchmark

$\begin{bmatrix}\lambda_1 & \lambda_2\end{bmatrix}$	$\begin{bmatrix}0.3 & 0.3\end{bmatrix}$	$\begin{bmatrix}0.5 & 0.5\end{bmatrix}$	$\begin{bmatrix}0.7 & 0.7\end{bmatrix}$
α^*	$\begin{bmatrix}-0.2266 & -0.0709 \\ 0.0173 & 0.3738\end{bmatrix}$	$\begin{bmatrix}-0.2414 & -0.0787 \\ -0.0110 & 0.3984\end{bmatrix}$	$\begin{bmatrix}-0.2610 & -0.0814 \\ 0.01956 & 0.4312\end{bmatrix}$
β^*	$\begin{bmatrix}-0.1201 & -0.0282 \\ 0.0092 & 0.2004\end{bmatrix}$	$\begin{bmatrix}-0.0963 & -0.0134 \\ 0.0011 & 0.1617\end{bmatrix}$	$\begin{bmatrix}-0.0594 & -0.0106 \\ 0.0029 & 0.1017\end{bmatrix}$
$\sigma^2_{opt}(y_1)$	2.8129	2.7473	2.6616
$\sigma^2_{opt}(y_2)$	3.4641	3.5821	3.8234
$\sigma^2_{opt}(y_3)$	2.9493	2.918	2.8721

For all of the three different LTVD benchmarks, with the increasing λ values, the optimal total variance of the first section is getting smaller, while the optimal total variance of the second section is becoming larger. It should be noted that for the third section, its optimal total variance is also decreasing with the increasing of λ values.

Table 3.3: Performance limit results based on the generalized LTVD benchmark

$\begin{bmatrix}\lambda_1 & \lambda_2\end{bmatrix}$	$\begin{bmatrix}0.3 & 0.3\end{bmatrix}$	$\begin{bmatrix}0.5 & 0.5\end{bmatrix}$	$\begin{bmatrix}0.7 & 0.7\end{bmatrix}$
α^*	$\begin{bmatrix}-0.1531 & -0.0503 \\ 0.0066 & 0.2725\end{bmatrix}$	$\begin{bmatrix}-0.1626 & -0.0525 \\ 0.0052 & 0.2895\end{bmatrix}$	$\begin{bmatrix}-0.1765 & -0.0275 \\ 0.0334 & 0.3143\end{bmatrix}$
β^*	$\begin{bmatrix}-0.0962 & -0.0182 \\ 0.0090 & 0.1713\end{bmatrix}$	$\begin{bmatrix}-0.0818 & -0.0188 \\ 0.0017 & 0.1456\end{bmatrix}$	$\begin{bmatrix}-0.0570 & 0.0056 \\ 0.0078 & 0.1018\end{bmatrix}$
$\sigma^2_{opt}(y_1)$	2.8727	2.8168	2.7499
$\sigma^2_{opt}(y_2)$	3.3128	3.3822	3.5197
$\sigma^2_{opt}(y_3)$	2.9291	2.8939	2.8466

With the same λ values, the smallest optimal total variance of the first section is always obtained on the regular LTVD benchmark. This is due to the fact that its objective function is aiming at minimizing the total variance of the first section only.

Comparing with the regular LTVD benchmark, the other two benchmarks lead to better trade-offs among different sections other than the transient section in the sense of total variance. As far as the regular LTVD benchmark is concerned, the total variance of the third section is not included in the objective function to be minimized, so its performance is not always guaranteed.

As for the weighted and generalized LTVD benchmarks, both of them can result in the trade-offs among different sections other than the transient one. However, the former one leads to a smaller sum of the weighted total variances of different sections while the latter results in a smaller maximum total variance of different sections.

3.5.2 Performance assessment calculation

For the case that the complete process model and disturbance model are all unavailable except for the closed-loop routine operating data and the interactor matrix, we can estimate the closed-loop model by time series analysis. The optimal output total variances can also be calculated with different user chosen λ values, based on the regular, weighted and generalized LTVD benchmarks respectively. The results are given in Table 3.4, Table 3.5 and Table 3.6 respectively. The optimal total variances are also shown in Figure 3.2, Figure 3.3 and Figure 3.4 respectively with dashed lines. Comparing the results of performance limit problems with those of performance assessment problems, it can be seen that the overall trends agree very well. The difference is likely due to the time series modelling error.

Table 3.4: Performance assessment results based on the regular LTVD benchmark

$\begin{bmatrix}\lambda_1 & \lambda_2\end{bmatrix}$	$\begin{bmatrix}0.3 & 0.3\end{bmatrix}$	$\begin{bmatrix}0.5 & 0.5\end{bmatrix}$	$\begin{bmatrix}0.7 & 0.7\end{bmatrix}$
α^*	$\begin{bmatrix}-0.2642 & -0.1369\\ 0.0084 & 0.4881\end{bmatrix}$	$\begin{bmatrix}-0.2859 & -0.1313\\ 0.0265 & 0.5302\end{bmatrix}$	$\begin{bmatrix}-0.3215 & -0.1406\\ -0.0187 & 0.5822\end{bmatrix}$
β^*	$\begin{bmatrix}-0.1400 & -0.0409\\ 0.0135 & 0.3127\end{bmatrix}$	$\begin{bmatrix}-0.1293 & 0.0216\\ 0.0352 & 0.2467\end{bmatrix}$	$\begin{bmatrix}-0.1026 & 0.0345\\ 0.0261 & 0.1446\end{bmatrix}$
$\hat{\sigma}^2_{opt}(y_1)$	2.8419	2.7701	2.6827
$\hat{\sigma}^2_{opt}(y_2)$	3.4803	3.696	4.1403
$\hat{\sigma}^2_{opt}(y_3)$	3.0205	2.9877	2.9623
$\hat{\eta}(y_1)$	0.9576	0.9334	0.9039
$\hat{\eta}(y_2)$	0.5103	0.5420	0.6071
$\hat{\eta}(y_3)$	0.8741	0.8646	0.8573

Table 3.5: Performance assessment results based on the weighted LTVD benchmark

$\begin{bmatrix}\lambda_1 & \lambda_2\end{bmatrix}$	$\begin{bmatrix}0.3 & 0.3\end{bmatrix}$	$\begin{bmatrix}0.5 & 0.5\end{bmatrix}$	$\begin{bmatrix}0.7 & 0.7\end{bmatrix}$
α^*	$\begin{bmatrix}-0.1929 & -0.0887\\ 0.0075 & 0.3860\end{bmatrix}$	$\begin{bmatrix}-0.2119 & -0.0552\\ -0.0286 & 0.4148\end{bmatrix}$	$\begin{bmatrix}-0.2407 & -0.1155\\ -0.0437 & 0.4548\end{bmatrix}$
β^*	$\begin{bmatrix}-0.1253 & 0.0840\\ -0.0097 & 0.2669\end{bmatrix}$	$\begin{bmatrix}-0.1174 & 0.0961\\ 0.0292 & 0.2120\end{bmatrix}$	$\begin{bmatrix}-0.0902 & 0.0866\\ -0.0039 & 0.1248\end{bmatrix}$
$\hat{\sigma}^2_{opt}(y_1)$	2.8841	2.8165	2.7231
$\hat{\sigma}^2_{opt}(y_2)$	3.2569	3.3897	3.6599
$\hat{\sigma}^2_{opt}(y_3)$	2.9237	2.8832	2.8357
$\hat{\eta}(y_1)$	0.9718	0.9490	0.9175
$\hat{\eta}(y_2)$	0.4776	0.4970	0.5367
$\hat{\eta}(y_3)$	0.8461	0.8344	0.8206

3.6 Conclusion

Three different LTVD benchmarks for MIMO processes are proposed and studied in this chapter for evaluating the controller performance in the sense of total variance. The process is subject to piecewise constant LTV disturbance dynamics. It can be concluded that the weighted and generalized LTVD benchmarks can gain better trade-offs between different sections of the disturbances than the regular LTVD benchmark. In the case there are only two different disturbances, these three LTVD benchmarks are exactly the same. But when there are more than 2 different disturbances, these three LTVD benchmarks are different. When only one representative disturbance is concerned, the regular LTVD benchmark is the right choice. However, when the sum of weighted total variances is to be minimized, we should choose the weighted LTVD benchmark. If we concern about the maximum (worst case) total variance of different disturbances, we would better choose the generalized LTVD

Table 3.6: Performance assessment results based on the generalized LTVD benchmark

$\begin{bmatrix}\lambda_1 & \lambda_2\end{bmatrix}$	$\begin{bmatrix}0.3 & 0.3\end{bmatrix}$	$\begin{bmatrix}0.5 & 0.5\end{bmatrix}$	$\begin{bmatrix}0.7 & 0.7\end{bmatrix}$
α^*	$\begin{bmatrix}-0.1602 & -0.0957\\ 0.0264 & 0.3294\end{bmatrix}$	$\begin{bmatrix}-0.1620 & -0.0246\\ -0.0672 & 0.3290\end{bmatrix}$	$\begin{bmatrix}-0.1681 & -0.1205\\ 0.0010 & 0.3343\end{bmatrix}$
β^*	$\begin{bmatrix}-0.1195 & 0.0717\\ -0.0412 & 0.2453\end{bmatrix}$	$\begin{bmatrix}-0.1144 & 0.0268\\ -0.0264 & 0.1948\end{bmatrix}$	$\begin{bmatrix}-0.0897 & 0.1247\\ -0.0255 & 0.1294\end{bmatrix}$
$\hat{\sigma}^2_{opt}(y_1)$	2.9118	2.8707	2.8073
$\hat{\sigma}^2_{opt}(y_2)$	3.1696	3.2291	3.3635
$\hat{\sigma}^2_{opt}(y_3)$	2.9118	2.8707	2.8098
$\hat{\eta}(y_1)$	0.9811	0.9673	0.9459
$\hat{\eta}(y_2)$	0.4648	0.4735	0.4932
$\hat{\eta}(y_3)$	0.8427	0.8308	0.8132

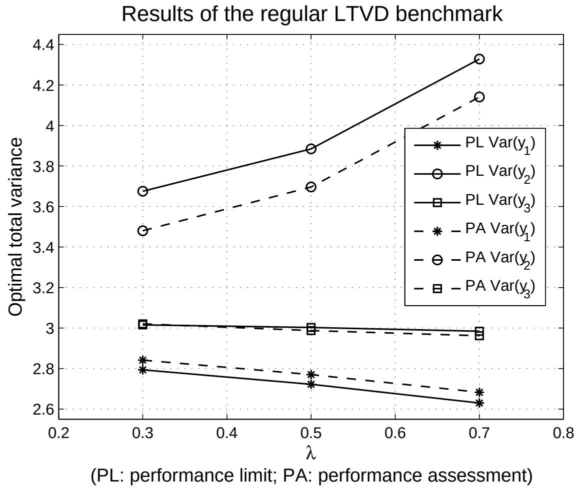

Figure 3.2: The trend of optimal variance vs user chosen λ value

benchmark to evaluate the controller performance.

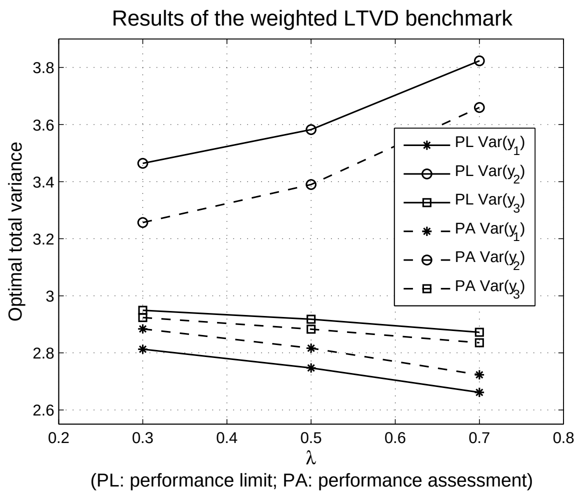

Figure 3.3: The trend of optimal variance vs user chosen λ value

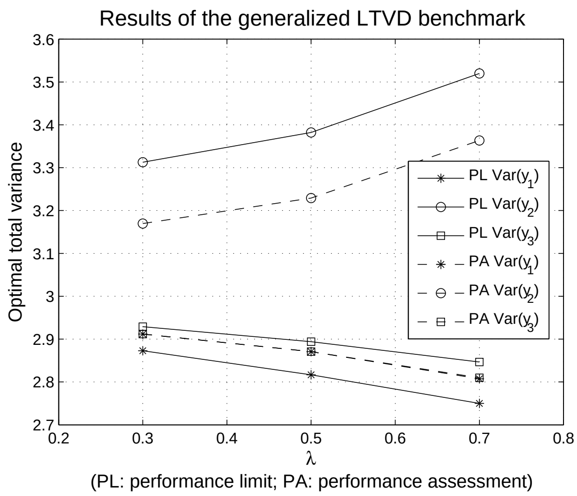

Figure 3.4: The trend of optimal variance vs user chosen λ value

4

Covariance Analysis Approach to Control Performance Assessment *

4.1 Introduction

Most of current controller performance assessment algorithms are based on the minimum variance control (MVC) benchmark and the performance index is calculated by comparing the minimum variance and the actual variance (Harris, 1989; Desborough and Harris, 1992; Huang and Shah, 1999). However, the minimum variance control law may not be the implementable one in practice due to its lack of robustness to model uncertainty and use of excessive input actions (Astrom and Wittenmark, 1997). More realistic user specified benchmark controls thus have been investigated. This class of benchmark compares the variance of current closed-loop dynamics with the variance of the user specified closed-loop dynamics directly (Kozub, 1996).

The user specified closed-loop dynamics can be determined in terms of performance requirement, such as time constant, settling time, decay ratio, desired variance, and so forth. Tyler and Morari (1996) have formulated an acceptable performance as constraints on the closed-loop impulse response coefficients. Performance assessment can be done by comparing the actual impulse response coefficients with those of acceptable performance.

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One useful practical specification can be expressed according to time constant or settling time (Kozub, 1996). A modified performance index proposed by Horch *et al.* (1999, 2000) is based on the user assigned closed-loop pole. Li *et al.* (2003) has developed a relative performance index which can be regarded as an alternative form of user specified performance benchmark. It is a measure that compares the variance of the output data from an acceptable reference model and that from the actual process, where the acceptable reference model can take nonparametric model form, like finite impulse response, or parametric models, like the first order transfer function. Huang and Shah (1998) have systematically described the user specified performance benchmark, from single-input single-output (SISO) case to multi-input multi-output (MIMO) case, and also from minimum phase system to non-minimum phase system. It was further extended to the controller performance assessment of linear time varying systems which are subject to piecewise constant disturbance dynamics (Huang, 1999; Olaleye, 2002; Olaleye *et al.*, 2004*b*; Xu and Huang, 2006).

However, the user specified closed-loop dynamics cannot be arbitrarily determined due to the process time delay. For the time delay systems, the closed-loop response can be divided into two parts, one is feedback control invariant and the other one feedback controller dependent (Harris, 1989). Therefore, only the latter part can be replaced by a user specified response trajectory. The obtained closed-loop response is referred to as the structured closed-loop response (Xu and Huang, 2006). The optimal structured closed-loop response can be used as an achievable control against which one can assess performance of control loops. In the previous work, the user specified response is given as a first order or second order transfer function or matrix with some specified performance requirement for the characteristic parameters, such as time constant and/or damping coefficient. However, the choice of the user specified response is not arbitrary. For example, in the closed-loop pole assignment, the choice of closed-loop pole is based on either robustness considerations or available additional process knowledge (Horch and Isaksson, 1999).

In this chapter, we attack this problem from viewpoint of a variance/covariance upper bound on the outputs. This is due to the fact that the variance typically represents product quality consistency (Shunta, 1995). The reduction of variances of some quality variables not only means the improved product quality but also makes it possible to operate to the constraints to increase throughput, reduce energy consumption and save raw materials. The covariance is the direct extension of the variance from SISO systems to MIMO systems. Therefore, if the output variance/covariance is not larger than its desired upper bound, the product quality consistency can be ensured. Meanwhile, the theoretical framework on the covariance control problems was proposed by Skelton *et al.* (1998). The necessary and sufficient conditions for a positive definite matrix to be assignable to the state vector and parametrization of all controllers that assign such a covariance to the state vector were obtained (Collins and Skelton, 1987; Hsieh *et al.*, 1989; Hsieh and Skelton, 1990; Grigoriadis and Skelton, 1997; Zhu *et al.*, 1997). However, the output variance/covariance constrained problem to be dealt with in this chapter is a multi-objective design problem and thus the covariance control framework presented in (Skelton *et al.*, 1998) is not applicable to this problem. Furthermore, in many control applications assigning a covariance to the state vector is not suitable in controller performance monitoring. Instead, specifying certain bounds on the covariance for the input and output vectors is often required (Hsieh *et al.*, 1989; Zhu *et al.*, 1997). Huang (2003) and Huang *et al.* (2003*b*) extended this idea to include the input in the controlled variables and showed that the generalized covariance constrained (GCC) problem can be converted into the feasibility problem via linear matrix inequalities (LMIs). If the GCC problem is feasible, the corresponding controller can be parameterized by the solution to these LMIs. This makes it possible to parameterize the structured closed-loop response subject to the output variance/covariance upper bound constraint. In this chapter, we present an effective framework and optimization methods to solve the variance/covariance upper bound constraint problems arising in controller performance monitoring via structured closed-loop response.

This chapter focuses on the performance assessment problem rather than the controller synthesis problem and the complete knowledge of models is not needed. The proposed benchmark can be used to evaluate the performance of the existing controllers. Performance measures showing how far the current controller is from the benchmark are presented by variance performance index and $\mathcal{H}_\infty$ norm of the difference between the desired structured closed-loop system and the existing closed-loop system. The main contribution of this chapter is therefore to solve the desired structured closed-loop response subject to the constraint of output variance/covariance upper bound. With closed-loop routine operating output data and process time delay (i.e., no complete process model information is available), the desired structured closed-loop response can be obtained directly via the estimated closed-loop time series model. It is useful for the controller performance assessment when both the chosen user specified closed-loop response with guaranteed stability and variance/covariance constraints are satisfied simultaneously. A significant feature is that the output variance/covariance upper bound constraint can be explicitly specified according to the product specifications and is always satisfied when the problem is feasible. This desired structured closed-loop response can thus be served as a benchmark against which the existing controller performance can be compared. In addition, two approaches, linearizing change of variables and Frank and Wolfe algorithm, are adopted to solve this problem, which result in a full order and a reduced order structured closed-loop response, respectively.

The outline of this chapter is as follows. Problem statement is discussed in Section 4.2. A framework for the structured closed-loop response design is given in Section 4.3. LMI-based optimization methods are presented for the structured closed-loop response design in Section 4.4. A guideline for the selection of the closed-loop response structure and the optimization strategies for performance assessment are illustrated in Section 4.5. A case study demonstrating the validity of our approaches is illustrated in Section 4.6, followed by concluding remarks in Section 4.7.

The notations throughout this chapter are standard. $\mathbf{R}^n$ denotes the n dimensional Euclidean space. $\mathbf{R}^{n\times m}$ is the set of all $n \times m$ real matrices. I is the identity matrix with appropriate dimensions. $*$ denotes the symmetric part or the block of no concern. $X \succ 0$ (resp. $X \succeq 0$) means that the matrix X is symmetric and positive definite (resp. positive semidefinite). Other notations are given in the Glossary.

4.2 Problem statement

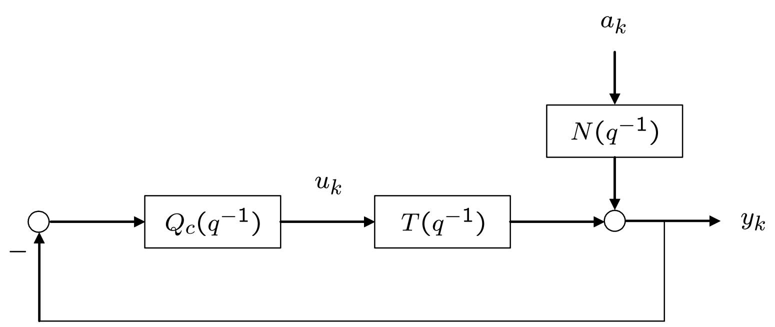

Figure 4.1: General feedback control framework

Consider a discrete linear time invariant system

$$y_k = T(q^{-1})u_k + N(q^{-1})a_k \tag{4.1}$$

where $T(q^{-1})$ is the process model and $N(q^{-1})$ is the disturbance model, as shown in

Figure 4.1. The state space realization of the system (4.1) is given by:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ga_k \\ y_k &= Cx_k + Fa_k \end{aligned} \tag{4.2}$$

where $x_k \in \mathbf{R}^n$ is the state of the system, $u_k \in \mathbf{R}^m$ the control signal, $y_k \in \mathbf{R}^p$ the measured output and $a_k \in \mathbf{R}^q$ the external disturbance that is a zero mean white noise sequence satisfying:

$$E(a_k) = \mathbf{0}, \quad E(a_k a_k^T) = \Omega$$

where Ω is a positive definite matrix.

Consider a dynamic output feedback controller $Q_c(q^{-1})$ described by:

$$\begin{aligned} x_{k+1}^C &= A_c x_k^C + B_c y_k \\ u_k &= C_c x_k^C + D_c y_k \end{aligned} \tag{4.3}$$

where $x_k^C \in \mathbf{R}^{n_c}$ is the state of the controller and n_c is a pre-assigned order of the controller. The closed-loop response is then given by the following state space equations:

$$\begin{aligned} X_{k+1} &= A_{cl} X_k + G_{cl} a_k \\ y_k &= C_{cl} X_k + Fa_k \end{aligned} \tag{4.4}$$

where

$$X_k = \begin{pmatrix} x_k \\ x_k^C \end{pmatrix}, A_{cl} = \begin{pmatrix} A + BD_cC & BC_c \\ B_cC & A_c \end{pmatrix}, G_{cl} = \begin{pmatrix} BD_cF + G \\ B_cF \end{pmatrix}, C_{cl} = \begin{pmatrix} C & 0 \end{pmatrix}$$

The steady state covariance of the closed-loop state vector X_k is defined as:

$$\Sigma_{cl} = \lim_{k\to\infty} E(X_k X_k^T) \tag{4.5}$$

If the closed-loop system (4.4) is asymptotically stable, the closed-loop steady state covariance matrix Σ_{cl} satisfies the following Lyapunov equation:

$$A_{cl}\Sigma_{cl}A_{cl}^T - \Sigma_{cl} + G_{cl}\Omega G_{cl}^T = 0 \tag{4.6}$$

In the following, the resultant closed-loop response is formulated for single-input single-output and multi-input multi-output systems, respectively.

4.2.1 Single-input single-output systems

Consider a SISO system of the form (4.1). The closed-loop response of the system (4.1) can be divided into the feedback control invariant part and the feedback controller dependent part owing to the process time delay, i.e.,

$$y_k = F(q^{-1})a_k + q^{-d}R_{cl}(q^{-1})a_k \tag{4.7}$$

where d is the process time delay. If the latter part $R_{cl}(q^{-1})$ is replaced by a user specified response trajectory, which is defined by the transfer function $L_R(q^{-1})$, we refer to the corresponding closed-loop response as the structured closed-loop response. It can be written as

$$y_k^* = F(q^{-1})a_k + q^{-d}L_R(q^{-1})a_k \tag{4.8}$$

Our purpose is to find a suitable $L_R(q^{-1})$ such that all the user specified requirements are satisfied, and at the same time the control performance is realizable by a linear time invariant controller. Therefore, this structured closed-loop response, if exists, has been used as a user specified benchmark to assess controller performance (Kozub, 1996; Xu and Huang, 2006). It is assumed that a feedback controller has been implemented to control the process, but the process model, disturbance model and controller model may all be unknown with exception of the process time delay d. A time series of closed-loop routine operating data can be sampled from the control loop and therefore a closed-loop time series model $\hat{G}_{cl}(q^{-1})$ can be estimated. Then the corresponding $F(q^{-1})$ and $R_{cl}(q^{-1})$ can be obtained from $\hat{G}_{cl}(q^{-1})$ and are denoted as $\hat{F}(q^{-1})$ and $\hat{R}_{cl}(q^{-1})$ respectively, i.e.,

$$\hat{y}_k = \hat{G}_{cl}(q^{-1})a_k, \qquad \hat{G}_{cl}(q^{-1}) = \hat{F}(q^{-1}) + q^{-d}\hat{R}_{cl}(q^{-1}) \tag{4.9}$$

By replacing $\hat{R}_{cl}(q^{-1})$ with $L_R(q^{-1})$, the structured closed-loop response is represented by

$$\hat{y}_k^* = \hat{F}(q^{-1})a_k + q^{-d}L_R(q^{-1})a_k \tag{4.10}$$

If $L_R(q^{-1})$ is chosen as zero, then the minimum variance control response is obtained. It is noted that, usually $L_R(q^{-1})$ is not chosen as zero for the user specified benchmark as proposed by Kozub (1996). We will solve the problem of selecting $L_R(q^{-1})$ in an optimal sense in this chapter.

4.2.2 Multi-input multi-output systems

Consider a MIMO system of the form (4.1). The generalization of the univariate process time delay is known as the interactor matrix. The unitary interactor matrix is an optimal factorization of time delays for multivariate systems (Huang and Shah, 1999). The output filtered by the unitary interactor matrix can be separated as

$$q^{-d}Dy_k = F(q^{-1})a_k + q^{-d}R_{cl}(q^{-1})a_k \tag{4.11}$$

which is equivalent to

$$y_k = q^{d}D^{-1}F(q^{-1})a_k + D^{-1}R_{cl}(q^{-1})a_k \tag{4.12}$$

where D is the unitary interactor matrix and d is its order. The first term is said to be feedback controller invariant and the second term is feedback controller dependent. By replacing $R_{cl}(q^{-1})$ with $L_R(q^{-1})$, the structured closed-loop response is then represented by

$$y_k^* = q^d D^{-1} F(q^{-1}) a_k + D^{-1} L_R(q^{-1}) a_k \tag{4.13}$$

Similarly, when only the closed-loop time series model and the unitary interactor matrix are known, we have

$$\hat{y}_k = \hat{G}_{cl}(q^{-1}) a_k, \qquad q^{-d} D \hat{G}_{cl}(q^{-1}) = \hat{F}(q^{-1}) + q^{-d} \hat{R}_{cl}(q^{-1}) \tag{4.14}$$

By replacing $\hat{R}_{cl}(q^{-1})$ with $L_R(q^{-1})$, the structured closed-loop response is thus represented by

$$\hat{y}_k^* = q^d D^{-1} \hat{F}(q^{-1}) a_k + D^{-1} L_R(q^{-1}) a_k \tag{4.15}$$

It is seen that, when $D = q^d$, the MIMO structured closed-loop response (4.15) is reduced to the same form as the SISO one.

Definition 4.2.1 *Given the discrete linear time invariant system (4.2), the output variance upper bound constraint is defined as*

$$\mathbf{trace}\left(\lim_{k \to \infty} E(y_k y_k^T) \right) < \sigma_y^2, \tag{4.16}$$

(resp. the output covariance upper bound constraint is defined as

$$\lim_{k \to \infty} E(y_k y_k^T) \prec \Phi_y,) \tag{4.17}$$

where σ_y^2 and Φ_y are user specified output variance and covariance upper bounds, respectively.

It is apparent that if the system matrices in (4.2) are all known, the output variance/covariance upper bound (4.16) or (4.17) can be satisfied by designing an appropriate dynamic output feedback controller $Q_c(q^{-1})$ in the state space formulation shown in (4.3). Once $Q_c(q^{-1})$ is solved, $L_R(q^{-1})$ is readily known. In this case, the problem to find $L_R(q^{-1})$ is a dual of controller design problem. However, the system matrices in (4.2) are often not available in practice. What we have is the closed-loop routine operating data from which the closed-loop time series model (4.9) or (4.14) can be identified. Then the controller performance assessment problem via structured closed-loop response subject to output variance/covariance upper bound can be stated as the following.

With known process time delay d for a SISO system or unitary interactor matrix D for a MIMO system, and given the estimated closed-loop time series model (4.9) or (4.14), find the parameterization of $L_R(q^{-1})$ such that the output of the closed-loop system (4.10) or (4.15) satisfies the output variance upper bound constraint (4.16) or output covariance upper bound constraint (4.17).

For this problem, if the structured closed-loop response can be parameterized successfully, then it can be utilized as a benchmark to evaluate the controller performance against which the output variance/covariance is compared. Therefore, a significant characteristic of this benchmark control is that its output variance/covariance is always constrained and no larger than the pre-specified upper bound. The extraction or estimation of the interactor matrix without knowing complete process models has been addressed by Huang and Shah (1999).

4.2.3 Controller performance estimation

Once $L_R(q^{-1})$ is solved, the existing closed-loop response can be compared against the structured closed-loop response in the sense of output variance. For a MIMO system, its overall performance index is defined as

$$\hat{\eta}_o = \frac{trace(cov(\hat{y}_k^*))}{trace(cov(\hat{y}_k))} \tag{4.18}$$

and its individual performance index for the $i-$th output is defined as

$$\hat{\eta}_i = \frac{var(\hat{y}_k^{*(i)})}{var(\hat{y}_k^{(i)})} \tag{4.19}$$

The estimated overall controller performance index should be greater than that of the minimum variance benchmark which is defined as the minimum output variance over the existing output variance. However, the individual indices are not necessary to be less than 1. If the performance index is smaller than 1, then the smaller the performance index, the more variance can be reduced or improved by tuning the existing controller. If the performance index is equal to or greater than 1, then the user specified output variance/covariance upper bound constraint has been satisfied by the existing controller.

4.3 Framework for structured closed-loop response design

Given the process and disturbance model (4.1), the following lemmas give the basic formulation for the synthesis of the controller subject to the output covariance upper bound constraint.

Lemma 4.3.1 *(Huang, 2003) The closed-loop system (4.4) satisfying the output covariance constraint (4.17) is asymptotically stable if and only if there exists a solution* $\{\Sigma, A_c, B_c, C_c, D_c\}$ *such that*

$$\begin{pmatrix} -\Sigma & A_{cl} & G_{cl} \\ A_{cl}^T & -\Sigma^{-1} & 0 \\ G_{cl}^T & 0 & -\Omega^{-1} \end{pmatrix} \prec 0 \tag{4.20}$$

$$\begin{pmatrix} -\Phi_y & C_{cl} & F \\ C_{cl}^T & -\Sigma^{-1} & 0 \\ F^T & 0 & -\Omega^{-1} \end{pmatrix} \prec 0 \tag{4.21}$$

Proof: Σ is a positive definite symmetric matrix satisfying $\Sigma \succ \Sigma_{cl}$, where Σ_{cl} satisfies (4.6). For proof, see (Huang, 2003).

□

Similarly, given the estimated closed-loop time series model (4.9) or (4.14), the following provides the basic formulation for performance assessment subject to the output variance/covariance upper bound. Since the SISO closed-loop time series model (4.9) is equivalent to letting $D = q^d$ of the MIMO one (4.14), only the MIMO case is considered in the sequel. In addition, we will focus on the output covariance upper bound constraint problem.

Define the state space realization of the following matrices

$$D^{-1} = \left(\begin{array}{c|c} A_D & G_D \\ \hline C_D & F_D \end{array}\right), q^d D^{-1}\hat{F}(q^{-1}) = \left(\begin{array}{c|c} A_F & G_F \\ \hline C_F & F_F \end{array}\right), L_R(q^{-1}) = \left(\begin{array}{c|c} A_R & G_R \\ \hline C_R & F_R \end{array}\right) \tag{4.22}$$

where $\hat{F}(q^{-1})$ satisfies the Diophantine equation (4.14), $A_F \in \mathbf{R}^{n_f \times n_f}$, $G_F \in \mathbf{R}^{n_f \times q}$, $C_F \in \mathbf{R}^{p \times n_f}$, $A_D \in \mathbf{R}^{n_d \times n_d}$, $G_D \in \mathbf{R}^{n_d \times p}$, and $C_D \in \mathbf{R}^{p \times n_d}$. The state space realization of the structured closed-loop response (4.15) can be written as

$$\begin{aligned} X_{k+1}^s &= A_S X_k^s + G_S a_k \\ y_k &= C_S X_k^s + F_S a_k \end{aligned} \tag{4.23}$$

where

$$A_S = \begin{pmatrix} A_F & 0 & 0 \\ 0 & A_D & G_D C_R \\ 0 & 0 & A_R \end{pmatrix}, G_S = \begin{pmatrix} G_F \\ G_D F_R \\ G_R \end{pmatrix}, C_S = \begin{pmatrix} C_F & C_D & F_D C_R \end{pmatrix}, F_S = F_F + F_D F_R \tag{4.24}$$

Lemma 4.3.2 *The structured closed-loop response (4.15) satisfying the output covariance upper bound constraint (4.17) with y_k replaced by $\hat{y}_k^*$ is feasible if and only if there exists a solution $\{\Sigma, A_R, G_R, C_R, F_R\}$ such that*

$$\begin{pmatrix} -\Sigma & A_S & G_S \\ A_S^T & -\Sigma^{-1} & 0 \\ G_S^T & 0 & -\Omega^{-1} \end{pmatrix} \prec 0 \tag{4.25}$$

$$\begin{pmatrix} -\Phi_y & C_S & F_S \\ C_S^T & -\Sigma^{-1} & 0 \\ F_S^T & 0 & -\Omega^{-1} \end{pmatrix} \prec 0 \tag{4.26}$$

Proof: The proof can be obtained by simply following Lemma 4.3.1 and omitted.

□

With Lemma 4.3.1 and 4.3.2, the optimal controller and the optimal structured closed-loop response can be solved respectively when the corresponding problems are feasible. The resultant output variances are always no larger than those of user specified constant covariance upper bound matrix, Φ_y. Therefore, these two problems are to be solved by verifying the feasibility of the corresponding set of constraints. On the other hand, if the output variances are to be minimized, e.g., by minimum variance control, we only need to replace the constant output covariance matrix Φ_y with a decision variable Φ and then minimize its trace, which is the sum of the output variances.

4.4 Solutions to the feasibility problems

4.4.1 Feasible solutions via full order synthesis

Consider a response trajectory $L_R(q^{-1})$ with $n_r = n_f + n_d$, which we call as full order. It can be directly solved from the closed-loop routine operating data according to the following theorem.

Theorem 4.4.1 *The structured closed-loop response (4.15) satisfying the output covariance upper bound constraint (4.17) is feasible via a full order response trajectory $L_R(q^{-1})$ if and only if there exists a solution $\{N_1, M_1, \hat{A}_R, \hat{G}_R, \hat{C}_R, \hat{F}_R\}$ such that*

$$\begin{pmatrix} -N_1 & -I & A_{FD} & A_{FD}N_1 + J_4\hat{C}_R & J_5 + J_4\hat{F}_R \\ * & -M_1 & M_1 A_{FD} & \hat{A}_R & M_1 J_5 + \hat{G}_R \\ * & * & -M_1 & -I & 0 \\ * & * & * & -N_1 & 0 \\ * & * & * & * & -\Omega^{-1} \end{pmatrix} \prec 0 \tag{4.27}$$

$$\begin{pmatrix} -\Phi_y & C_{FD} & C_{FD}N_1 + F_D\hat{C}_R & F_F + F_D\hat{F}_R \\ * & -M_1 & -I & 0 \\ * & * & -N_1 & 0 \\ * & * & * & -\Omega^{-1} \end{pmatrix} \prec 0 \tag{4.28}$$

If there exists a solution, the full order $L_R(q^{-1})$ can be parameterized as:

$$\begin{cases} F_R = \hat{F}_R \\ C_R = \hat{C}_R N_2^{-T} \\ G_R = M_2^{-1}(\hat{G}_R - M_1 J_4 F_R) \\ A_R = M_2^{-1}(\hat{A}_R - M_1 A_{FD} N_1 - M_1 J_4 C_R N_2^T) N_2^{-T} \end{cases} \tag{4.29}$$

where

$$A_{FD} = \begin{pmatrix} A_F & 0 \\ 0 & A_D \end{pmatrix}, \quad C_{FD} = \begin{pmatrix} C_F & C_D \end{pmatrix}, \quad J_4 = \begin{pmatrix} 0 \\ G_D \end{pmatrix}, \quad J_5 = \begin{pmatrix} G_F \\ 0 \end{pmatrix}$$

and N_2, M_2 are any matrices satisfying

$$N_2 M_2^T = I - N_1 M_1 \tag{4.30}$$

Proof: ($\Rightarrow$) According to Lemma 4.3.2, the structured closed-loop response (4.14) satisfying the output covariance upper bound constraint (4.17) is feasible if and only if (4.25) and (4.26) are satisfied, where A_S, G_S, C_S and F_S are defined in (4.24). Partition Σ and its inverse Σ^{-1} as

$$\Sigma = \begin{pmatrix} N_1 & N_2 \\ N_2^T & N_3 \end{pmatrix}, \quad \Sigma^{-1} = \begin{pmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{pmatrix}$$

where $N_1, M_1 \in \mathbf{R}^{(n_f+n_d)\times(n_f+n_d)}$ and $N_3, M_3 \in \mathbf{R}^{n_r\times n_r}$ with $n_r = n_f + n_d$. Define Π_1 and Π_2 as

$$\Pi_1 = \begin{pmatrix} I & 0 \\ M_1 & M_2 \end{pmatrix}, \quad \Pi_2 = \begin{pmatrix} I & 0 \\ N_1 & N_2 \end{pmatrix}$$

Then the following identities hold:

$$\Pi_1 \Sigma \Pi_1^T = \begin{pmatrix} N_1 & I \\ I & M_1 \end{pmatrix}$$

$$\Pi_2 \Sigma^{-1} \Pi_2^T = \begin{pmatrix} M_1 & I \\ I & N_1 \end{pmatrix}$$

$$\Pi_1 A_S \Pi_2^T = \begin{pmatrix} A_{FD} & A_{FD}N_1 + G_{DR}N_2^T \\ M_1 A_{FD} & M_1 A_{FD} N_1 + M_1 G_{DR} N_2^T + M_2 A_R N_2^T \end{pmatrix}$$

$$\Pi_1 G_S = \begin{pmatrix} G_{FR} \\ M_1 G_{FR} + M_2 G_R \end{pmatrix}$$

$$C_S \Pi_2^T = \begin{pmatrix} C_{FD} & C_{FD}N_1 + C_{FR}N_2^T \end{pmatrix}$$

where G_{DR}, G_{FR}, C_{FR} and F_{DR} are defined as follows:

$$G_{DR} = \begin{pmatrix} 0 \\ G_D C_R \end{pmatrix}, \quad G_{FR} = \begin{pmatrix} G_F \\ G_D F_R \end{pmatrix}, \quad C_{FR} = F_D C_R, \quad F_{DR} = F_D F_R$$

Define Γ and Υ as

$$\Gamma = \begin{pmatrix} \Pi_1 & 0 & 0 \\ 0 & \Pi_2 & 0 \\ 0 & 0 & I \end{pmatrix}, \quad \Upsilon = \begin{pmatrix} I & 0 & 0 \\ 0 & \Pi_2 & 0 \\ 0 & 0 & I \end{pmatrix}$$

Pre- and post-multiply both sides of (4.25) with Γ and Γ^T, respectively, and pre- and post-multiply both sides of (4.26) with Υ and Υ^T, respectively. Using the definition of the following matrices

$$\begin{cases} \hat{F}_R = F_R \\ \hat{C}_R = C_R N_2^T \\ \hat{G}_R = M_2 G_R + M_1 J_4 F_R \\ \hat{A}_R = M_2 A_R N_2^T + M_1 A_{FD} N_1 + M_1 J_4 C_R N_2^T \end{cases}$$

LMIs (4.27) and (4.28) are readily obtained with the identity (4.30).

($\Leftarrow$) By $\begin{pmatrix} N_1 & I \\ I & M_1 \end{pmatrix} \succ 0$ which is implicitly included in LMIs (4.27) and (4.28), we infer $M_1 \succ 0, N_1 - M_1^{-1} \succ 0$ such that $I - N_1 M_1$ is nonsingular. Hence, we can always find square and nonsingular matrices N_2 and M_2 satisfying (4.30). By reversing matrix manipulations, we can obtain LMIs (4.25) and (4.26). This completes the proof.

$\square$

4.4.2 Feasible solutions via reduced order synthesis

Note that, for any matrices $X \succ 0$ and $Y \succ 0$, $X, Y \in \mathbf{R}^{n \times n}$, if the LMI

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} \succeq 0 \tag{4.31}$$

is feasible, then $\mathbf{trace}(XY) \geq n$, and $\mathbf{trace}(XY) = n$ if and only if $XY = I$ (Ghaoui *et al.*, 1997; Zhang *et al.*, 2003).

A reduced order $L_R(q^{-1})$ (i.e. with $n_r < n_f + n_d$) is to be solved under the inequality constraints (4.25) and (4.26). Similarly, define $X \triangleq \Sigma$ and $Y \triangleq \Sigma^{-1}$ such that $XY = I$. Also define the following matrices:

$$\bar{A}_S = \begin{pmatrix} A_F & 0 & 0 \\ 0 & A_D & 0 \\ 0 & 0 & 0 \end{pmatrix}, \bar{G}_S = \begin{pmatrix} G_F \\ 0 \\ 0 \end{pmatrix}, \bar{C}_S = \begin{pmatrix} C_F & C_D & 0 \end{pmatrix}, \bar{F}_S = F_F, K_R = \begin{pmatrix} A_R & G_R \\ C_R & F_R \end{pmatrix}$$

$$H_1 = \begin{pmatrix} 0 & 0 \\ 0 & G_D \\ I & 0 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 0 & 0 & I \\ 0 & 0 & 0 \end{pmatrix}, \quad H_3 = \begin{pmatrix} 0 & F_D \end{pmatrix}, \quad H_4 = \begin{pmatrix} 0 \\ I \end{pmatrix}$$

Then the inequalities (4.25) and (4.26) are converted into the following inequalities

$$\begin{pmatrix} -X & \bar{A}_S + H_1 K_R H_2 & \bar{G}_S + H_1 K_R H_2 \\ * & -Y & 0 \\ * & * & -\Omega^{-1} \end{pmatrix} \prec 0 \tag{4.32}$$

$$\begin{pmatrix} -\Phi_y & \bar{C}_S + H_3 K_R H_4 & \bar{F}_S + H_3 K_R H_4 \\ * & -Y & 0 \\ * & * & -\Omega^{-1} \end{pmatrix} \prec 0 \tag{4.33}$$

$$XY = I \tag{4.34}$$

Hence, a feasible solution of the problem (4.32)-(4.34) can be obtained by solving the following concave minimization problem:

$$\text{Minimize}_{\{X,Y,K_R\}}\ \mathbf{trace}(XY) \text{ subject to } (4.31), (4.32), (4.33) \tag{4.35}$$

To find a local optimal solution of the concave minimization problems, the popular conditional gradient algorithm, also called the Frank and Wolfe algorithm (Frank and Wolfe, 1956; Bertsekas, 1995), can be used, such as in (de Oliveira and Geromel, 1997; Ghaoui *et al.*, 1997; Zhang *et al.*, 2003). A computational algorithm for obtaining a local solution of the concave minimization problems is given next. To this end, define a convex set by the set of LMIs as

$$\mathscr{C}_{(X,Y,K_R)} \triangleq \{(X, Y, K_R) : (4.31), (4.32), (4.33), X \succ 0, Y \succ 0\}$$

Algorithm 4.4.1 *For given* $\Phi_y \succ 0$,

(1) *Find an initial feasible solution* $(X^0, Y^0) \in \mathscr{C}_{(X,Y,K_R)}$. *Set* $k = 0$.

(2) *Set* $V^k = Y^k$, $W^k = X^k$. *Linearize the concave objective function of the problem (4.35) at a given point* (V^k, W^k) *and define a linear function*

$$f_k(X, Y) \triangleq \mathbf{trace}(V^k X + W^k Y) \tag{4.36}$$

(3) *Find* (X^{k+1}, Y^{k+1}) *by solving the following convex programming:*

$$\text{Minimize}_{\{X,Y,K_R\}}\ f_k(X, Y) \textit{ subject to } (4.31), (4.32), (4.33)$$

(4) *If* $f_{k+1}(X^{k+1}, Y^{k+1}) < \epsilon$, *where* ϵ *is a pre-determined tolerance, is satisfied, then exit. Otherwise, set* $k = k + 1$, *and return to Step 2.*

Theorem 4.4.2 *(Ghaoui* et al.*, 1997) The algorithm has the following properties:*

(i) $2(n_f + n_d + n_r) \leq f_{k+1} \leq f_k$.

(ii) $\lim_{k\to\infty} f_k = 2(n_f + n_d + n_r)$ *if and only if* $XY = I$ *at the optimum.*

The above theorem shows that Algorithm 4.4.1 can ensure the sequence f_k converge $f^{opt} \geq 2(n_f + n_d + n_r)$, and if $f^{opt} = 2(n_f + n_d + n_r))$, for a given Φ_y, a feasible solution of the problem (4.32)-(4.34) is found, which implies $L_R(q^{-1})$ is obtained.

4.5 Selection of $L_R(q^{-1})$ structure and optimization strategies

4.5.1 Selection of $L_R(q^{-1})$ structure

As shown in Section 4.4.1, by the full order synthesis using linearizing change of variables, a full order $L_R(q^{-1})$ can be obtained from Theorem 4.4.1 if and only if the problem is feasible. In this case, the order of $L_R(q^{-1})$ is $n_r = n_f + n_d$. The reduced order synthesis has also been dealt with in Section 4.4.2. A reduced order $L_R(q^{-1})$ can be obtained by Algorithm 4.4.1. It is noted that both the order and the structure of $L_R(q^{-1})$ can be specified *a priori* according to certain control requirements. In particular, the first order or second order form is preferred in practice. The rationale is that most process dynamic behavior may be approximated by either a first order or a second order dynamics. Specifically, an underdamped dynamic response may be represented by a second order dynamics. Therefore, for SISO systems, a typical $L_R(q^{-1})$ can be chosen as one of the following formulations,

$$L_R(q^{-1}) = \frac{\alpha + \beta q^{-1}}{1 + \lambda q^{-1}} \qquad \lambda = -e^{-\frac{\Delta T}{\tau}} \tag{4.37}$$

where α and β are the unknown free parameters to be determined, λ is specified according to the desired closed-loop time constant τ with the sampling period ΔT (Kozub, 1996). Or

$$L_R(q^{-1}) = \frac{\alpha_0 + \alpha_1 q^{-1} + \alpha_2 q^{-2}}{1 + \lambda_1 q^{-1} + \lambda_2 q^{-2}} \tag{4.38}$$

where

$$\lambda_1 = \begin{cases} -e^{-\frac{\xi\Delta T}{\tau}}\left(e^{\frac{\sqrt{\xi^2-1}\Delta T}{\tau}} + e^{\frac{-\sqrt{\xi^2-1}\Delta T}{\tau}}\right) & \text{when} \quad \xi \geq 1, \\ -2e^{-\frac{\xi\Delta T}{\tau}}\cos\left(\frac{\sqrt{1-\xi^2}\Delta T}{\tau}\right) & \text{when} \quad 0 < \xi < 1. \end{cases}$$

$$\lambda_2 = e^{-\frac{2\xi\Delta T}{\tau}}$$

ξ is the damping coefficient, τ is the natural period or the inverse natural frequency (Ogunnaike and Ray, 1994), and α_0, α_1 and α_2 are the decision variables to be determined. Specifically, for the second order system, ξ and τ can be determined according to the desired characteristic of the underdamped response, such as rise time, overshoot, decay ratio, settling time, and so forth. In the following, we will take the second order form (4.38) of $L_R(q^{-1})$ as an example and demonstrate that a feasible $L_R(q^{-1})$ can be obtained by simply solving two LMIs.

The transfer function of $L_R(q^{-1})$ (4.38) is readily represented by its state space form

$$\left(\begin{array}{c|c} A_R & G_R \\ \hline C_R & F_R \end{array}\right) = \left(\begin{array}{cc|c} -\lambda_1 & 1 & \alpha_1 - \lambda_1\alpha_0 \\ -\lambda_2 & 0 & \alpha_2 - \lambda_2\alpha_0 \\ \hline 1 & 0 & \alpha_0 \end{array}\right)$$

Once λ_1 and λ_2 are assigned values, A_R and C_R become constant matrices, and the decision variables α_0, α_1 and α_2 remain in the matrices G_R and F_R. With the definition of the following matrices

$$\bar{G}_S = \begin{pmatrix} G_F \\ 0 \\ 0 \end{pmatrix}, \quad \bar{F}_S = F_F, \quad K_R = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{pmatrix}$$

$$J_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & G_D \\ 1 & 1 & 0 \end{pmatrix}, \quad J_2 = \begin{pmatrix} -\lambda_1 & 1 & 0 \\ -\lambda_2 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad J_3 = \begin{pmatrix} F_D & 0 \end{pmatrix}$$

then the inequalities (4.25) and (4.26) can be directly converted into the following two LMIs:

$$\begin{pmatrix} -\Sigma & A_S\Sigma & \bar{G}_S + J_1J_2K_R \\ * & -\Sigma & 0 \\ * & * & -\Omega^{-1} \end{pmatrix} \prec 0, \quad \begin{pmatrix} -\Phi_y & C_S\Sigma & \bar{F}_S + J_3K_R \\ * & -\Sigma & 0 \\ * & * & -\Omega^{-1} \end{pmatrix} \prec 0 \tag{4.39}$$

where A_S and C_S are constant matrices. Therefore, $L_R(q^{-1})$ can be selected as either a full order one by Theorem 4.4.1 or a reduced order one by Algorithm 4.4.1. In particular, when $L_R(q^{-1})$ is selected as a first order or a second order transfer function with a given denominator but an unknown numerator, the feasibility problem can be solved directly by the LMI formulation in (4.39).

In the case that $L_R(q^{-1})$ is selected as a first order transfer function (4.37), the following matrices should be redefined in (4.39),

$$J_1J_2 = \begin{pmatrix} 0 & 0 \\ G_D & 0 \\ -\lambda & 1 \end{pmatrix}, \quad K_R = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

4.5.2 Selection of optimization strategies

In Section 4.4, two synthesis approaches are provided to obtain a feasible solution of $L_R(q^{-1})$. However, the results from feasibility problems are not unique and may not always be practical in applications. In order to overcome this problem, further design strategies need to be considered such that the resulting $L_R(q^{-1})$ is not only feasible but also practical. In the following, the one that is closest to $\hat{R}_{cl}(q^{-1})$ will be suggested among those feasible solutions.

With closed-loop time series models being identified from routine operating data, we can take full advantage of the identified model information. Since $\hat{R}_{cl}(q^{-1})$ is available from the identified closed-loop model and reflects part of the existing closed-loop dynamics, the response of $L_R(q^{-1})$ should be chosen to be close to that of $\hat{R}_{cl}(q^{-1})$ such that less tuning effort on the existing controller is required while still satisfying the variance/covariance upper bound. Theoretically if the output variance/covariance upper bound is already satisfied by the existing control, $\hat{R}_{cl}(q^{-1})$ will be the ideal solution of the structured closed-loop response $L_R(q^{-1})$ in the sense that no effort needs to tune the existing controller. On the other hand, if the existing controller does not result in a satisfactory response $\hat{R}_{cl}(q^{-1})$, which means that the existing output (co-)variance is larger than the specified upper bound, it is likely that the closer $L_R(q^{-1})$ is to $\hat{R}_{cl}(q^{-1})$, the less effort is needed to tune the controller to satisfy the (co-)variance upper bound. This idea leads to the model approximation or the closed-loop response matching problem, with given $\hat{R}_{cl}(q^{-1})$ but unknown $L_R(q^{-1})$. Once the closed-loop time series model is identified, $\hat{R}_{cl}(q^{-1})$ is fixed while $L_R(q^{-1})$ is flexible in either its order or its structure.

In the model reduction literature, the $\mathcal{H}_\infty$ norm has been considered one of the most meaningful measures (Xu *et al.*, 2001; Zhang *et al.*, 2003; Ebihara and Hagiwara, 2004). In the following, the $\mathcal{H}_\infty$ norm is introduced to our closed-loop response matching problem. The objective of our problem is to obtain a practical $L_R(q^{-1})$ such that $L_R(q^{-1})$ approximates $\hat{R}_{cl}(q^{-1})$ as close as possible in the sense of the $\mathcal{H}_\infty$ norm, i.e.,

$$\|L_R(q^{-1}) - \hat{R}_{cl}(q^{-1})\|_\infty \leq \gamma \tag{4.40}$$

By the following state space realizations

$$L_R(q^{-1}) = \left(\begin{array}{c|c} A_R & G_R \\ \hline C_R & F_R \end{array}\right), \quad \hat{R}_{cl}(q^{-1}) = \left(\begin{array}{c|c} \hat{A}_R & \hat{G}_R \\ \hline \hat{C}_R & \hat{F}_R \end{array}\right)$$

We have

$$L_R(q^{-1}) - \hat{R}_{cl}(q^{-1}) = \left(\begin{array}{cc|c} A_R & 0 & G_R \\ 0 & \hat{A}_R & \hat{G}_R \\ \hline C_R & -\hat{C}_R & F_R - \hat{F}_R \end{array}\right) = \left(\begin{array}{c|c} A_M & G_M \\ \hline C_M & F_M \end{array}\right)$$

From the bounded real lemma (Gahinet and Apkarian, 1994), $\|L_R(q^{-1}) - \hat{R}_{cl}(q^{-1})\|_\infty \leq \gamma$ if and only if there exist a symmetric positive definite matrix $\Pi \in \mathbf{R}^{(n_r+\hat{n}_r)\times(n_r+\hat{n}_r)}$ and $L_R(q^{-1})$ satisfying

$$\begin{pmatrix} -\Pi^{-1} & A_M & G_M & 0 \\ * & -\Pi & 0 & C_M^T \\ * & * & -\gamma I & F_M^T \\ * & * & * & -\gamma I \end{pmatrix} \prec 0 \tag{4.41}$$

For convenience, define $P \triangleq \Pi^{-1}$ and $Q \triangleq \Pi$ such that $PQ = I$. Also define the following matrices

$$\bar{A}_M = \begin{pmatrix} 0 & 0 \\ 0 & \hat{A}_R \end{pmatrix}, \quad \bar{G}_M = \begin{pmatrix} 0 \\ \hat{G}_M \end{pmatrix}, \quad \bar{C}_M = \begin{pmatrix} 0 & -\hat{C}_R \end{pmatrix}, \quad \bar{F}_M = -\hat{F}_R,$$

$$H_5 = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \quad H_6 = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \quad H_7 = \begin{pmatrix} 0 & I \end{pmatrix}$$

then the inequality (4.41) is converted into the following form

$$\begin{pmatrix} -P & \bar{A}_M + H_5K_RH_6 & \bar{G}_M + H_5K_RH_4 & 0 \\ * & -Q & 0 & (\bar{C}_M + H_7K_RH_6)^T \\ * & * & -\gamma I & (\hat{F}_M + H_7K_RH_4)^T \\ * & * & * & -\gamma I \end{pmatrix} \prec 0 \tag{4.42}$$

$$PQ = I \tag{4.43}$$

This closed-loop response matching problem is readily combined with the problem (4.32)-(4.34) for finding a practical solution $L_R(q^{-1})$ of a specified order $\hat{n}_r$. Hence a practical $L_R(q^{-1})$ can be obtained by solving the following concave minimization problem:

$$\text{Minimize}_{\{X,Y,P,Q,K_R\}}\ \mathbf{trace}(XY) + \mathbf{trace}(PQ) \tag{4.44}$$

subject to

$$\text{LMIs } (4.31), (4.32), (4.33), (4.42) \text{ and } \begin{pmatrix} P & I \\ I & Q \end{pmatrix} \succeq 0$$

Define a convex set by the set of LMIs as

$$\mathscr{C}_{(X,Y,P,Q,K_R)} \triangleq$$
$$\{(X,Y,P,Q,K_R) : (4.31), (4.32), (4.33), (4.42), \begin{pmatrix} P & I \\ I & Q \end{pmatrix} \succeq 0, X \succ 0, Y \succ 0, P \succ 0, Q \succ 0\}$$

Applying Algorithm 4.4.1 to the concave minimization problem leads to a practical solution $L_R(q^{-1})$.

In particular, if $L_R(q^{-1})$ takes a second order form as in (4.38), then the inequality (4.41) is directly converted into the following form

$$\begin{pmatrix} -\Pi^{-1} & A_M\Pi^{-1} & \bar{G}_M + K_1K_R & 0 \\ * & -\Pi^{-1} & 0 & \Pi^{-1}C_M^T \\ * & * & -\gamma I & (\bar{F}_M + K_2K_R)^T \\ * & * & * & -\gamma I \end{pmatrix} \prec 0 \tag{4.45}$$

where

$$A_M = \begin{pmatrix} -\lambda_1 & 1 & 0 \\ -\lambda_2 & 0 & 0 \\ 0 & 0 & \hat{A}_R \end{pmatrix}, \quad C_M = \begin{pmatrix} 1 & 0 & -\hat{C}_R \end{pmatrix}, \quad \bar{G}_M = \begin{pmatrix} 0 \\ 0 \\ \hat{G}_R \end{pmatrix}, \quad \bar{F}_M = -\hat{F}_R,$$

$$K_1 = \begin{pmatrix} -\lambda_1 & 1 & 0 \\ -\lambda_2 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \quad K_R = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{pmatrix}$$

This is exactly an LMI with the decision variables Π and K_R. Therefore, in this special case, a second order $L_R(q^{-1})$ that approximates a closed-loop time series model can be obtained by solving LMIs as follows:

$$\text{Minimize}_{\{\Sigma,\Pi,Q,K_R\}}\ \gamma \text{ subject to } (4.39), (4.45) \tag{4.46}$$

In the case that $L_R(q^{-1})$ is selected as a first order transfer function (4.37), the following matrices should be redefined in (4.45),

$$A_M = \begin{pmatrix} -\lambda & 0 \\ 0 & \hat{A}_R \end{pmatrix}, \quad C_M = \begin{pmatrix} 1 & -\hat{C}_R \end{pmatrix}, \quad \bar{G}_M = \begin{pmatrix} 0 \\ \hat{G}_R \end{pmatrix}, \quad \bar{F}_M = -\hat{F}_R,$$

$$K_1 = \begin{pmatrix} -\lambda & 1 \\ 0 & 0 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 1 & 0 \end{pmatrix}, \quad K_R = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

4.5.3 Discussion of time delay mismatch

The structured closed-loop response is derived by minimizing the difference between the impulse response of $L_R(q^{-1})$ and that of actual one while satisfying the variance/covariance upper bound constraint which is specified by the user. Obviously if the specified variance/covariance upper bound constraint is already satisfied by the existing control, then the benchmark structured closed-loop response would be the same as the existing closed-loop response. That means $\hat{R}_{cl}(q^{-1})$ is the right choice of $L_R(q^{-1})$. Note that the impulse response of the existing closed-loop system is calculated from the closed-loop routine operating data without using any time delay information. In this special case, the controller performance is not sensitive to the time delay mismatch at all and as a result the performance indices will be equal to 1. In practice, if the concerned specified variance/covariance upper bound constraint is satisfied, which is readily calculated directly from the closed-loop routine operating data, then no further effort is worthwhile to calculate the structured closed-loop response.

However, it is very often that the specified variance/covaraince upper bound constraint is not satisfied and the structured closed-loop response is thus needed to be investigated. In this case, the (co-)variance of the existing closed-loop response is larger than that of the specified upper bound constraint and $L_R(q^{-1})$ is not the same as $\hat{R}_{cl}(q^{-1})$. As a result, the obtained performance indices will be sensitive to time delay mismatch. Fortunately, this sensitivity can be monitored and reduced by minimizing the $\mathcal{H}_\infty$ norm of the difference between $L_R(q^{-1})$ and $\hat{R}_{cl}(q^{-1})$. This $\mathcal{H}_\infty$ norm gap is denoted as γ and is calculated by the algorithms together with the benchmark structured closed-loop response. Therefore, the resultant (co-)variance of the benchmark structured closed-loop response is always trying to approach its specified upper bound even there is time delay mismatch. If the time delay used is smaller than the real one, the estimated minimum variance becomes smaller and its difference with corresponding upper bound is thus getting larger. This is equivalent to relatively more degrees of freedom in the approximating of the structured closed-loop response to the existing one and the γ value may become smaller. On the other hand, if the time delay used is larger than the real one, the difference between the estimated minimum variance and its upper bound will become smaller. As a consequence, some degrees of freedom are lost in the calculation of the structured closed-loop response and the γ value may become larger. Therefore, γ value is an indicator to show that gap between $L_R(q^{-1})$ and $\hat{R}_{cl}(q^{-1})$ in the sense of $\mathcal{H}_\infty$ norm. By the minimization of this $\mathcal{H}_\infty$ norm gap, the benchmark (co-)variance is always, more or less, close to its upper bound and thus the solution of the benchmark (co-)variance may not be very sensitive to the delay mismatch.

It should be noted that, if there is time delay mismatch and the time delay is getting too large, caution must be taken due to the inflation of the minimum variance part and the optimization problem may become infeasible if this inflated minimum variance is greater than the specified variance upper bound.

4.6 Case studies

4.6.1 A dry process rotary cement kiln

Consider a dry process rotary cement kiln with a capacity of 1000 t of clinker a day, which is taken from Westerlund (1981) and Mäkilä *et al.* (1984). The model was given as

$$y_k + \begin{pmatrix} -0.914 & -0.08 \\ 0.126 & -0.917 \end{pmatrix} y_{k-1} = \begin{pmatrix} 2.091 & -0.0744 \\ -0.211 & -0.0156 \end{pmatrix} u_{k-1} + a_k + \begin{pmatrix} 0 & 0 \\ 0 & 0.715 \end{pmatrix} a_{k-1} \tag{4.47}$$

where $E(a_k a_k^T) = \begin{pmatrix} 0.0644 & 0.000257 \\ 0.000257 & 0.0214 \end{pmatrix}$ and the sampling time is 5 min. It is required that large variations in $y_k^{(1)}$ should be avoided in order to ensure steady state operation of the plant. Small variance of $y_k^{(2)}$ will make it possible to operate the process closer to the limit which specifies the maximum free lime content of the clinker. This will result in reduced energy consumption.

The system (4.47) can be written into transfer matrix form as

$$\begin{aligned} y_k =& \frac{\begin{pmatrix} 2.091q^{-1} - 1.934q^{-2} & -0.0744q^{-1} + 0.06698q^{-2} \\ -0.211q^{-1} - 0.07061q^{-2} & -0.0156q^{-1} + 0.02363q^{-2} \end{pmatrix}}{1 - 1.831q^{-1} + 0.8482q^{-2}} u_k \\ &+ \frac{\begin{pmatrix} 1 - 0.917q^{-1} & 0.08q^{-1} + 0.0572q^{-2} \\ -0.126q^{-1} & 1 - 0.199q^{-1} - 0.6535q^{-2} \end{pmatrix}}{1 - 1.831q^{-1} + 0.8482q^{-2}} a_k \end{aligned} \tag{4.48}$$

and the unitary interator matrix is calculated as $D = \begin{pmatrix} 0 & q \\ q & 0 \end{pmatrix}$.

A state space realization of the model (4.47) is

$$\begin{cases} x_{k+1} = \begin{pmatrix} 0.914 & 0.08 \\ -0.126 & 0.917 \end{pmatrix} x_k + \begin{pmatrix} 2.091 & -0.0744 \\ -0.211 & -0.0156 \end{pmatrix} u_k + \begin{pmatrix} 0.914 & 0.08 \\ -0.126 & 1.632 \end{pmatrix} a_k \\ y_k = x_k + a_k \end{cases} \tag{4.49}$$

In this example, we assume that the following output feedback controller has been implemented to control the dry process rotary cement kiln (4.49).

$$u_k = \begin{pmatrix} -0.177 & 0.125 \\ 1.84 & 2.09 \end{pmatrix} y_k$$

Four thousand routine operating output data points are collected for time series analysis and the corresponding closed-loop time series model $\hat{G}_{cl}(q^{-1})$ is identified as

$$\hat{G}_{cl}(q^{-1}) = \frac{\begin{pmatrix} 1 - 1.609q^{-1} + 0.6363q^{-2} + 0.0079q^{-3} & 0.1651q^{-1} + 0.05098q^{-2} - 0.1293q^{-3} \\ -0.1132q^{-1} + 0.08767q^{-2} - 0.0067q^{-3} & 1 - 0.4236q^{-1} - 0.524q^{-2} + 0.2173q^{-3} \end{pmatrix}}{1 - 2.025q^{-1} + 1.332q^{-2} - 0.2801q^{-3}}$$

The output variances are calculated from $\hat{G}_{cl}(q^{-1})$ as $var(\hat{y}_k^{(1)}) = 0.0969$, $var(\hat{y}_k^{(2)}) = 0.2123$, both of which are greater than the specified variance upper bounds. In addition, the output variances of $\hat{y}_k^{(1)}$ and $\hat{y}_k^{(2)}$ under minimum variance control are verified as 0.0644 and 0.0214, respectively, by minimizing the sum of output variances according to Theorem 4.4.1.

Consider to find a second order closed-loop response $L_R(q^{-1})$ satisfying the user specified output variance upper bounds, i.e., $var(\hat{y}_k^{*(1)}) \leq 0.0939$, $var(\hat{y}_k^{*(2)}) \leq 0.193$. Solving the problem (4.44) using Algorithm 4.4.2, with $\gamma = 0.87$, we have

$$L_R(q^{-1}) = \frac{\begin{pmatrix} -0.1466 + 0.0383q^{-1} + 0.0038q^{-2} & 1.212 + 0.2015q^{-1} - 0.2592q^{-2} \\ 0.305 - 0.4214q^{-1} + 0.1186q^{-2} & 0.07465 + 0.505q^{-1} - 0.2046q^{-2} \end{pmatrix}}{1 - 1.186q^{-1} + 0.3097q^{-2}} \tag{4.50}$$

and

$$var(\hat{y}_k^{*(1)}) = 0.092, \quad var(\hat{y}_k^{*(2)}) = 0.1877$$

$$\hat{\eta}_1 = 0.9494, \quad \hat{\eta}_2 = 0.8841, \quad \hat{\eta}_o = 0.9046$$

If both $\hat{y}_k^{*(1)}$ and $\hat{y}_k^{*(2)}$ are required to have second order dynamics with the same time constant τ as 5 sampling units and the same damping coefficient ξ as 0.707, then $L_R(q^{-1})$ is solved by the LMI problem (4.46). With $\gamma = 0.905$, we have

$$L_R(q^{-1}) = \frac{\begin{pmatrix} -0.1088 + 0.0835q^{-1} + 0.0083q^{-2} & 1.622 - 1.7q^{-1} + 0.3576q^{-2} \\ 0.1016 - 0.0975q^{-1} - 0.0052q^{-2} & 0.2665 - 0.0078q^{-1} - 0.1507q^{-2} \end{pmatrix}}{1 - 1.719q^{-1} + 0.7537q^{-2}} \tag{4.51}$$

$$var(\hat{y}_k^{*(1)}) = 0.0879, \quad var(\hat{y}_k^{*(2)}) = 0.193$$

$$\hat{\eta}_1 = 0.9064, \quad \hat{\eta}_2 = 0.9091, \quad \hat{\eta}_o = 0.9083$$

It is noted that even though (4.50) and (4.51) are different in time constants and damping coefficients, they are obtained under the same output variance upper bound constraints. Actually, $L_R(q^{-1})$ (4.50) has the resultant $\tau = 1.7969$ and $\xi = 1.0656$. However, for the second $L_R(q^{-1})$, both τ and ξ are specified *a priori* and meet some dynamic characteristic requirements as well. In particular, $L_R(q^{-1})$ of the latter case is obtained via LMIs without iterations.

The impulse responses of the two resultant $L_R(q^{-1})$ and $\hat{R}_{cl}(q^{-1})$ are given in Figure 4.2. It is as expected that the impulse response of the first $L_R(q^{-1})$ is much closer to that of $\hat{R}_{cl}(q^{-1})$ than the second one in the sense of $\mathcal{H}_\infty$ norm. For the first $L_R(q^{-1})$, only the order is specified while for the second $L_R(q^{-1})$, both the damping coefficient and the

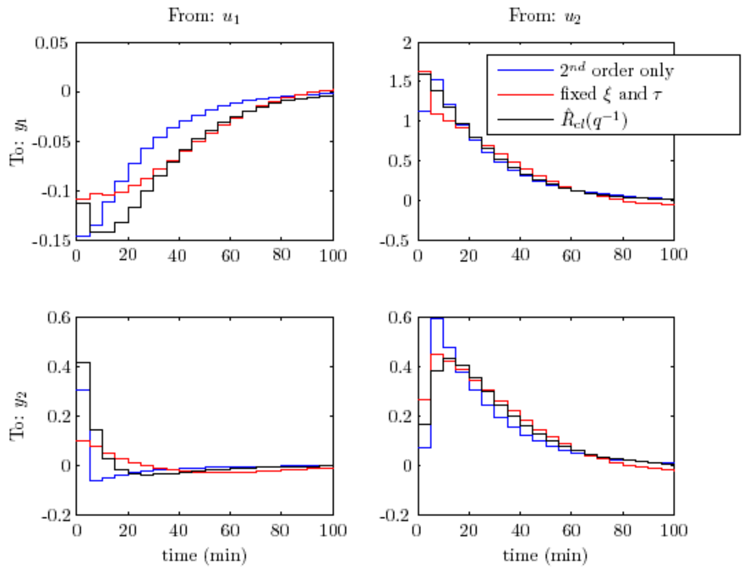

Figure 4.2: Impulse responses of two $L_R(q^{-1})$ and $R_{cl}(q^{-1})$

time constant are fixed *a priori*. Therefore, some degrees of freedom are lost for the second $L_R(q^{-1})$ and the resultant γ value is also larger than that of the first one. This can be readily seen and verified from maximum singular values in the frequency domain (Figure 4.3).

In the following the impact of the benchmark to the time delay mismatch is to be investigated by taking the second case with fixed ξ and τ as an example. The order of the unitary interactor matrix, d, is assumed to be an integer variable varying from 1 to 6. The performance indices and the corresponding γ values are calculated for different interactor orders (a measure of time delay of MIMO systems). The result is given in Table 4.1. The γ values and performance indices versus interactor orders are also plotted in Figure 4.4 and 4.5, respectively. It can be seen that the performance index of the second control loop is not sensitive to the changing of interactor order at all and this happens for the first control loop when $d \geq 2$. Even though there is a jump for the first control loop when d is changing from 1 to 2, the difference is rather small. For the $\mathcal{H}_\infty$ norm gap, its value is smaller than 1 when $d \leq 4$. With approaching of the estimated minimum variance to the specified variance upper bound, this gap reduction is more difficult and the γ value is significantly increased when d is changing from 4 to 6. This verifies our remarks in Section 4.5.3.

4.6.2 A sulphur recovery unit process

A sulphur recovery unit (SRU) process was studied by Olaleye *et al.* (2004*b*) and Xu and Huang (2006) for control loop performance assessment of linear time varying systems.

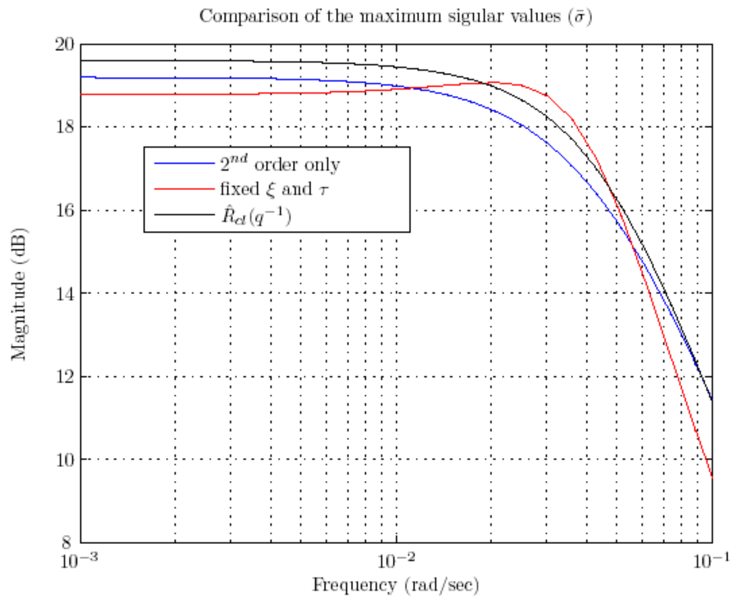

Figure 4.3: Maximum singular values of two $L_R(q^{-1})$ and $R_{cl}(q^{-1})$

Table 4.1: Impact of time delay mismatch

d	γ	$\hat{\eta}_0$	$\hat{\eta}_1$	$\hat{\eta}_2$
1	0.9050	0.9083	0.9064	0.9091
2	0.8778	0.9274	0.9678	0.9090
3	0.8836	0.9272	0.9675	0.9089
4	0.9045	0.9274	0.9678	0.9090
5	1.0035	0.9272	0.9671	0.9089
6	1.2879	0.9272	0.9669	0.9090

In this example, a PID controller is applied to control the difference, $2SO_2 - H_2S$, by manipulating the flowrate of the trim air. This real-time data set inludes 740 data points with three data sections subject to three different disturbances (Figure 4.6) and the time delay is 2 sampling units. The objective of this study is to find a linear time invariant control (benchmark) that can be used to assess performance of the existing control in the presence of time varying disturbance dynamics.

The time series analysis of the process output data for three different sections gives the

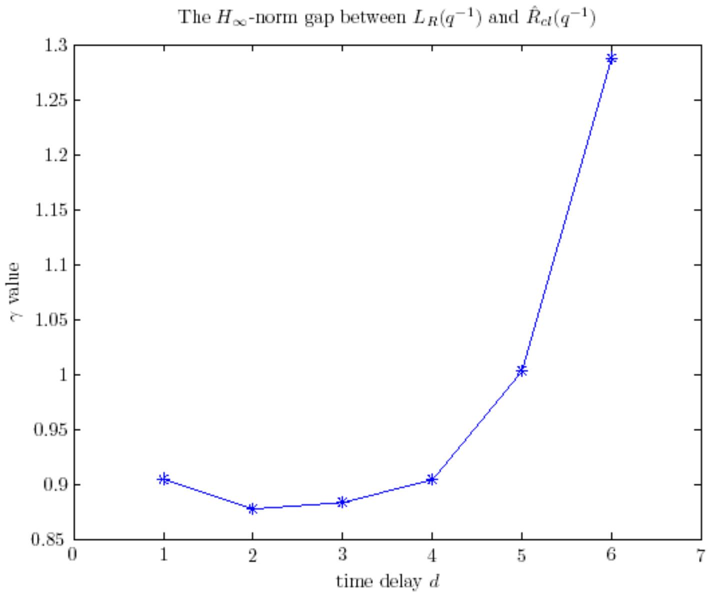

Figure 4.4: H_∞ norm gap between $L_R(q^{-1})$ and $R_{cl}(q^{-1})$ versus time delay

result of three closed-loop models,

$$\hat{G}_{cl}^{(1)}(q^{-1}) = \frac{1 + 1.556q^{-1} + 0.6336q^{-2}}{1 + 1.062q^{-1} + 0.3564q^{-2}}$$
$$\hat{G}_{cl}^{(2)}(q^{-1}) = \frac{1 + 0.06696q^{-1} - 0.6938q^{-2}}{1 - 1.218q^{-1} + 0.2267q^{-2}}$$
$$\hat{G}_{cl}^{(3)}(q^{-1}) = \frac{1 - 0.1859q^{-1} - 0.7604q^{-2}}{1 - 1.044q^{-1} + 0.04666q^{-2}}$$

It is known that the second section was subject to a major disturbance. By Diophantine equation of $\hat{G}_{cl}^{(2)}(q^{-1})$ with process time delay 2, the feedback controller invariant part is calculated as $\hat{F}^{(2)}(q^{-1}) = 1 + 1.2846q^{-1}$, and $\hat{R}_{cl}^{(2)}(q^{-1}) = \frac{0.6438 - 0.2912q^{-1}}{1 - 1.218q^{-1} + 0.2267q^{-2}}$. The output of the second section is required to follow the structured closed-loop response as

$$\hat{y}_k^{*(2)} = \left(\hat{F}^{(2)}(q^{-1}) + q^{-2}L_R(q^{-1})\right) a_k$$

According to Xu and Huang (2006), the closed-loop responses of three sections subject to the user specified response in the second section can be formulated as the following,

$$Y_k = \begin{pmatrix} \hat{y}_k^{*(1)} \\ \hat{y}_k^{*(2)} \\ \hat{y}_k^{*(3)} \end{pmatrix} = \begin{pmatrix} \frac{\hat{G}_{cl}^{(1)}(q^{-1})}{\hat{G}_{cl}^{(2)}(q^{-1})}\hat{F}^{(2)}(q^{-1}) \\ \hat{F}^{(2)}(q^{-1}) \\ \frac{\hat{G}_{cl}^{(3)}(q^{-1})}{\hat{G}_{cl}^{(2)}(q^{-1})}\hat{F}^{(2)}(q^{-1}) \end{pmatrix} a_k + \begin{pmatrix} q^{-2}\frac{\hat{G}_{cl}^{(1)}(q^{-1})}{\hat{G}_{cl}^{(2)}(q^{-1})} \\ q^{-2} \\ q^{-2}\frac{\hat{G}_{cl}^{(3)}(q^{-1})}{\hat{G}_{cl}^{(2)}(q^{-1})} \end{pmatrix} L_R(q^{-1})a_k$$
$$\triangleq \left(\begin{array}{c|c} A_F & G_F \\ \hline C_F & F_F \end{array}\right) a_k + \left(\begin{array}{c|c} A_D & G_D \\ \hline C_D & F_D \end{array}\right) L_R(q^{-1})a_k$$

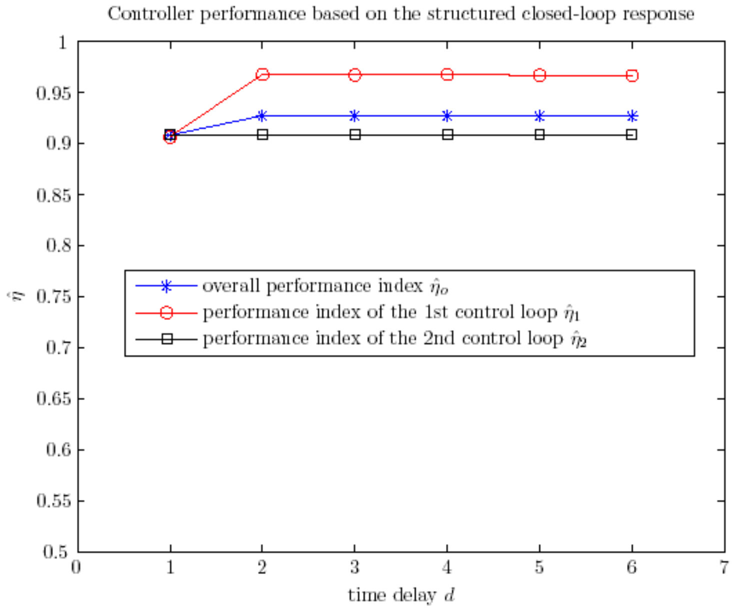

Figure 4.5: Performance indices versus time delay

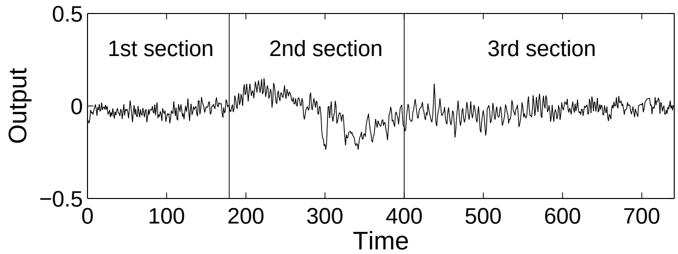

Figure 4.6: Time series plot of process output

If $L_R(q^{-1})$ takes a first order transfer function as (4.37) and let $\lambda = -0.3679$ with $\Omega = 1$, by replacing the user specified output covariance upper bound, Φ_y, which is a constant matrix, with Φ being the decision variable in (4.39), the minimization of the sum of the weighted output variances of the first and third sections can be formulated as

$$\text{Minimize}_{\{\Sigma,\Phi,K_R\}}\ \Phi(1,1) + \Phi(3,3) \text{ subject to } (4.39)$$

The solution of this optimization problem is exactly the same as that achieved in (Xu and Huang, 2006), i.e.,

$$L_R(q^{-1}) = \frac{0.6343 + 0.0802q^{-1}}{1 - 0.3679q^{-1}}$$

and

$$var(\hat{y}_k^{(1)*}) = 1.4388, var(\hat{y}_k^{(2)*}) = 3.1662, var(\hat{y}_k^{(3)*}) = 1.9256$$

$$\hat{\eta}_1 = 1.0933, \quad \hat{\eta}_2 = 0.2695, \quad \hat{\eta}_3 = 0.9052$$

If, however, we add the following variance upper bound constraints on the output of the first and third sections respectively,

$$\Phi(1,1) \leq 1.5, \quad \Phi(3,3) \leq 1.9$$

then we can get the following results,

$$L_R(q^{-1}) = \frac{0.5222 + 0.0994q^{-1}}{1 - 0.3679q^{-1}}$$

and

$$var(\hat{y}_k^{*(1)}) = 1.4962, var(\hat{y}_k^{*(2)}) = 3.0212, var(\hat{y}_k^{*(3)}) = 1.8997$$

$$\hat{\eta}_1 = 1.1369, \quad \hat{\eta}_2 = 0.2561, \quad \hat{\eta}_3 = 0.8930$$

It shows that by adding the output variance upper bound constraints, the structured closed-loop response (benchmark) satisfies all of the user's specifications in all sections simultaneously. Thus, the solution proposed in this chapter shows a great advantage over previous results in the sense that the variance upper bound of each output or each section of the output can be explicitly specified simultaneously.

4.7 Conclusions

The controller performance monitoring problem has been studied from the perspective of the structured closed-loop response subject to the user explicitly specified output variance/covariance upper bound constraint in this chapter. The desired structured closed-loop response can be solved via the approach of linearizing change of variables for a full order formulation or via the Frank and Wolfe algorithm for a reduced order formulation. With closed-loop routine operating data and *a priori* knowledge of process time delay for SISO systems or unitary interactor matrix for MIMO systems, the desired structured closed-loop response can be obtained with the user specified output variance/covariance upper bound constraint satisfied. The gap with the existing closed-loop response is measured by $\mathcal{H}_\infty$ norm. The resultant feasible structured closed-loop response can be served as a practical variance/covariance benchmark against which the existing controller performance can be compared. The controller performance can be evaluated on its variance performance index. The time delay mismatch is also discussed. The case studies illustrate the solution of the structured closed-loop response that can be achieved by a linear time invariant controller.

5 Performance assessment of model predictive control for variability and constraint tuning *

5.1 Introduction

Model predictive control (MPC) has found wide spread industrial applications in the last few decades. It has been proven as one of the most effective advanced process control (APC) strategies to deal with multivariable constraint control problems. There is a large amount of literature on MPC research and applications, including some recent survey articles (Morari and Lee, 1999; Rawlings, 2000; Qin and Badgwell, 2003) and books (Camacho and Bordons, 1998; Maciejowski, 2002). However, less effort has been made on the performance assessment of the existing MPC applications.

Since MPC technology is multivariate by nature, in this sense minimum variance control (MVC) benchmark as a theoretical lower bound of variance for multivariate controller performance assessment (Harris *et al.*, 1996; Huang *et al.*, 1997*a*; Ko and Edgar, 2001*b*; Ko and Edgar, 2001*a*) may be applied to evaluate the performance of MPC applications. The MVC benchmark has been proven useful for controller performance assessment in practice (Qin, 1998; Huang and Shah, 1999; Harris and Seppala, 2001)

*This chapter has been published in Xu, F., Huang, B. and Akande, S., Performance assessment of model pedictive control for variability and constraint tuning, Ind. Eng. Chem. Res., 46, 1208-1219, 2007 since Harris' earlier work (Harris, 1989). The significance of the MVC benchmark lies in the fact that it does give a theoretical absolute lower bound on variance. It has been applied in process industry to evaluate the performance of MPC applications (Gao *et al.*, 2003; Schäfer and Cinar, 2004; Yang *et al.*, 2004). Besides the MVC benchmark, some alternative benchmarks have also been proposed, such as linear quadratic Gaussian (LQG) benchmark (Huang and Shah, 1999), user specified benchmark (Kozub, 1996), model validation (Huang *et al.*, 2003*a*), designed versus achieved controller performance (Patwardhan *et al.*, 2002; Schäfer and Cinar, 2004), Normalized Multivariate Impulse Response (NMIR) curve (Huang and Shah, 1999), NMIR_{wof} curve (Shah *et al.*, 2001), tree mapping visualization (Shah *et al.*, 2005), and so on. These alternative benchmarks are more practical than the MVC benchmark to some extent. However, two key properties of the MPC performance have not been considered in these methods. They are namely economic performance and constraint handling.

According to Prett and Garcia(Prett and Garcia, 1988), modern decision making process has different technology layers relating to control, optimization and logistics, respectively. Logistics is the high level scheduling to respond to external market changes for profit maximization. Optimization is the manipulation of process degrees of freedom for the satisfaction of plant economic objectives. Control is the manipulation of process degrees of freedom for the satisfaction of operating criteria. It is expected that most profit improvement comes from the optimization layer where the economic target is received from the logistics layer and the optimal operating target is solved and sent to the control system for realization. The likely economic benefit that depends on the structure of the on-line optimizer could be estimated by the method of the average deviation from optimum (de Hennin *et al.*, 1994; Loeblein and Perkins, 1998). A steady state back off from the active constraints can be calculated for every structure with the consideration of the measurement error, the parametric uncertainty and the structural modeling error. However, the profit improvement will never be obtained without collaborative integration of logistics, optimization and control. A dynamic back off could be estimated due to the likely disturbances and uncertain parameters (Lear *et al.*, 1995; Figueroa *et al.*, 1996). This leads directly to the studies of the back off approach to simultaneous design and control (Loeblein and Perkins, 1999; Seferlis and Grievink, 2001; Kookos and Perkins, 2004). The essence of back off approach is a move that is required in the operating point away from the nominal optimal one, which is usually located on the constraint limit, in order to maintain the feasibility and operability, due to the likely disturbances and uncertainty.

The idea of back off approach can be applied into the economic performance evaluation of the run-time applications (Zhou and Forbes, 2003). The steady state economic benefit comes from operating mean shift (Muske, 2003). Just reducing the variability does not always credit to the economic benefit, but it does result in some hidden intangible benefits (Latour, 1992), such as improved product quality consistency (Shunta, 1995). It is claimed that the variance can typically be reduced by at least 50% due to the advance process control (Latour *et al.*, 1986). Since the base case operation is to be compared for the economic benefit quantification, it should be a period of typical closed-loop operation with the existing control system. The target optimal steady state operation condition can be calculated by specifying a reasonable back off away from the constraint limit. It is obvious that the target operation is too conservative if the constraint limit is never violated (Latour, 1992). There are many different rules in the literature for the allowable constraint limit violation (Latour *et al.*, 1986; Martin *et al.*, 1991; Muske and Finegan, 2001). A reasonable rule should be applied in terms of base case condition and desired specifications. In the case of excessive violation of base case condition, the same limit rule can be used to set a reasonable percentage of violation, such as 5%. If the base case violation is acceptable, the same percentage rule is suggested. Once the base case operation and the optimal operation condition are both identified, the economic benefit potential is readily obtained when the economic objective function is explicitly established (Anderson, 1996).

For the latest generation of MPC products, MPC application itself can be further divided into a steady state optimization driven by economics and a dynamic optimization (Kassmann *et al.*, 2000). The separate steady state optimization is performed at each control cycle in order to drive steady state inputs and outputs as closely as possible to their optimal economic targets (Qin and Badgwell, 2003). For example, Dynamic Matrix Control (DMC) integrates a linear programming (LP) for the optimum economic steady state (Sorensen and Cutler, 1998), and Robust Model Predictive Control Technology (RMPCT) also includes a quadratic programming (QP) for profit optimization (Krishnan *et al.*, 1998). Since this LP or QP reflects economic objective explicitly, it can be utilized to evaluate the economic performance of MPC applications.

This chapter aims at multivariate control performance assessment and tuning of model predictive control by considering the relationship between economic performance and process variance. The contributions of this chapter can be summarized as: (1) A systematic approach is proposed to evaluate the economic performance of existing run-time MPC applications; (2) The MVC benchmark is combined directly with benefit potential, which leads to an optimal benefit potential that is achieved by MVC; (3) Variability and constraint tuning guidelines are devised for better tuning settings of MPC applications.

The remainder of this chapter is organized as follows. In Section 5.2 several different scenarios for MPC performance assessment are described in the form of constrained quadratic optimization problems. Section 5.3 presents and explains a systematic approach for MPC economic performance assessment. The minimum energy control is introduced and the QP problem in the steady state optimization is formulated as LMIs in Section 5.4. A simulation example of shell control problem and a pilot-scale experiment are provided to demonstrate the feasibility of the proposed approach in Section 5.5, followed by concluding remarks in Section 5.6.

5.2 Problem description

5.2.1 Benefit potential analysis

For illustration, assume a multivariable process that consists of two manipulated variables (MVs) and two controlled variables (CVs), where y_1 (i.e., CV_1) is a quality variable that has direct impact on profit and y_2 (i.e., CV_2) is a constrained variable. The base case operation is shown in Figure 5.1. Because of the disturbances, there is variability on both y_1 and y_2.

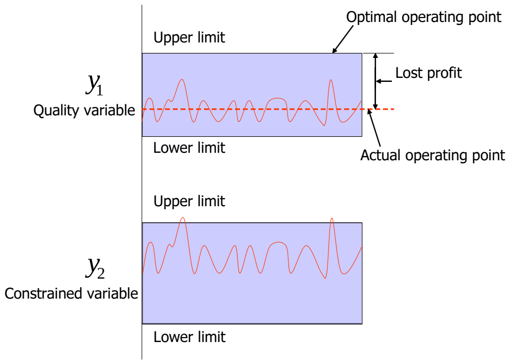

Figure 5.1: Base case operation

Assume that the optimal operating point of y_1 is located on its upper limit; thus the actual operating point (dash line) is far away from its optimal operating point, which means lost profit. The purpose of this study is to assess the possibility to move its operating point as close as possible to its optimal operating point. In this chapter, the base case operation is characterized by its current mean values and variances. The optimal operation condition is obtained by solving the economic steady state optimization problem subject to the current constraint limit settings and process variability. A reasonable percentage of constraint limit violation, 5%, is allowed such that 95% of operation falls within the range of ± 2 times standard deviation (Latour *et al.*, 1986; Martin *et al.*, 1991). Economic benefit potential can be investigated by comparing optimal operation with base case operation. Since all the scenarios are compared against the same base case operation, in the following we list the problem formulations of optimal operations for different scenarios relative to the base case operation.

For a general $p \times m$ process G with steady state gain matrix K, controlled by an MPC controller, it is assumed that $\{\bar{y}_{i0}, \bar{u}_{j0}\}$ is the current operating point and $\{\bar{y}_i, \bar{u}_j\}$ is the optimal operating point for the i-th output and the j-th input, respectively. With a real-time input/output data collection of length N_L, the quadratic economic objective function may be formulated as follows,

$$J = \frac{1}{N_L} \sum_{k=1}^{N_L} J_k \tag{5.1}$$

where

$$J_k = \sum_{i=1}^{p} \left[b_{ki} \times \bar{y}_i + a_{ki}^2 (\bar{y}_i - y_{dki})^2 \right] + \sum_{j=1}^{m} \left[b_{kj} \times \bar{u}_j + a_{kj}^2 (\bar{u}_j - u_{dkj})^2 \right]$$

and a_{ki} (resp. a_{kj}) and b_{ki} (resp. b_{kj}) are the quadratic and linear coefficients respectively for the controlled variables (resp. the manipulated variables), y_{dki} and u_{dkj} are the corresponding target values.

Consider that the current operating point at $[\bar{y}_{i0}, \bar{u}_{j0}]$. Moving the current operating point by $[\Delta\bar{y}_i, \Delta\bar{u}_j]$, the new optimal operating point is $[\bar{y}_i, \bar{u}_j]$. Then $[\Delta\bar{y}_i, \Delta\bar{u}_j]$ must satisfy the steady state gain function described by the following equations:

$$\begin{aligned} &\sum_{j=1}^{m} [K_{ij} \times \Delta\bar{u}_j] = \Delta\bar{y}_i, i = 1, \cdots, p \\ &\bar{y}_i = \bar{y}_{i0} + \Delta\bar{y}_i, \bar{y}_{i0} = \frac{\sum_{k=1}^{N_L} y_{ki0}}{N_L}, i = 1, \cdots, p \\ &\bar{u}_j = \bar{u}_{j0} + \Delta\bar{u}_j, \bar{u}_{j0} = \frac{\sum_{k=1}^{N_L} u_{kj0}}{N_L}, j = 1, \cdots, m \end{aligned} \tag{5.2}$$

Obviously the new operating point $[\bar{y}_i, \bar{u}_j]$ cannot be arbitrary. Considering allowable 5% violation of constraints, the following inequalities must be satisfied:

$$Y_{Lki} - s_{yi} \times Y_{holki} + 2 \times Y_{stdi0}(1 + r_{yi}) \leq \bar{y}_i \leq Y_{Hki} + s_{yi} \times Y_{holki} - 2 \times Y_{stdi0}(1 + r_{yi}) \tag{5.3}$$

$$U_{Lkj} - s_{uj} \times U_{holkj} + 2 \times U_{stdj} \leq \bar{u}_j \leq U_{Hkj} + s_{uj} \times U_{holkj} - 2 \times U_{stdj} \tag{5.4}$$

where $i = 1, \cdots, p$ and $j = 1, \cdots, m$.

In the above two inequalities, Y_{Lki} (resp. U_{Lkj}) and Y_{Hki} (resp. U_{Hkj}) are the low limit and high limit of CV_i (resp. MV_j), Y_{holki} (resp. U_{holkj}) is the half of the constraint range of CV_i (resp. MV_j), Y_{stdi0} is the standard deviation of CV_i which is calculated from routine operating data of current operation, s_{yi} (resp. s_{uj}) is the percentage of constraint limit change of CV_i (resp. MV_j), r_{yi} is the percentage of variability change of CV_i, and U_{stdj} is the standard deviation of MV_j. In this representation, Y_{Lki}, Y_{Hki}, U_{Lkj} and U_{Hkj} are collected at time stamp k.

The actual constraint limits of both CVs and MVs are to be adjusted by the percentages of constraint limit changes s_{yi} and s_{uj}, respectively. The magnitude of constraint limit adjustment depends on s_{yi} and Y_{holki} for the controlled variables (resp. s_{uj} and U_{holkj} for the manipulated variables). Positive values of s_{yi} and s_{uj} lead to relaxation of the constraint limits with increased operating ranges while negative ones result in reduction of the constraint limits with decreased operating ranges. In addition, the size of the back off due to disturbances is described by the standard deviations. For the controlled variables in (5.3), the standard deviations are set to be adjusted by the percentage of variability change r_{yi}. This is due to the fact that the variability of the controlled variables may be further reduced by tuning the cascaded regulatory control loops. For the manipulated variables in (5.4), however, the standard deviations should be calculated according to the controller transfer function. The calculation of the standard deviation U_{stdj} in (5.4) will be discussed in detail in the next section.

Now the economic optimization problem for the benefit potential analysis of different scenarios can be combined into the following form,

$$\text{Minimize}_{\{\bar{y}_i,\bar{u}_j\}} \; J \text{ subject to } (5.2), (5.3) \text{ and } (5.4) \tag{5.5}$$

For the base case operation, the economic objective function value is readily calculated by replacing $\{\bar{y}_i, \bar{u}_j\}$ with the current operating point $\{\bar{y}_{i0}, \bar{u}_{j0}\}$ in (5.1). This objective function value is denoted as J_0, which is a value to be compared in the calculation of benefit potentials. In the sequel, we will discuss and explain benefit potential calculations under different scenarios.

- **Ideal operation scenario**: $r_{yi} = -1, s_{yi} = 0, s_{uj} = 0$
  In this scenario, the disturbance influence is not considered and a strict steady state operation is assumed. There is no back off due to the disturbances and the constraint limits are kept unchanged. To illustrate this scenario, consider again the two-output example. For the system with outputs shown in Figure 5.1, the trajectories of y_1 and y_2 are now two straight lines in this scenario as shown in Figure 5.2. Therefore, it is

possible to move the operating line of the quality variable y_1 to its optimal operating upper bound, only if the steady state operation condition (5.2) is satisfied and there

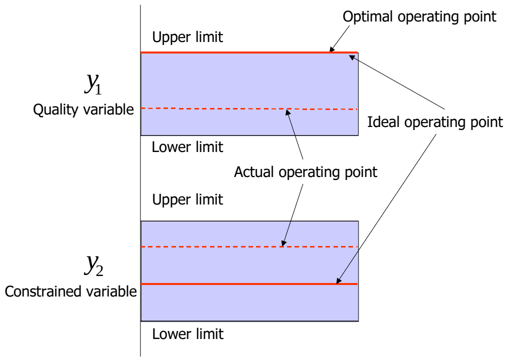

Figure 5.2: Optimal operation under ideal scenario

is no constraint violation of y_2 and all manipulated variables. The solution of (5.5) gives rise to an ideal optimal operating condition $\{\bar{y}_{Ii}, \bar{u}_{Ij}\}$ and the corresponding objective function is denoted as J_I. Then *the ideal benefit potential* ΔJ_I can be calculated simply by

$$\Delta J_I = J_I - J_0 \tag{5.6}$$

- **Existing variability scenario**: $r_{yi} = 0, s_{yi} = 0, s_{uj} = 0$
  In this scenario, the present level of disturbance is taken into account. The purpose is to move the operating point as close as possible to the ideal optimum.

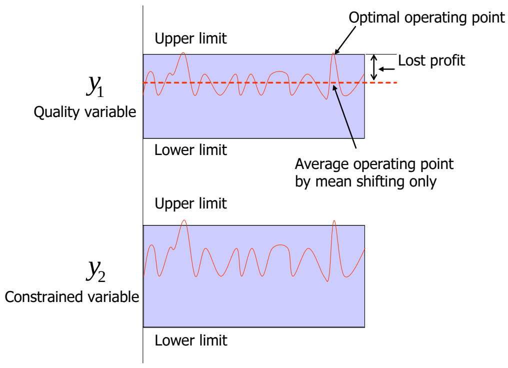

Figure 5.3: Optimal operation by mean shifting only

For the base case operation in Figure 5.1, the actual operating point of y_1 can be moved up closer to its ideal optimal operating point with the consideration of back off caused by the present level of disturbance as shown in Figure 5.3. By shifting mean only, the distance between the average operating point and the ideal operating point could be reduced, which means increased profit. The resultant optimal operating point is denoted as $\{\bar{y}_{Ei}, \bar{u}_{Ej}\}$ and the corresponding objective function as J_E. *The existing benefit potential* ΔJ_E can be calculated as the following,

$$\Delta J_E = J_E - J_0 \tag{5.7}$$

- **Reducing variability scenario**: $r_{yi} = R_{yi}, s_{yi} = 0, s_{uj} = 0$
  In the existing operation scenario, no action is taken to reduce the variability of the controlled variables and the existing benefit potential is obtained by shifting mean values only. If the variability of one or some of the controlled variables can be reduced to certain percentage by tuning the cascaded regulatory control loops, then the back off can also be reduced, which will allow further mean values shifting in the direction of ideal operating point. If a significant increase of the benefit potential is observed by variability reduction on one or more of the controlled variables, it is worth to tune the corresponding regulatory control loops. The variability reduction percentage is denoted as R_{yi} and can be assigned a negative value but should never be smaller than -1 by nature, i.e., $-1 \leq R_{yi} \leq 0$. Considering the case in Figure 5.3, if the variability of the quality variable y_1 is to be reduced, its mean value can be moved closer to the ideal optimal operating point as indicated in Figure 5.4 and thus gives rise to increased benefit potential. If the optimal operating point by variability reduction is denoted as $\{\bar{y}_{Vi}, \bar{u}_{Vj}\}$ with the corresponding objective function as J_V, then *the optimal benefit potential by reducing variability*, ΔJ_V, can be obtained by

$$\Delta J_V = J_V - J_0 \tag{5.8}$$

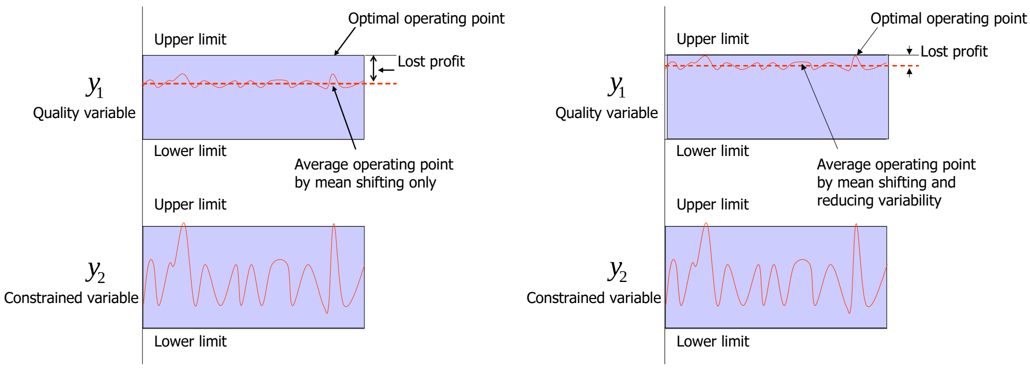

Figure 5.4: Optimal operation by mean shifting and variability reduction

It is noted that in general the reduced variability of one variable transfers to increased variability of other variables. The variability of the quality variables is of main concern since they have direct impact on profit, and their variability could be considered to be reduced by transferring to the constrained variables. For the constrained variables, their variability is typically not concerned as long as they stay within their constraint limits.

- **Relaxing constraint scenario**: $r_{yi} = 0, s_{yi} = S_{yi}, s_{uj} = S_{uj}$
  The constraint limits of both controlled and manipulated variables are usually determined in the design phase when an MPC application is commissioned, but some of them may be allowed to adjust afterward. This leaves an alternative way to improve the MPC economic performance. For the quality variables, the constraint relaxation may allow their mean values to be moved closer to the ideal optimal operating point even if there is no reduction of variability as indicated in Figure 5.5. For the constrained variables, however, the increase on their operating ranges will

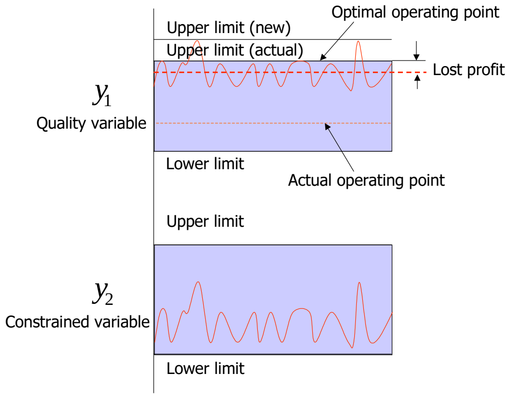

Figure 5.5: Optimal operation by relaxing constraint

allow more variability to be transferred from the quality variables to them, which may create a room for the quality variables to reduce their variability. In Figure 5.4, for example, if a larger magnitude of variability is allowed with the constraint relaxation on y_2, then it is possible to move y_1 closer to its optimum. The constraint percentage change is denoted as S_{yi} for the controlled variables and S_{uj} for the manipulated variables. S_{yi} and S_{uj} should usually be assigned positive values in order to increase the profit. If the optimal result due to the constraint relaxation is represented by $\{\bar{y}_{Ci}, \bar{u}_{Cj}\}$ and the corresponding objective function is J_C, then *the optimal benefit* *potential by relaxing constraint*, ΔJ_C, is given by

$$\Delta J_C = J_C - J_0 \tag{5.9}$$

- **Simultaneous reducing variability and relaxing constraint scenario**: $r_{yi} = R_{yi}, s_{yi} = S_{yi}, s_{uj} = S_{uj}$
If both variability and constraint relaxation are considered simultaneously, additional benefit potential may be achieved. Different combinations of R_{yi}, S_{yi} and S_{uj} may result in different values on the objective function and also different benefit potentials. For a given set of R_{yi}, S_{yi} and S_{uj}, we denote the optimal operating point as $\{\bar{y}_{Bi}, \bar{u}_{Bj}\}$ and the corresponding objective function as J_B. As a consequence, *the optimal benefit potential by reducing variability and relaxing constraint simultaneously*, ΔJ_B, is obtained by

$$\Delta J_B = J_B - J_0 \tag{5.10}$$

Notice that the benefit potentials ΔJ_V, ΔJ_C and ΔJ_B are all relative to the present operating condition, but include the benefit potential of existing variability scenario, ΔJ_E. Therefore, the contribution of any change that is due to variability reduction or constraint relaxation should be obtained by subtracting *the existing benefit potential*, ΔJ_E.

5.2.2 Tuning guidelines to achieve the target benefit potential

The ideal benefit potential ΔJ_I can be achieved only when the variability of all variables can be reduced to zero, which is rarely possible in practice. Therefore, by variability reduction, ΔJ_I can not be reached in practice. However, it is the maximum benefit potential by tuning the variability, against which *the target benefit potential* could be established by assigning an appropriate percentage. The target benefit potential can be achieved by either tuning variability or tuning constraint limits. The variability and the constraint tuning problems are formulated in the following.

- **Variability tuning problem**: $s_{yi} = s_{uj} = 0$
In the benefit potential analysis, the percentages of variability change r_{yi} are given by the user as R_{yi}. If, instead, we use them as decision variables, the optimal r_{yi} can be found from the optimization problem accordingly for a given target benefit potential. A target benefit potential ratio by tuning variability, R_V, is defined as the ratio between the targeted benefit potential and the ideal benefit potential, where R_V

should be within 0 and 1. With a given R_V, r_{yi} may be calculated but the solutions may not be unique. To minimize tuning effort, we would want variability adjustment percentages r_{yi} as small as possible. If we define the maximum variability adjustment percentage of r_{yi} across all variables as r, then the optimal variability adjustment percentages r_{yi}, in the sense of minimizing the maximum variability adjustment percentage r, may be found through the optimization of the following problem:

$$\text{Minimize}_{\{\bar{y}_i, \bar{u}_j, r_{yi}, r, u_{stdj}\}} \ -r$$

subject to

$$\begin{aligned}
& r_{yi} > r > -1 \\
& J - J_0 = R_V \times \Delta J_I \\
& s_{yi} = s_{uj} = 0 \\
& \text{Equalities } (5.2) \\
& \text{Inequalities } (5.3) \text{ and } (5.4) \\
& \text{Input-output relation through an optimal control (see Section 5.3.1)}
\end{aligned} \tag{5.11}$$

It is noted that in the above formulation, the standard deviations of the manipulated variables in the constraint inequalities (5.4) are set as decision variables, u_{stdj}, rather than constant values, U_{stdj}. This is because the variabilities of the controlled variables are related to the variabilities of the manipulated variables through the controller. This controller should be an optimal controller as a benchmark. Thus the standard deviations of the manipulated variables should also be decision variables of a controller design problem. This means both the standard deviations of the controlled variables and the standard deviations of the manipulated variables should be determined simultaneously within two optimization problems. This will be discussed in detail in the next section.

- **Constraint tuning problem**: $r_{yi} = 0$

  If the variability could not be reduced further, we may also achieve the target benefit potential by tuning the constraint limits. Similarly, a target benefit potential ratio by tuning constraint, R_C, is defined as the target benefit potential against the ideal benefit potential. We would also want the percentages of the constraint change, s_{yi} and s_{uj}, to be as small as possible. If we define the maximum constraint change percentage of s_{yi} and s_{uj} across all variables as s, then the optimal constraint tuning percentages s_{yi} and s_{uj}, in the sense of minimizing the maximum tuning percentage, can be solved through

$$\text{Minimize}_{\{\bar{y}_i,\bar{u}_j,s_{yi},s_{uj},s\}}\ s$$

subject to

$$\begin{aligned}
&s \geq s_{yi} \geq 0\\
&s \geq s_{uj} \geq 0\\
&J - J_0 = R_C \times \Delta J_I\\
&r_{yi} = 0\\
&\text{Equalities } (5.2)\\
&\text{Inequalities } (5.3) \text{ and } (5.4)
\end{aligned} \tag{5.12}$$

5.3 Solution to the optimization problems

5.3.1 Calculation of standard deviations of the input variables due to the variability change of the output variables

In this section, we consider the following problem: given the output variance upper bound for each outputs, what are the minimum inputs variance? The objective is to identify a benchmark control so that the increase of input variance is minimized when the output variance is reduced.

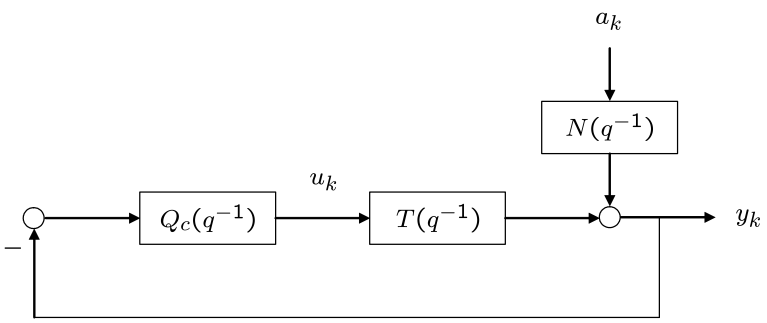

Figure 5.6: General feedback control framework

Consider a discrete linear system

$$y_k = T(q^{-1})u_k + N(q^{-1})a_k \tag{5.13}$$

where $T(q^{-1})$ is the process model and $N(q^{-1})$ is the disturbance model, as shown in Figure 5.6. For a typical industrial MPC application, the process model $T(q^{-1})$ is known, but the disturbance model $N(q^{-1})$ is usually unknown and needs to be estimated.

The state space realization of system (5.13) is given by:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ga_k \\ y_k &= Cx_k + Fa_k \end{aligned} \tag{5.14}$$

where $x_k \in \mathbf{R}^n$ is the state of the system, $u_k \in \mathbf{R}^m$ the control signal, $y_k \in \mathbf{R}^p$ the measured output, and $a_k \in \mathbf{R}^q (q = p)$ the external disturbance that is a zero mean white noise sequence satisfying:

$$E(a_k) = 0, \quad E(a_k a_k^T) = \Omega$$

where, $\Omega \succ 0$, is the covariance matrix of a_k.

Assume that the system (5.13) or (5.14) is controlled by a full order dynamic output feedback controller $Q_c(q^{-1})$, which is described as

$$\begin{aligned} x_{k+1}^C &= A_c x_k^C + B_c y_k \\ u_k &= C_c x_k^C + D_c y_k \end{aligned} \tag{5.15}$$

where $x_k^C \in \mathbf{R}^{n_c}$ is the state of the controller and n_c is a pre-assigned order of the controller. For a full order controller, n_c is equal to n. Then we have the following theorem.

Theorem 5.3.1 *The minimum input variance (energy) control satisfying output variance constraint via a full order dynamic output feedback control law (5.15) can be solved by the following semi-definite programming problem:*

$$\begin{aligned} &\text{Minimize}_{\{\Sigma_1, M_1, \hat{A}_c, \hat{B}_c, \hat{C}_c, \hat{D}_c, \Phi_y, \Phi_u\}} \ \mathbf{trace}(\Phi_u) \\ &\textit{subject to} \end{aligned} \tag{5.16}$$

$$\begin{pmatrix} -\Sigma_1 & -I & A + B\hat{D}_c C & A\Sigma_1 + B\hat{C}_c & G + B\hat{D}_c F \\ -I & -M_1 & M_1 A + \hat{B}_c C & \hat{A}_c & M_1 G + \hat{B}_c F \\ A^T + C^T \hat{D}_c^T B^T & A^T M_1 + C^T \hat{B}_c^T & -M_1 & -I & 0 \\ \Sigma_1 A^T + \hat{C}_c^T B^T & \hat{A}_c^T & -I & -\Sigma_1 & 0 \\ G^T + C^T \hat{D}_c^T B^T & G^T M_1 + F^T \hat{B}_c^T & 0 & 0 & -\Omega^{-1} \end{pmatrix} \prec 0 \tag{5.17}$$

$$\begin{pmatrix} \Phi_y & C & C\Sigma_1 & F \\ C^T & M_1 & I & 0 \\ \Sigma_1 C^T & I & \Sigma_1 & 0 \\ F^T & 0 & 0 & \Omega^{-1} \end{pmatrix} \succ 0 \tag{5.18}$$

$$\begin{pmatrix} \Phi_u & \hat{D}_c C & \hat{C}_c & \hat{D}_c F \\ * & M_1 & I & 0 \\ * & * & \Sigma_1 & 0 \\ * & * & * & \Omega^{-1} \end{pmatrix} \succ 0 \tag{5.19}$$

$$\Phi_y(i,i) \leq \sigma_{Byi}^2, i = 1, \cdots, p \tag{5.20}$$

where Φ_y is the output covariance matrix and Φ_u is the input covariance matrix, both of them are decision variables, and σ_{Byi}^2 is the pre-specified i-th output variance upper bound. If there exists a feasible solution, a full order dynamic output feedback controller (5.15) can be parameterized as:

$$\begin{cases} D_c = \hat{D}_c \\ C_c = (\hat{C}_c - \hat{D}_c C \Sigma_1)\Sigma_2^{-T} \\ B_c = M_2^{-1}(\hat{B}_c - M_1 B \hat{D}_c) \\ A_c = M_2^{-1}(\hat{A}_c - M_1 A \Sigma_1 - M_1 B D_c C \Sigma_1 - M_2 B_c C \Sigma_1 - M_1 B C_c \Sigma_2^T)\Sigma_2^{-T} \end{cases} \tag{5.21}$$

where $M_2 \in \mathbf{R}^{n\times n}$ and $\Sigma_2 \in \mathbf{R}^{n\times n}$ are any matrices satisfying

$$\Sigma_2 M_2^T = I - \Sigma_1 M_1 \tag{5.22}$$

Proof: The proof can be found in Huang(Huang, 2003) and Scherer *et al.*(Scherer *et al.*, 1997), and is omitted here.

□

With Theorem 5.3.1, the optimal standard deviations of the manipulated variables can be obtained via minimum energy control for the variability reduction related optimization problems. When the variability reduction percentages of the controlled variables, r_{yi}, are specified *a priori* as R_{yi}, i.e., $\sigma_{Byi} = (1 + R_{yi})Y_{stdi0}$, then the standard deviations of the manipulated variables, u_{stdj}, can be calculated according to Theorem 5.3.1, and the optimal benefit potential can be obtained thereafter by solving the optimization problem (5.5). This two-step procedure can be applied to solve the problems for two scenarios which are reducing variability scenario, and simultaneous reducing variability and relaxing constraint scenario, respectively. For the variability tuning problem (5.11), the variability tuning guideline can not be obtained via this simple sequential two-step procedure since both the variability reduction percentages, r_{yi}, and the standard deviations of the manipulated variables, u_{stdj}, are decision variables and should be solved simultaneously. However, this problem can be solved by several iterations of this two-step procedure until the constraints in (5.11) and constraints (5.17), (5.18), (5.19) and (5.20) are satisfied simultaneously. The corresponding algorithm for the tuning variability problem is given below:

Algorithm 5.3.1 *For a discrete linear system (5.14), the variability tuning problem can be solved by the following steps:*

(1) *Set* $U_{stdj}^{(0)} = U_{stdj0}, j = 1, \cdots, m$ *and* $k = 0$*, where* U_{stdj0} *is the current standard deviation of the* j*-th manipulated variable.*

(2) *Solve the variability tuning problem (5.11) with* u_{stdj} *replaced by* $U_{stdj}^{(k)}$ *in the constraint inequalities (5.4).*

(3) *Solve the minimum energy control problem (5.16) and the solution gives the covariance matrix* Φ_u*.*

(4) *If* $max(|U_{stdj}^{(k)} - \sqrt{\Phi_u(j,j)}|) < \epsilon$*, where* $j = 1, 2, ..., m$*, and* ϵ *is a pre-determined tolerance, is satisfied, then exit. Otherwise, set* $k = k + 1$ *and* $U_{stdj}^{(k)} = \sqrt{\Phi_u(j,j)}$*, go to step 2.*

5.3.2 Reformulation of the economic objective function

In Section 5.2, five problems for benefit potential analysis and two tuning problems for variability and constraint have been presented. All of the these optimization problems can be formulated as LMIs and thus solved efficiently. The key to these problems is to transform the quadratic term in the economic objective function (5.1) into LMI. In the following we will show this procedure.

It can be seen that the minimization of the objective function (5.1) can be further formulated as

$$\begin{array}{c} \text{Minimize}_{\{\bar{y}_i, \bar{u}_j\}} \ \gamma \\ \text{subject to} \end{array} \tag{5.23}$$

$$\sum_{k=1}^{N_L} \left\{ \sum_{i=1}^{p} \left[b_{ki} \times \bar{y}_i + a_{ki}^2 (\bar{y}_i - y_{dki})^2 \right] + \sum_{j=1}^{m} \left[b_{kj} \times \bar{u}_j + a_{kj}^2 (\bar{u}_j - u_{dkj})^2 \right] \right\} < \gamma \tag{5.24}$$

Apparently, the constraint is quadratic and nonlinear. For the first term on the controlled variables, we can further derive as follows.

$$\begin{aligned} &\sum_{k=1}^{N_L} \left\{ \sum_{i=1}^{p} \left[b_{ki} \times \bar{y}_i + a_{ki}^2 (\bar{y}_i - y_{dki})^2 \right] \right\} \\ =& \sum_{i=1}^{p} \left\{ \left(\sum_{k=1}^{N_L} b_{ki} \right) \bar{y}_i + \sum_{k=1}^{N_L} \left[a_{ki}^2 (\bar{y}_i - y_{dki})^2 \right] \right\} \\ =& \sum_{i=1}^{p} \left\{ \left(\sum_{k=1}^{N_L} b_{ki} \right) \bar{y}_i + \left(\sum_{k=1}^{N_L} a_{ki}^2 \right) \bar{y}_i^2 - \left(2 \sum_{k=1}^{N_L} \left(a_{ki}^2 y_{dki} \right) \right) \bar{y}_i + \sum_{k=1}^{N_L} \left(a_{ki}^2 y_{dki}^2 \right) \right\} \end{aligned}$$

$$= Y_{lin} + \sum_{i=1}^{p} \left[\left(\sqrt{\sum_{k=1}^{N_L} a_{ki}^2} \right) \bar{y}_i \right]^2$$

where

$$Y_{lin} = \sum_{i=1}^{p} \left\{ \sum_{k=1}^{N_L} \left(b_{ki} - 2a_{ki}^2 y_{dki} \right) \bar{y}_i \right\} + \sum_{i=1}^{p} \left\{ \sum_{k=1}^{N_L} \left(a_{ki}^2 y_{dki}^2 \right) \right\}$$

Similarly, for the second term of the manipulated variables, we have the following result:

$$\sum_{k=1}^{N_L} \left\{ \sum_{j=1}^{m} \left[b_{kj} \times \bar{u}_j + a_{kj}^2 (\bar{u}_j - u_{dkj})^2 \right] \right\} = U_{lin} + \sum_{j=1}^{m} \left[\left(\sqrt{\sum_{k=1}^{N_L} a_{kj}^2} \right) \bar{u}_j \right]^2$$

where

$$U_{lin} = \sum_{j=1}^{m} \left\{ \sum_{k=1}^{N_L} \left(b_{kj} - 2a_{kj}^2 y_{dkj} \right) \bar{u}_j \right\} + \sum_{j=1}^{m} \left\{ \sum_{k=1}^{N_L} \left(a_{kj}^2 y_{dkj}^2 \right) \right\}$$

Then the constraint (5.24) becomes

$$\gamma - Y_{lin} - U_{lin} - \sum_{i=1}^{p} \left[\left(\sqrt{\sum_{k=1}^{N_L} a_{ki}^2} \right) \bar{y}_i \right]^2 - \sum_{j=1}^{p} \left[\left(\sqrt{\sum_{k=1}^{N_L} a_{kj}^2} \right) \bar{u}_j \right]^2 > 0$$

According to *Schur complement* , the corresponding LMI can be obtained as the following.

$$\begin{pmatrix} \gamma - Y_{lin} - U_{lin} & X_{lin}^T \\ X_{lin} & I \end{pmatrix} \succ 0 \tag{5.25}$$

where $X_{lin} =$

$$\left[\left(\sqrt{\sum_{k=1}^{N_L} a_{k1}^2} \right) \bar{y}_1 \quad \cdots \quad \left(\sqrt{\sum_{k=1}^{N_L} a_{kp}^2} \right) \bar{y}_p \quad \left(\sqrt{\sum_{k=1}^{N_L} a_{k1}^2} \right) \bar{u}_1 \quad \cdots \quad \left(\sqrt{\sum_{k=1}^{N_L} a_{km}^2} \right) \bar{u}_m \right]^T$$

In addition, all the constraints are linear. Therefore, we can conveniently solve these problems via LMI toolboxes, such as SeDuMi (Sturm, 1998-2001) together with YALMIP interface (Löfberg, 2004). Actually, since the objective functions are all quadratic for the original problems, we can also adopt some quadratic programming approaches, such as *quadprog* function in Matlab optimization toolbox (Mathworks, 1990 - 2005).

5.4 A systematic approach

The economic performance assessment of MPC applications includes economic performance assessment, sensitivity analysis and tuning guidelines, which is named as a systematic approach in this section. Two MPC economic performance indices are proposed to reflect the benefit potential of any current run-time MPC application. Sensitivity analysis is proposed to check the impact on the benefit potential by either variability reduction of quality output/controlled variables or constraint relaxation of constrained variables. The tuning guidelines give rise to suggested tuning operation on either variability or constraint limits for the desired target benefit potential.

5.4.1 Economic performance assessment

In the ideal scenario, disturbances are not considered in the optimal benefit potential calculation. The optimal operating point of the quality variables is expected to be pushed directly towards the constraint bound subject to the constraints of other variables and there is no back off due to variability. However, in the existing variability scenario, the present disturbances are taken into account. The benefit potential is obtained by only shifting the mean values of the quality variables in the direction of increasing benefit potential without reducing variability and hence the back off from the constraint bound depends on the magnitude of the present level of disturbances. By comparing these two scenarios, an economic performance index without tuning can be defined as

$$\eta_{wot} = \frac{\Delta J_E}{\Delta J_I}$$

It is obvious that $0 \leq \eta_{wot} \leq 1$, which indicates the benefit potential ratio that could be realized by just pushing the mean values without reducing the variability, while $1 - \eta_{wot}$ indicates the benefit potential ratio that is due to no variability. If $\eta_{wot} = 0$, no benefit potential could be obtained without reducing the variability, and it implies that sufficient economic performance has arrived if the variability cannot be further reduced by the existing control. If $\eta_{wot} = 1$, there is no disturbance and the economic performance can be improved significantly by simply moving the operating point. If the economic objective function (5.1) is only related to the output/controlled variables and all the coefficients, a_{kj} and b_{kj}, that are related to the input/manipulated variables, are assigned as zeros, the MVC benchmark for variance performance assessment can be introduced. It gives a theoretical absolute variance lower bound. A theoretical economic performance index with MVC as the benchmark can thus be defined as

$$\eta_T = \frac{\Delta J_{MVC}}{\Delta J_I}$$

where ΔJ_{MVC} is *the theoretical benefit potential*, ΔJ_T, that could be achieved by MVC. ΔJ_{MVC} can be calculated by employing variability reduction percentage R_{yi} based simply on the variance improvement potential from the MVC benchmark. It is in part due to the mean value shifting and in part due to the variability reduction. It can be seen that $0 \leq \eta_T \leq 1$. By comparing with the existing economic performance index, the following inequality holds,

$$0 \leq \eta_{wot} \leq \eta_T \leq 1$$

Therefore, if no variability could be reduced, η_{wot} (or ΔJ_E) could be adopted to evaluate the economic performance of current MPC applications. $\eta_{wot} = 0$ (or $\Delta J_E = 0$) shows that no benefit potential could be further obtained without reducing the variability. On the other hand, if the MVC benchmark is available, η_T (or ΔJ_{MVC}) can be utilized instead, which gives an optimal economic benefit potential that could be realized by reducing the variability through the minimum variance control. The positive value of η_T (or ΔJ_{MVC}) does not mean that this benefit potential could be surely achieved since MVC itself is rarely implemented and there are always constraints on the manipulated variables in practice so that MVC is not always realizable. However, the benefit potential given by a positive value of η_{wot} (or ΔJ_E) could be actually achieved in practice by simply moving the operating point to the optimal one.

5.4.2 Sensitivity analysis

Sensitivity analysis is used to investigate the impact of benefit potential to the variability or constraint change of each individual variable. The result shows the importance of variability reduction or constraint relaxation of different variables in terms of their contributions to the benefit potential. In the variability sensitivity analysis, only one of r_{yi} is specified a small value (e.g., -1%) but all other r_{yi}, s_{yi} and s_{uj} are set as 0, then the benefit potential is observed to see the effect of variability reduction of this controlled variable. Similarly, in the constraint sensitivity analysis, only one of s_{yi} or s_{uj} is specified a small value (e.g., $+1\%$) but all other s_{yi}, s_{uj} and r_{yi} are set as 0, then the benefit potential is calculated to see the impact of this small constraint relaxation of this variable. If a small change on either variability reduction or constraint relaxation leads to a large change on the benefit potential, we say that the benefit potential is sensitive to variability or constraint change of this variable. It is worthwhile to reduce the variability or relax the constraints of those variables that have great contributions to the benefit potentials.

5.4.3 Tuning guidelines

As analyzed in the economic performance assessment, the benefit potential from the ideal scenario or from the MVC benchmark may not be achieved in practice. Nevertheless, ΔJ_I or ΔJ_T can be regarded as a benchmark on the benefit potential against which other scenarios could be compared. The desired target benefit potential that can be chosen by users should never be greater than ΔJ_I by just tuning variability of the controlled variables. Here a target benefit potential ratio by tuning variability, R_V, has been defined as the ratio between the target benefit potential and the one calculated from ideal scenario. It can be seen that R_V can never be greater than 1 because the variability can not be reduced more than 100%. Likewise, the target benefit potential due to the constraint range change relative to that of ideal scenario has been defined as the target benefit ratio by tuning constraint, R_C. Since constraint range of some variable may be increased to a great extent, the value of R_C may be specified as greater than 1. Once the desired target benefit potential ratio by tuning variability R_V or the target benefit potential ratio by tuning constraint R_C is specified, the corresponding optimization problem will result in the variability tuning guideline or the constraint tuning guideline. The variability or constraint tuning guideline tells directly which variables should be tuned on either variability or constraint, and how much should be tuned for each of these variables in order to reach the desired target benefit potential.

5.5 Case studies

5.5.1 Simulation example

5.5.1.1 Process and controller description

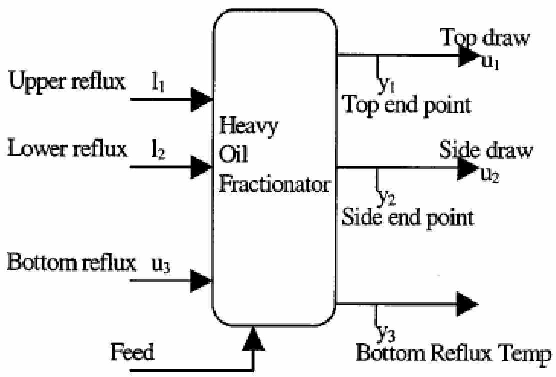

Figure 5.7: Reduced Shell heavy oil fractionator (Ying and Joseph, 1999)

The Shell control problem has been studied by many researchers (Prett *et al.*, 1990). It is a heavy oil fractionator control problem with 7 controlled variables, 3 manipulated variables and 2 unmeasured disturbance variables (Ying and Joseph, 1999). Three key controlled variables labeled y_1,y_2 and y_3 are considered in this study (see Figure 5.7). They are top end point (y_1), side end point (y_2) and bottom reflux temperature (y_3). The process is subject to the disturbances from upper reflux (l_1) and lower reflux (l_2) which are assumed as white noises. The controlled variables are expected to remain within their respective production specifications by adjusting the manipulated variables: top draw (u_1), side draw (u_2) and bottom reflux (u_3). The nominal process model and disturbance model are given below in the form of continuous transfer matrices, respectively.

$$T(s) = \begin{pmatrix} \frac{4.05}{50s+1}e^{-27s} & \frac{1.77}{60s+1}e^{-28s} & \frac{5.88}{50s+1}e^{-27s} \\ \frac{5.39}{50s+1}e^{-18s} & \frac{5.72}{60s+1}e^{-14s} & \frac{6.90}{40s+1}e^{-15s} \\ \frac{4.38}{33s+1}e^{-20s} & \frac{4.42}{44s+1}e^{-22s} & \frac{7.20}{19s+1} \end{pmatrix}, N(s) = \begin{pmatrix} \frac{1.20}{45s+1}e^{-27s} & \frac{1.44}{40s+1}e^{-27s} \\ \frac{1.52}{25s+1}e^{-15s} & \frac{1.83}{20s+1}e^{-15s} \\ \frac{1.14}{27s+1} & \frac{1.26}{32s+1} \end{pmatrix}$$

The MPC design problem has the following formulation (Ying and Joseph, 1999)

$$\text{Minimize}_{\{\substack{u_{1,k+j},u_{2,k+j},u_{3,k+j} \\ j=0,\cdots,H_u-1}\}} \sum_{i=1}^{H_p} \left[w_{y_1}^2 \left(y_{1,k+i} - y_{1sp}\right)^2 + w_{y_2}^2 \left(y_{2,k+i} - y_{2sp}\right)^2 \right] + \sum_{j=0}^{H_u-1} w_{u_3}^2 \left(u_{3,k+j} - u_{3sp}\right)^2 \tag{5.26}$$

subject to

$$\begin{aligned} &-0.5 \leqslant y_{1,k+j} \leqslant 0.5, \quad 1 \leqslant j \leqslant H_p \\ &-10 \leqslant y_{2,k+j} \leqslant 10, \quad 1 \leqslant j \leqslant H_p \\ &-0.5 \leqslant y_{3,k+j} \leqslant 10, \quad 1 \leqslant j \leqslant H_p \\ &-0.5 \leqslant u_{1,k+j} \leqslant 0.5, \quad 0 \leqslant j \leqslant H_u - 1 \\ &-0.5 \leqslant u_{2,k+j} \leqslant 0.5, \quad 0 \leqslant j \leqslant H_u - 1 \\ &-0.5 \leqslant u_{3,k+j} \leqslant 0.5, \quad 0 \leqslant j \leqslant H_u - 1 \\ &-0.05 \leqslant \Delta u_{1,k+j} \leqslant 0.05, \quad 0 \leqslant j \leqslant H_u - 1 \\ &-0.05 \leqslant \Delta u_{2,k+j} \leqslant 0.05, \quad 0 \leqslant j \leqslant H_u - 1 \\ &-0.05 \leqslant \Delta u_{3,k+j} \leqslant 0.05, \quad 0 \leqslant j \leqslant H_u - 1 \end{aligned} \tag{5.27}$$

The Matlab MPC toolbox (Bemporad *et al.*, 2005) is used to design MPC controller, where control interval is set as $Ts = 1$ min, prediction horizon as $H_p = 30$, control horizon as $H_u = 2$, weighting coefficients as $w_{y_1} = w_{y_2} = 1.414$ and $w_{u_3} = 1$. With this

MPC controller, the base case operation with given constraint limits is shown in Figure 5.8, where we can see that all of the controlled variables and manipulated variables are located within their corresponding constraint limits.

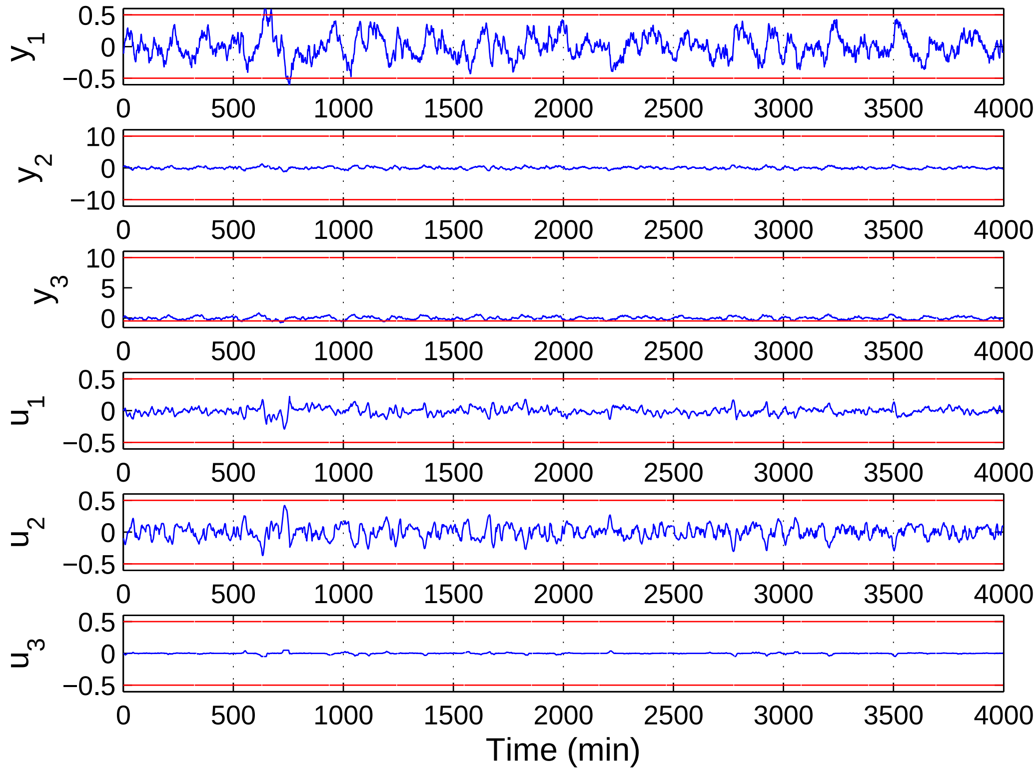

Figure 5.8: Base case operation of Shell system

5.5.1.2 Economic performance evaluation and verification

The economic objective function is set as maximization of the top end point y_1. It takes the same form as (5.1), where $J_k = -\bar{y}_1$ and the low/high limits (Y_{Lki}, U_{Lkj},Y_{Hki}, U_{Lkj}) in (5.3) and (5.4) are set the same as those in (5.27). According to the systematic approach discussed in the previous section, the benefit potentials and operating conditions of different scenarios are calculated and verified respectively. The results are listed in Table 5.1. The details will be further analyzed and discussed shortly.

Economic performance assessment

The benefit potentials of different scenarios are given in Figure 5.9. It shows that

Table 5.1: Results of Shell system (Var.=Variability, Con.=Constraint,Tun.=Tuning)

Different scenarios	optimal operating point						Benefit potentials	
	$\bar{y}_1$	$\bar{y}_2$	$\bar{y}_3$	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$	Calculated	Verified
Ideal	0.5000	2.3077	1.6614	0.0016	0.4721	-0.0635	0.5016	0.5016
Existing	0.1395	0.3187	0.2718	-0.0190	0.0458	0.0177	0.1411	0.1395
Var. Tun.	0.2994	2.9374	2.1153	-0.1675	0.7391	-0.0615	0.3010	0.2879
Con. Tun.	0.2994	0.5442	0.4776	-0.0053	0.0538	0.0331	0.3010	0.2994

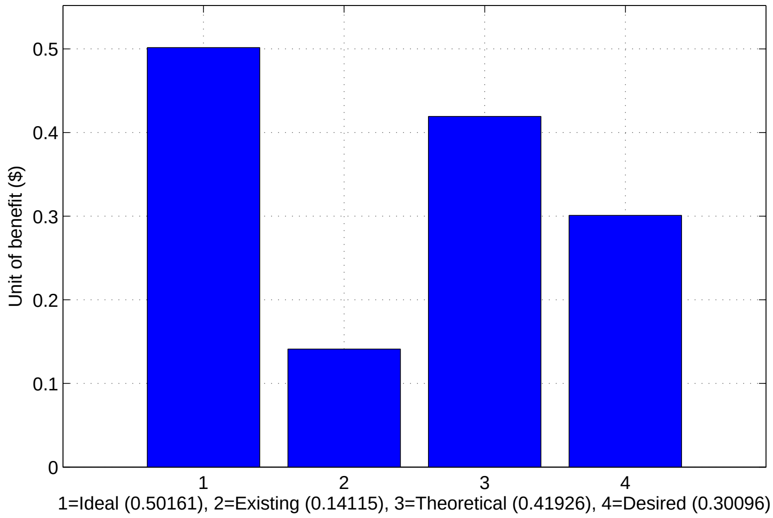

Figure 5.9: Benefit potentials of Shell system

$\Delta J_I = 0.5016$ and $\Delta J_E = 0.1411$. With MVC as the benchmark, the variance performance is calculated according to Huang and Shah (Huang and Shah, 1999) and the performance indices of different control loops are shown in Figure 5.10. Since the economic objective function is only related to the output/controlled variable, if the constraints of the input/manipulated variables can be relaxed sufficiently the variance potential resulting from the MVC benchmark can be applied to calculate the benefit potential and the result, ΔJ_{MVC}, can be served as the theoretical one, ΔJ_T. Therefore, $\Delta J_T = \Delta J_{MVC} = 0.4193$ and this is the benefit potential that could be achieved if MVC is implemented. The economic performance index without tuning is calculated as $\eta_{wot} = 28\%$ and the theoretical economic performance index as $\eta_T = 84\%$. This means that: (1) 28% of the ideal benefit potential (η_{wot}) can be achieved by mean shifting only without any other controller tuning effort; (2) 56% of the ideal benefit potential ($\eta_T - \eta_{wot}$) is possibly achieved by further tuning variability of the output/controlled variables. Thus one can conclude that the economic performance of this MPC application can be improved

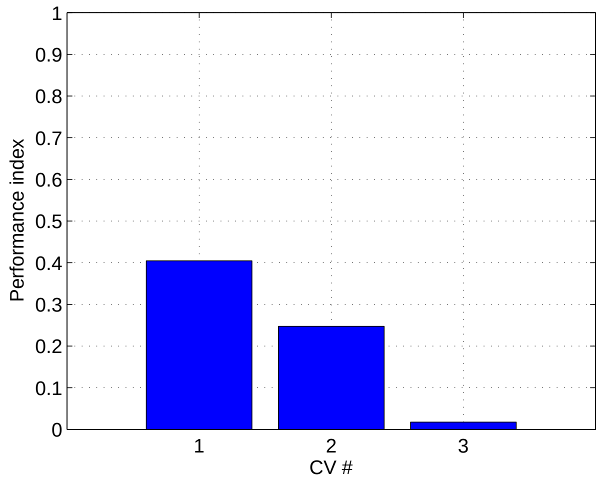

Figure 5.10: Variance performance assessment result of Shell system

significantly by shifting mean values as well as reducing variability.

Optimal tuning guidelines for desired benefit potential

The desired target benefit potential ratios by tuning variability and by tuning constraint

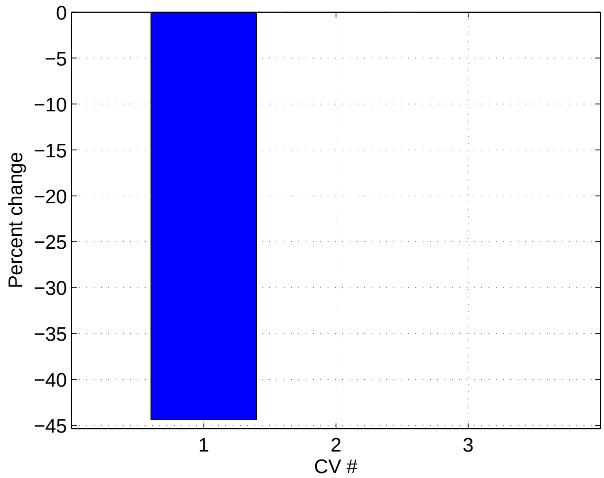

Figure 5.11: Suggested CV variability tuning guideline for desired benefit potential

respectively, are both specified as $R_V = R_C = 60\%$ and the desired target benefit potential is equal to 0.3010 which is smaller than $\Delta J_T(i.e., 0.4193)$ and larger than $\Delta J_E(i.e., 0.1411)$ (Figure 5.9). This target benefit potential is expected to be achieved by either tuning variability or relaxing constraints. The variability tuning guideline suggests

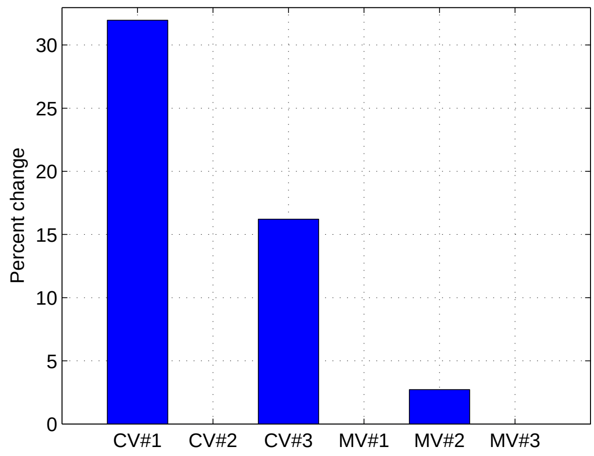

Figure 5.12: Suggested constraint tuning guideline for desired benefit potential

to reduce the standard deviation of y_1 by 44.34% (Figure 5.11), and the constraint tuning guideline tells that the present constraints of y_1, y_3 and u_2 could be relaxed by 31.96%, 16.21% and 2.72%, respectively (Figure 5.12) to achieve the same desired target benefit potential.

The calculated benefit potentials, ΔJ_I, ΔJ_E and the desired benefit potentials are verified by setting the setpoint as the corresponding optimal operating point in the MPC. When there is no disturbance, the operating point of the MPC is pushed to the one as suggested in Table 5.1 for the ideal potential scenario, the achieved benefit potential is exactly the same as the calculated one. By only moving the operating point in the presence of present level of disturbance, the existing benefit potential is verified as 0.1395, which is very close to the calculated one (i.e., 0.1411). This implies that the existing benefit potential is indeed achievable by simply changing the operating point without any controller tuning effort. In terms of variability or constraint tuning guideline, the achieved benefit potentials are verified as 0.2879 by variability reduction and 0.2994 by constraint relaxation. Both of them are very close to the desired target benefit potential (i.e., 0.3010). This shows that the calculated benefit potentials agree well with the actually achieved ones through the tuning and demonstrates the feasibility of the proposed systematic approach for MPC economic performance assessment.

5.5.2 Multi-tank experiment

5.5.2.1 Experimental setup and controller design

The Multi-tank system consists of upper, middle, and lower tanks each equipped with drain valves, as illustrated in Figure 5.13. A variable speed pump fills the upper tank. The liquid outflows the tanks due to gravity. The control goal is to keep the desired liquid levels in each of the tanks. The level sensors mounted in the tanks measure these levels. There are manual and automatic valves (that act as flow resistors) to stabilize the desired levels. Through Real-Time Workshop (RTW) and Real-Time Windows Target (RTWT), MATLAB and Simulink are used to develop and run real-time control. The setup can be used to verify and validate performance assessment strategies for model predictive control.

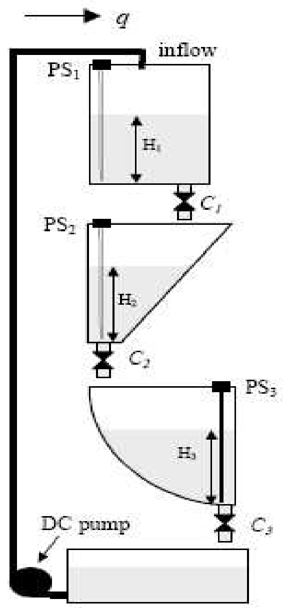

Figure 5.13: Experimental system configuration

The pilot experiment is designed and performed when all of the three drain valves are fully open with appropriate inflow q. The three tank levels, H_1, H_2 and H_3, are controlled variables and the three automatic valves, C_1, C_2 and C_3, are manipulated variables. The disturbance comes from inflow q due to the non-steadiness of the feeding pump. In addition, noises are directly added on the output measurements, H_1, H_2 and H_3, through the computer.

At first, the following process model is identified through identification experiment:

$$T(s) = \begin{pmatrix} \frac{-0.1971}{48.22s+1}e^{-4.58s} & 0 & 0 \\ \frac{10.82s+0.02552}{2434s^2+72.81s+1} & \frac{-0.2105}{33.82s+1}e^{-1.14s} & 0 \\ \frac{12.54s+0.03078}{8960s^2+92.52s+1}e^{-10.1s} & \frac{12.01s+0.01055}{1622s^2+157.9s+1}e^{-6.04s} & \frac{-0.3558}{94.49s+1}e^{-3s} \end{pmatrix}$$

This process model is employed for the MPC controller design. The constraints are set as $0.5 \leq C_i \leq 1.0$, $-0.1 \leq \triangle C_i \leq 0.1$ and $0.05 \leq H_i \leq 0.25$, where $i = 1, 2, 3$. The input rate weights are given as $[0.5, 0.5, 0.5]$ and the output weights $[1, 0.8, 0.9]$. The control interval is 2 second, the prediction and control horizons are designed as 15 and 2, respectively.

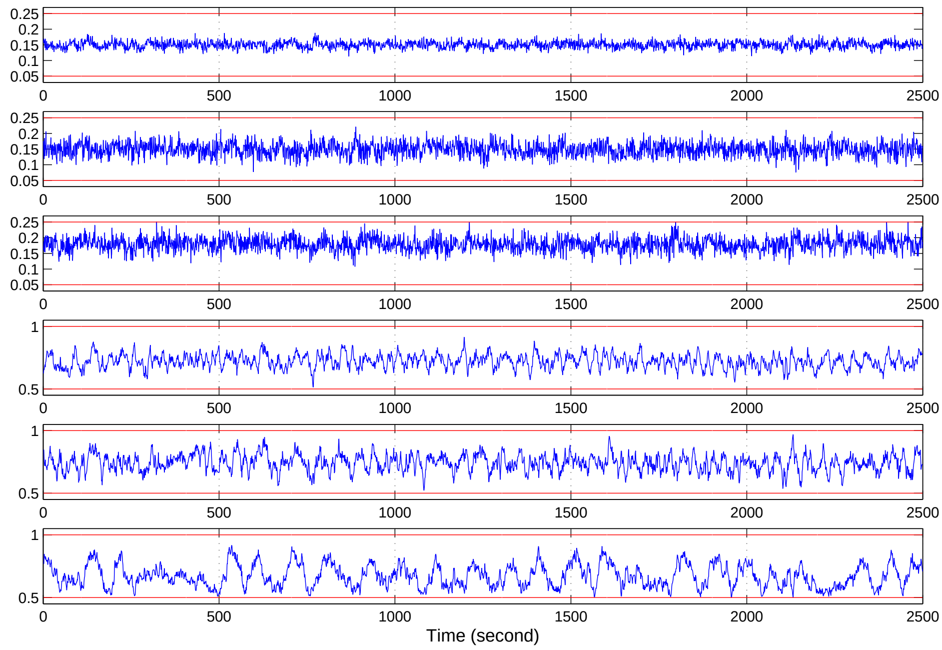

Figure 5.14: Base case operation of Multi-Tank system

5.5.2.2 Data collection and analysis

By applying the designed MPC controller, the real time data are collected and regarded as base case operation in this study, as shown in Figure 5.14. It is observed that all of the controlled variables and manipulated variables are running within their corresponding constraints.

If the economic objective function is set, for example, as the maximization of the third tank level subject to its constraint, say, $max(100H_3)$ or $min(-100H_3)$, and all the conditions are kept unchanged except specified otherwise, then the benefit potentials and optimal operating conditions of different scenarios are calculated on the basis of the base case operation data set. For the existing scenario or constraint tuning guideline scenario, the MPC controller is adjusted of operating point or tuned in terms of its optimal operating condition and the experiment is conducted again using the new controller to see whether the corresponding estimated benefit potential can be realized. The main results are shown in Table 5.2. The calculated and realized benefit potentials of different scenarios are also shown in Figure 5.15. The details are explained next.

Table 5.2: Results of Multi-Tank system (Con.=Constraint,Tun.=Tuning)

Different scenarios		operating point						Benefit potentials
		$\bar{H}_1$	$\bar{H}_2$	$\bar{H}_3$	$\bar{C}_1$	$\bar{C}_2$	$\bar{C}_3$	
Ideal	Calculated	0.0977	0.1189	0.2500	0.9882	0.9189	0.5013	7.0874
Existing scenario	Calculated	0.1174	0.1281	0.1833	0.8881	0.8632	0.6786	0.4152
	Realized	0.1334	0.1469	0.1835	0.7841	0.9907	0.7098	0.4374
Con. Tun. scenario	Calculated	0.0984	0.1102	0.2217	0.9847	0.9599	0.5819	4.2524
	Realized	0.1039	0.1120	0.2208	0.9462	0.9155	0.6387	4.1700

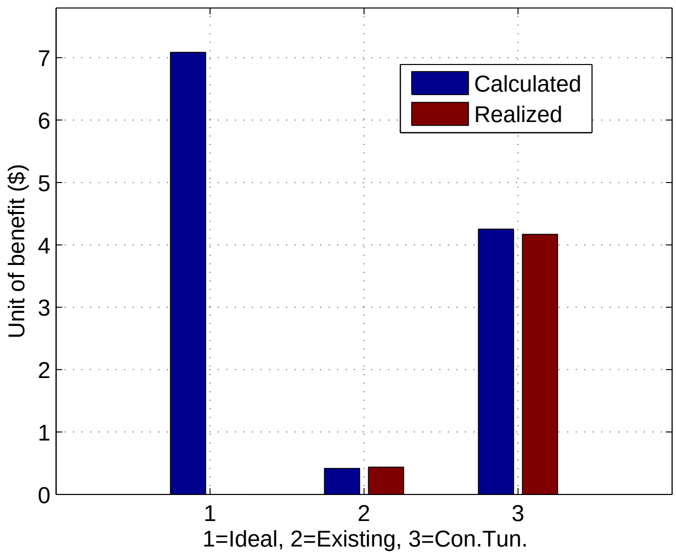

Figure 5.15: Benefit potentials of Multi-Tank system

For the base case operation, the MVC benchmark shows that this MPC controller performs very well in the sense of output variance. With minimum variance as the benchmark, its multivariate performance index is calculated as 0.9887 and its individual control loop performance indices are 0.9634, 1.0087 and 0.9755, for the three tanks respectively. Thus, improvement through variance reduction is unlikely. The further calculation shows that $\Delta J_I = 7.0874$ and $\Delta J_E = 0.4152$. The economic performance without tuning is thus 5.8%, which means that the MPC controller has also good economic performance if no tuning on the variance or the constraints is considered. This is because the benefit potential by mean shifting is rather small.

For the existing scenario, the realized benefit potential is very close to that of the calculated one. This indicates that the calculated existing benefit potential is indeed achievable in practice. For the constraint tuning guidelines, the target benefit potentials are set as 60% of the calculated ideal benefit potential (7.0874), i.e., 4.2524. The constraint tuning guideline shows that the target benefit potential (4.2524) can be achieved if the constraints of the controlled variables could be relaxed by 11.68%, 15.50%, 26.06% and all of the constraints of the manipulated variables by 38.65%. This means $0.0383 \leq H_1 \leq 0.2617$, $0.0345 \leq H_2 \leq 0.2655$, $0.0239 \leq H_3 \leq 0.2761$ and $0.3067 \leq C_i \leq 1.0$, where $i = 1, 2, 3$. The result shows, once again, that the realized benefit potential (4.1700) is very close to the target benefit potential (4.2524).

5.6 Conclusion

The MPC performance assessment with considerations of economic benefit and constraint is studied. Performance assessment algorithms are developed, which can be used to evaluate the benefit potentials by either reducing variability or relaxing constraints. Case studies on a simulated industrial process and a laboratory Multi-Tank experiment show the feasibility of the proposed algorithms and demonstrate that the proposed systematic approach is beneficial for process control engineers in their routine maintenance of MPC applications, especially on the evaluation of economic performance and providing variance or constraint tuning guidelines. The proposed approach is also integrated with multivariate controller performance assessment (MVPA) which is based on the MVC benchmark and gives the relationship between variance based performance assessment and MPC economic performance assessment. Part of the proposed algorithms has been integrated into a plant-oriented solution package for MPC performance monitoring and they are scalable to larger MPC applications.

6

APC Performance Monitoring: Industrial Practice and Application *

6.1 Introduction

During the last few years, a software package, which is called Performance Analysis Technology and Solutions (PATS), has been developed under collaboration with process industries. Many algorithms developed from this thesis have been integrated into the software. It includes many independent components and each of them has its own initiative from industries. All of these components of this software can be classified into three different categories in terms of overall functionality, namely: interface components, assistant components and application components (Figure 6.1). The interface components were designed for retrieving raw process data from different sources, such as data historian with historical data (1 component), OPC (OLE for Process Control, where OLE is referred to as Object Linking and Embedding technique) server with real-time data (2 components), and virtual plant with simulated data (1 component). The assistant components are used to deal with these raw data and then provide the application components with consistent and cleaned data. Some routine works of these assistant components include

*Part of this chapter has been published in Xu, F. , Huang, B. and Tamayo, E.C., Assessment of variance/constraint induced economic performance for model predictive control, IFAC International Symposium on Advanced Control of Chemical Processes (ADCHEM2006), Gramado, Brazil, April 2-5, 899-904, 2006 data preparation, data preprocessing, etc.. The main task of process data analysis is carried out by different application components with different design objectives driven by process industries. They cover the most state-of-the-art technologies which have been transferred directly from recent research work in the fields of controller performance monitoring (3 components), process model identification (2 components), fault detection and isolation (1 component), and controller design (1 component). PATS bridges the gap between academia and industries, and make it possible to apply the research outcome directly into process industries.

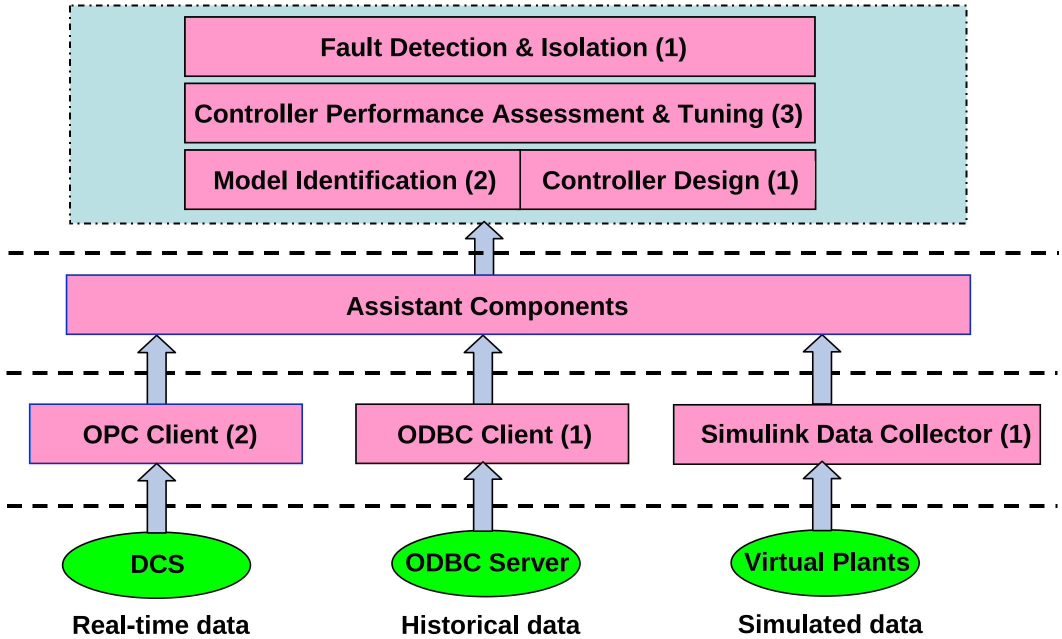

Figure 6.1: An overview of the PATS package

One of the main purposes of PATS is to perform controller performance monitoring in the process industries, especially on the widely used model predictive control (MPC) applications. MPC has been proven as one of the most effective advanced process control (APC) strategies to deal with multivariable constraint control problems (Qin and Badgwell, 2003). Even though the rewards of MPC applications can be great, most of them are not used to their full capacity in practice due to lack of maintenance and conservative/tight operations on the CV/MV limits (Singh and Seto, 2002). In many cases, only a small portion of the available benefits has been realized and half of the refineries have captured about only 40% of the benefits (King, 1999). Unfortunately, less effort has been made on the performance evaluation of existing MPC applications, especially on the economic performance. Therefore, a systematic and standardized approach is demanded by the process industries to monitor the performance of MPC applications and facilitate the task of MPC maintenance.

The outline of this chapter is as follows. Section 6.2 gives a summary of the algorithms for multivariate controller performance assessment and MPC economic performance assessment, which are utilized by the PATS. A plant-oriented solution for APC performance monitoring is then proposed on the basis of industrial implementation background in Section 6.3. An industrial MPC application is described thereafter in Section 6.4, and its performance is analyzed by using the PATS with routine operating data in Section 6.5. It is ended with concluding remarks in Section 6.6.

6.2 Algorithms for APC performance assessment

6.2.1 FCOR algorithm for multivariate controller performance assessment

A standard multivariate process can be represented by

$$y_k = T(q^{-1})u_k + N(q^{-1})a_k \tag{6.1}$$

where $T(q^{-1})$ is a proper, rational $p \times m$ transfer function matrix, $N(q^{-1})$ is a disturbance model, y_k, u_k and a_k are output, input and white noise vectors of appropriate dimensions. The difficulty in multivariate controller performance assessment is the factorization of the time delay matrix, which is known as the interactor matrix. The interactor matrix is an equivalent form or generalization of the time delay in multivariate systems. It needs process model (or at least the first few Markov Matrices) to capture the delay terms. The interactor matrix can be factored out by using QR factorization (Rogozinski *et al.*, 1987; Peng and Kinnaert, 1992), or SVD (Singular Value Decomposition) (Huang *et al.*, 1997*b*), or direct definition (Lu, 2005). Once the interactor matrix is known, the minimum variance control benchmark can be readily extended to evaluate the performance of multivariable control systems. The overall and individual performance indices can be calculated by multivariate FCOR (filtering and correlation analysis) algorithm (Huang and Shah, 1999).

Algorithm 6.2.1 *The FCOR algorithm (Huang and Shah, 1999) can be summarized as the following steps:*

(1) *Estimate the unitary interactor matrix* D *with delay order* d *that satisfies* $D^T(q^{-1})D(q) = I$ *and* $D(q) = D_0q^d + D_1q^{d-1} + D_2q^{d-2} + \cdots + D_{d-1}q$.

(2) *Identify and filter the time series model $\hat{G}_{cl}(q^{-1})$ from closed-loop routine operating output data, calculate the covariance matrix of the whitened sequence, $\hat{\Sigma}_a$.*

$$\hat{y}_k = \hat{G}_{cl}(q^{-1})\hat{a}_k, \quad q^{-d}D\hat{G}_{cl}(q^{-1}) = \hat{F}(q^{-1}) + q^{-d}\hat{R}_{cl}(q^{-1}), \quad \hat{\Sigma_a} = E(\hat{a}_k\hat{a}_k^T)$$

(3) *Calculate the benchmark output covariance matrix as*

$$E(\hat{y}_k\hat{y}_k^T)|_{mv} = \hat{E}_0\hat{\Sigma}_a\hat{E}_0^T + \hat{E}_1\hat{\Sigma}_a\hat{E}_1^T + \cdots + \hat{E}_{d-1}\hat{\Sigma}_a\hat{E}_{d-1}^T$$

where

$$(\hat{E}_0, \hat{E}_1, \cdots, \hat{E}_{d-1}) = (D_0^T, D_1^T, \cdots, D_{d-1}^T)\begin{pmatrix} \hat{F}_0 & \hat{F}_1 & \cdots & \hat{F}_{d-1} \\ \hat{F}_1 & \hat{F}_2 & \cdots & \\ \vdots & \vdots & & \\ \vdots & \hat{F}_{d-1} & & \\ \hat{F}_{d-1} & & & \end{pmatrix}$$

and

$$\hat{F}(q^{-1}) = \hat{F}_0 + \hat{F}_1 q^{-1} + \cdots + \hat{F}_{d-1}q^{d-1}$$

(4) *Obtain the overall and individual performance indices as the following,*

$$\hat{\eta}_o = \frac{trace(E(\hat{y}_k\hat{y}_k^T)|_{mv})}{trace(E(\hat{y}_k\hat{y}_k^T))} = \frac{trace(\hat{E}_0\hat{\Sigma}_a\hat{E}_0^T + \hat{E}_1\hat{\Sigma}_a\hat{E}_1^T + \cdots + \hat{E}_{d-1}\hat{\Sigma}_a\hat{E}_{d-1}^T)}{trace(E(\hat{y}_k\hat{y}_k^T))}$$

and

$$\hat{\eta}_i = \frac{diag(E(\hat{y}_k\hat{y}_k^T)|_{mv})}{diag(E(\hat{y}_k\hat{y}_k^T))} = \frac{diag(\hat{E}_0\hat{\Sigma}_a\hat{E}_0^T + \hat{E}_1\hat{\Sigma}_a\hat{E}_1^T + \cdots + \hat{E}_{d-1}\hat{\Sigma}_a\hat{E}_{d-1}^T)}{diag(E(\hat{y}_k\hat{y}_k^T))}$$

6.2.2 MPC economic performance assessment and tuning guidelines

The algorithms for MPC economic performance assessment and tuning guidelines have been introduced with given process model and disturbance model in the previous chapter. The back-offs of the controlled variables are directly calculated on the basis of their standard deviations of the existing operation. For the manipulated variables, their back-offs are optimized as their standard deviations are minimized in terms of minimum energy control such that the range of optimal operation is expanded to the greatest extent. The variability relation between the controlled variables and the manipulated variables is thus explicitly established.

However, the disturbance model from the white noise to the process output, $N(q^{-1})$, is usually not available in practice, and thus the input standard deviation, U_{stdj}, can not be calculated based on the process model and disturbance model with the output variance constraint. From the observation of the distinction between the controlled variables and the manipulated variables, it is found that for most control problems the output variance is of greater concern in terms of product quality. This is due to the fact that the key quality variables are usually selected from the controlled variables. On the other hand, the operating ranges of the manipulated variables are often strictly limited by some physical hard constraints. Therefore, it is reasonable to calculate their back-offs on the basis of the existing operating ranges instead of the standard deviations.

By considering practical implementation, the algorithms for the MPC economic performance assessment can be further simplified in that there is no need to calculate the standard deviations of the input variables and their back-offs are directly based on the operating ranges of the existing operation. Therefore, for practical implementation algorithms, the variability changes of the controlled variables are performed in terms of their existing standard deviations while the variability changes of the manipulated variables are established in terms of their existing operating ranges.

It must be noted that since the relation between the variability changes of the controlled variables based on the existing standard deviations and the variability changes of the manipulated variables based on the existing operating ranges is not well established, unless the disturbance models are exactly known. They are dealt with independently in the practical implementation algorithms. Thus, for one specific component of the software that deals with variability tuning guideline, the results obtained from a specific component for the variability tuning are only a guideline for the targeted distributions of variances among the controlled variables if certain benefit potential is desired. The variance distribution serves as a tuning direction or an ideal benchmark for control tuning, but depending on the process dynamics and tuning methods, this ideal target may or may not be practically achieved.

Therefore, the main difference between the practical implementation algorithms and the ones proposed in the previous chapter lies in the simplification of input/output inequality descriptions. These distinct inequality descriptions of the input/output variables were first presented by Xu *et al.* (2006) and are summarized below for different scenarios. In the following, $i = 1, \cdots, p$, $j = 1, \cdots, m$, and U_{qorj0} is referred to as a quarter of moving range of the j-th input variable in the existing operation.

- **Ideal operation scenario**:

$$\begin{aligned} Y_{Lki} \leq \bar{y}_i \leq Y_{Hki} \\ U_{Lkj} \leq \bar{u}_j \leq U_{Hkj} \end{aligned} \tag{6.2}$$

The disturbance is not considered and thus a strict steady state optimum can be calculated subject to the constraints. The obtained benefit potential is the maximum one that can be ideally achieved without constraint violations. This *ideal benefit potential* is denoted as ΔJ_I.

- **Existing variability scenario**:

$$\begin{aligned} Y_{Lki} + 2 \times Y_{stdi0} \leq \bar{y}_i \leq Y_{Hki} - 2 \times Y_{stdi0} \\ U_{Lkj} + 2 \times U_{qorj0} \leq \bar{u}_j \leq U_{Hkj} - 2 \times U_{qorj0} \end{aligned} \tag{6.3}$$

The present level of disturbance is considered. The purpose is to investigate whether there is benefit potential by moving mean values only. This benefit potential is called as *the existing benefit potential*, ΔJ_E.

- **Reducing variability scenario**:

$$\begin{aligned} Y_{Lki} + 2 \times Y_{stdi0}(1 + R_{yi}) \leq \bar{y}_i \leq Y_{Hki} - 2 \times Y_{stdi0}(1 + R_{yi}) \\ U_{Lkj} + 2 \times U_{qorj0}(1 + R_{uj}) \leq \bar{u}_j \leq U_{Hkj} - 2 \times U_{qorj0}(1 + R_{uj}) \end{aligned} \tag{6.4}$$

where R_{yi} and R_{uj} are specified *a priori*. If the variability can be reduced, then the mean values can be moved closer to their ideal optimal ones. This benefit potential is defined as *the optimal benefit potential by reducing variability*, ΔJ_V.

- **Relaxing constraint scenario**:

$$\begin{aligned} Y_{Lki} - S_{yi} \times Y_{holki} + 2 \times Y_{stdi0} \leq \bar{y}_i \leq Y_{Hki} + S_{yi} \times Y_{holki} - 2 \times Y_{stdi0} \\ U_{Lkj} - S_{uj} \times U_{holkj} + 2 \times U_{qorj0} \leq \bar{u}_j \leq U_{Hkj} + S_{uj} \times U_{holkj} - 2 \times U_{qorj0} \end{aligned} \tag{6.5}$$

where S_{yi} and S_{uj} are specified *a priori*. The constraint relaxation also creates opportunities for the quality variables to approach their ideal optimal values. This potential is defined as *the optimal benefit potential by relaxing constraint*, ΔJ_C.

- **Simultaneous reducing variability and relaxing constraint scenario**:

$$\begin{aligned} & Y_{Lki} - S_{yi} \times Y_{holki} + 2 \times Y_{stdi0}(1 + R_{yi}) \leq \bar{y}_i \leq \\ & \qquad Y_{Hki} + S_{yi} \times Y_{holki} - 2 \times Y_{stdi0}(1 + R_{yi}) \\ & U_{Lkj} - S_{uj} \times U_{holkj} + 2 \times U_{qorj0}(1 + R_{uj}) \leq \bar{u}_j \leq \\ & \qquad U_{Hkj} + S_{uj} \times U_{holkj} - 2 \times U_{qorj0}(1 + R_{uj}) \end{aligned} \tag{6.6}$$

where R_{yi}, R_{uj}, S_{yi} and S_{uj} are specified *a priori*. This scenario considers both the variability reduction and constraint relaxation. The corresponding benefit potential is thus called as *the optimal benefit potential by reducing variability and relaxing constraint simultaneously*. It is denoted as ΔJ_B.

- **Variability tuning problem**:

$$\begin{aligned} Y_{Lki} + 2 \times Y_{stdi0}(1 + r_{yi}) \leq \bar{y}_i \leq Y_{Hki} - 2 \times Y_{stdi0}(1 + r_{yi}) \\ U_{Lkj} + 2 \times U_{qorj0}(1 + r_{uj}) \leq \bar{u}_j \leq U_{Hkj} - 2 \times U_{qorj0}(1 + r_{uj}) \end{aligned} \tag{6.7}$$

  where r_{yi} and r_{uj} are unknown variability tuning rates. This is to solve the variables and how much should be reduced on their variability in order to achieve the target benefit potential.

- **Constraint tuning problem**:

$$\begin{aligned} Y_{Lki} - s_{yi} \times Y_{holki} + 2 \times Y_{stdi0} \leq \bar{y}_i \leq Y_{Hki} + s_{yi} \times Y_{holki} - 2 \times Y_{stdi0} \\ U_{Lkj} - s_{uj} \times U_{holkj} + 2 \times U_{qorj0} \leq \bar{u}_j \leq U_{Hkj} + s_{uj} \times U_{holkj} - 2 \times U_{qorj0} \end{aligned} \tag{6.8}$$

  where s_{yi} and s_{uj} are unknown constraint tuning rates. This is to find the variables and how much should be relaxed on their constraint limits such as to achieve the target benefit potential.

For the economic performance assessment, we have the following two indices.

$$\eta_{wot} = \frac{\Delta J_E}{\Delta J_I} \qquad and \qquad \eta_T = \frac{\Delta J_{MVC}}{\Delta J_I}$$

where η_{wot} is the economic performance index without tuning, η_T is the theoretical economic performance index with MVC as the benchmark, and ΔJ_{MVC} is *the theoretical benefit potential*, ΔJ_T, that could be achieved by MVC. For these two economic performance indices, the following inequality relation holds.

$$0 \leq \eta_{wot} \leq \eta_T \leq 1$$

6.3 Industrial APC performance monitoring framework

This section will introduce our industrial practice and experience for APC performance monitoring. The PATS software package has been tested and implemented in industry in particular, for APC performance monitoring. Up-to-date there are more than 10 MPC applications implemented in different plants of this industry. Unfortunately it's rather hard for the process control engineers to routinely monitor the performance of these MPC applications. By PATS implementation, a practical plant-oriented solution has been carried out for APC performance monitoring which is based directly on the existing distributed control systems (DCS). It tells not only the output variance performance but also the economic performance and MPC tuning guidelines, by taking full advantage of existing available process data information.

6.3.1 Implementation background

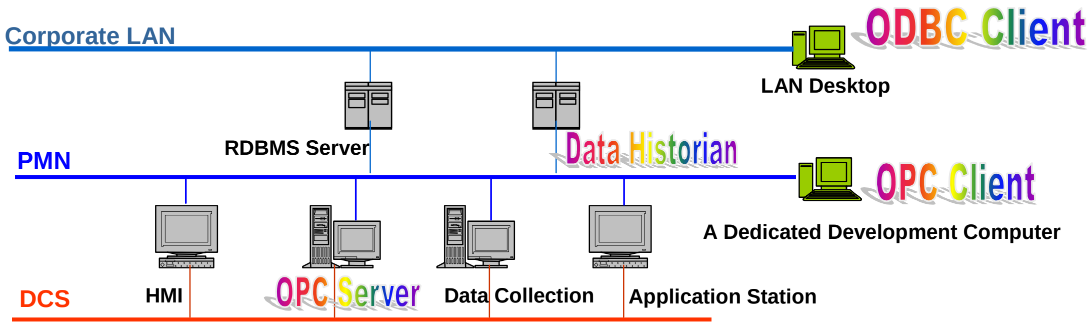

Figure 6.2: Implementation framework for APC performance monitoring

The overall implementation framework is illustrated in Figure 6.2. There are three network layers, including DCS network at the bottom, process management network (PMN) in the middle and corporate local area network (LAN) on the top. The distributed controllers running regulatory control loops are all connected to the DCS network via network interfaces. There are human-machine interface (HMI), application station, OPC server and data collection between DCS and PMN, and relational database management system (RDBMS) server and data historian between PMN and LAN. The APC applications are implemented and running on the application station. The real-time data can be collected from the OPC server via OPC client. The data collection collects the real-time data from DCS with the background supported by the RDBMS server and at the same time transfers the collected data to the data historian in a batch mode. The data historian keeps the historical data information with compression option, which can be accessed from any corporate LAN desktop computers.

The PATS package includes two interface components, one is an OPC client which is also named as real-time data collector, and the other one is ODBC client, where ODBC stands for open database connectivity protocol. The PATS package can be loaded in any LAN desktop computers with ODBC client as the data interface and data historian as the data source. Note that by this way the historical data could be retrieved for analysis. For the real-time data, the OPC client is installed on a dedicated development computer which has a direct connection with PMN such that the OPC client could have access to the OPC server. Therefore, the PATS package can also be implemented for real-time data analysis.

It should be mentioned that this framework is built on the basis of existing control system infrastructure, but it is readily transplanted to and integrated with other control systems.

6.3.2 A plant-oriented solution for APC performance monitoring

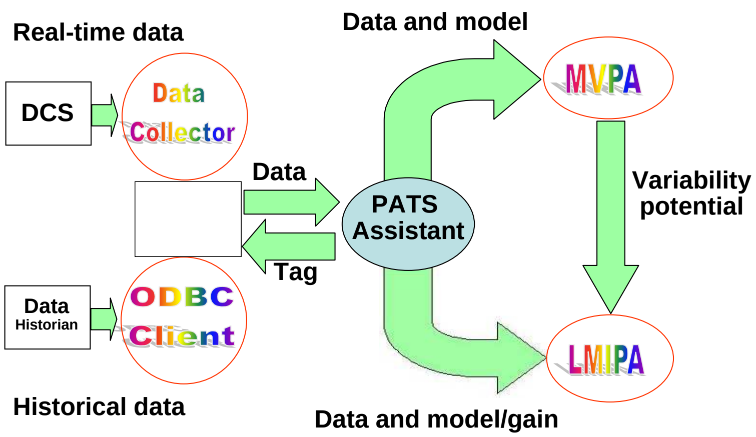

Figure 6.3: Structure of plant-oriented solution for APC performance monitoring

With the implementation background in Figure 6.2, a plant-oriented solution (Figure 6.3) has been proposed, especially for APC performance monitoring. It is composed of two main application components, two interface components and some assistant components of the PATS package.

The historical process data can be retrieved from data historian by ODBC client and the real-time process data from DCS by real-time data collector via OPC technique. Both of historical data and real-time data are then transferred and supplied by the assistant components to the application components. The assistant components serve as a process information bridge or hub between the interface components and the application components. On the one hand, they provide convenient utilities to enter process tag names for the interface components and process model information for the application components if it is required. On the other hand, the collected process data are preprocessed, such as detecting and removing outliers, and then standardized in the format such that the data set is guaranteed to be consistent with the application components.

There are two main application components in this solution for APC performance monitoring, which are namely briefed as MVPA and LMIPA. MVPA is referred to as multivariate controller performance assessment component with MVC as the benchmark. It employs FCOR algorithm (Huang and Shah, 1999) to calculate the individual and overall variance performance indices. The variance performance indices quantify how good the existing controller is compared with the MVC benchmark, and give rise to the improvement potential in the sense of output variance. By the FCOR algorithm, both process model and routine operating output data are required. LMIPA stands for linear matrix inequality approach for performance assessment with an objective for providing APC economic performance assessment and tuning guidelines. It takes process gain, process input/output data and the relevant control parameters such as hard constraints as the input and then calculates the benefit potentials of different scenarios, economic performance indices, suggested MPC variability and constraint tuning guidelines. The variability improvement potential from the MVPA component can be provided to the LMIPA component for a theoretical benefit potential estimation with MVC as the benchmark. This bridges the MVPA component and the LMIPA component within this plant-oriented solution.

Therefore, this integrated solution provides not only the variance performance result with MVC as the benchmark, but also the economic performance information and MPC tuning guidelines, and hence facilitates the task of APC application performance monitoring and maintenance.

6.4 Process description and data collection

6.4.1 Process description

The gas oil hydrotreating unit (GOHTU) is designed to produce high quality treated gas oil product. It receives a blend of raw gas oil and then treats it with hydrogen at high pressure and moderate temperature in fixed catalyst beds. Olefinic chemical bonds are saturated to a more stable form and the sulphur and nitrogen contents are reduced. The reactor effluent is fractionated into product gas oil and partially treated naphtha. The simplified schematic process diagram is shown in Figure 6.4.

The main parts of the GOHTU are feed section, reactor section, reactor effluent section and fractionator section. In the feed section, the raw gas oil is filtered in feed filters to remove particulate matter and flows into the surge drum C-1, from where it is pumped to the reactor section. In the reactor section, the raw gas oil is preheated and combined with treat gas before it is sent to the reactors. The treat gas is composed of recycle gas and fresh makeup hydrogen. The hydrogenation reactions occur in the reactors with catalyst. The reactor effluent is then cooled and separated into a liquid stream and a vapour stream in the hot high pressure separator, C-6. The liquid stream is sent to the fractionator section. The recycle gas is compressed after the removal of light oils, NH_3 and H_2S. In the fractionator section, the liquid stream from the reactor section is fractionated into product gas oil (fractionator bottoms) and the partially treated naphtha (fractionator overhead).

An MPC controller has been designed and applied in the reactor section with the following objectives: (1) optimize reactor equivalent isothermal temperatures (EITs) to

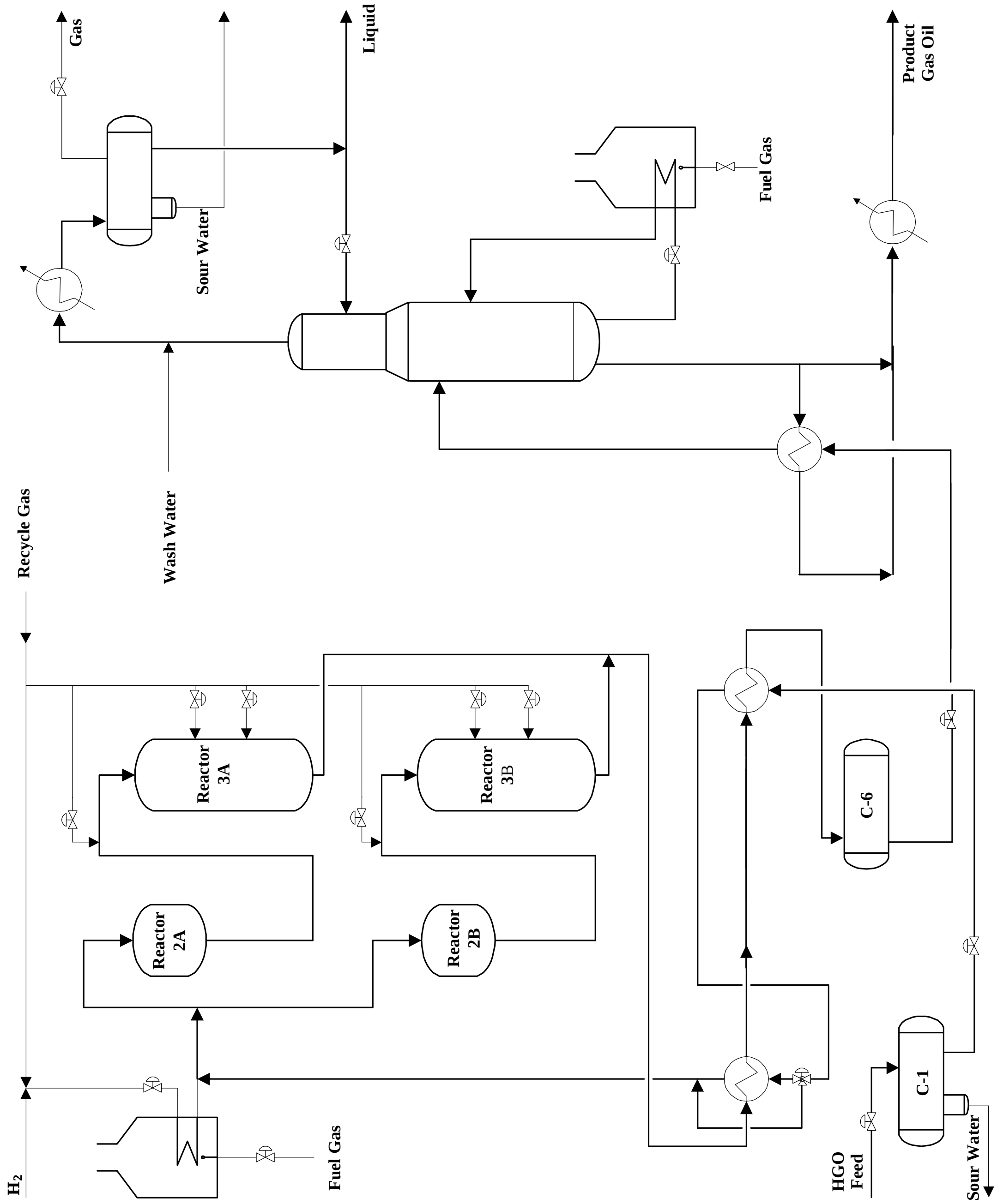

Figure 6.4: Schematic diagram of the GOHTU process

maintain nitrogen and sulphur specifications, extend the catalyst run length, and reduce hydrogen consumption; (2) optimize hydrogen partial pressure to extend the catalyst run length by adjusting treat gas ratio and recycle gas purity; (3) minimize hydrogen and fuel gas consumption; (4) improve operation safety. This controller includes 41 controlled variables (CVs), 15 manipulated variables (MVs) and 5 disturbance variables (DVs).

6.4.2 Process data collection

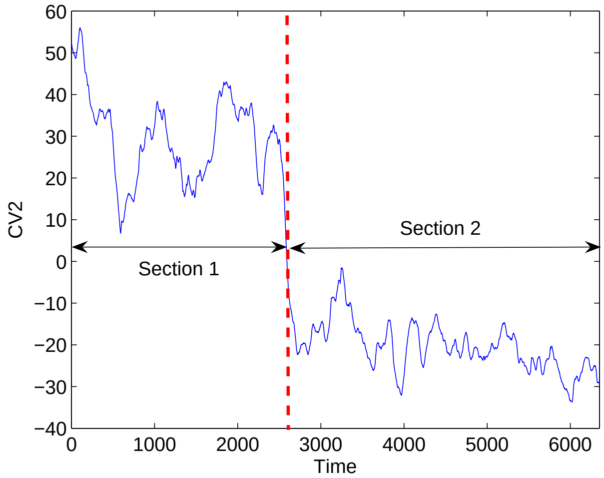

Figure 6.5: Time series plot of CV2

The real-time data collected for this analysis include controlled variables, manipulated variables and their related parameter variables, such as high/low limits, linear coefficients, quadratic coefficients and resting values. The data collection lasted for approximately 26.5 hours with sampling time 15 second and totally 6350 data points. There is a significant operating condition transition between data point number 2570 and 2600. This can be seen clearly from CV2 (Figure 6.5). For proprietary reason, data shown in the figure have been normalized. Owing to two different operating conditions, it is necessary to divide it into two data sections. The first section includes 2570 data points (about 10.71 hours) and the second section includes 3750 data points (about 15.63 hours). From these two data sets, the total variances were calculated as 206.6849 and 81.8848, respectively. The data analysis will be carried out with respect to these two data sections respectively.

6.5 Data analysis for APC performance assessment

6.5.1 Variance performance assessment using the MVC benchmark

The FCOR algorithm employs the process model to factorize the interactor matrix and the closed-loop routine operating data to calculate the performance indices. With the given process model, the unitary interactor matrix was obtained as a 41×41 anti-diagonal matrix.

$$\begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & q \\ 0 & 0 & 0 & \cdots & q & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & q & \cdots & 0 & 0 \\ 0 & q & 0 & \cdots & 0 & 0 \\ q & 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \tag{6.9}$$

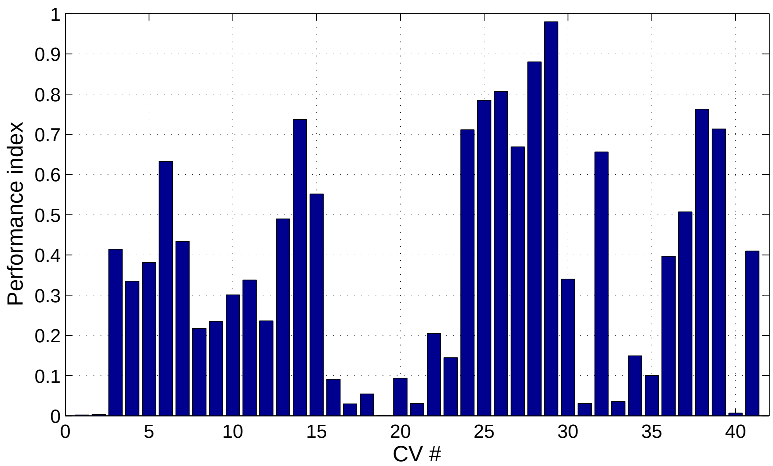

Figure 6.6: Variance performance of data section 1

By using the FCOR algorithm, the variance performance indices from MVPA with MVC as the benchmark were calculated as shown in Figure 6.6 for the first data section and Figure 6.7 for the second data section. The overall variance performance indices were obtained as $\hat{\eta}_{MVC}(y_1) = 0.1310$ for data section 1 and $\hat{\eta}_{MVC}(y_2) = 0.3682$ for data section 2. For both sections, they have similar relative performance among different control loops. CV3-CV15, CV22-CV30, CV31, CV36-CV39 and CV41 have relatively higher performance index values than those of other control loops. CV1, CV2, CV16, CV17, CV19, CV21 and CV40 show poor performance. In general, this MPC application performed much better in section 2 and most of performance index values of section 2 are significantly larger than

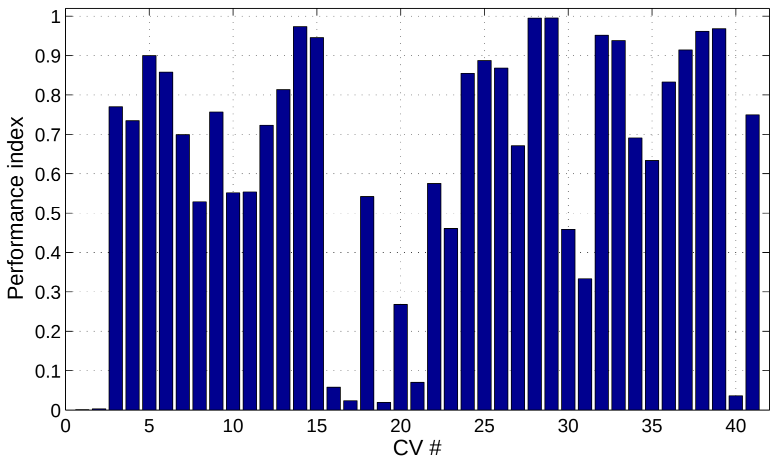

Figure 6.7: Variance performance of data section 2

the corresponding ones of section 1 except a few control loops like CV16 and CV17. The individual control loop CV29 of the data section 1, and the individual control loops CV14, CV15, CV28, CV29, CV38, CV39 of the data section 2, performed very well, and their performance indices are very close to 1. However, the variance performance index values of CV1 and CV2 in both data sections are small, which may imply possibility of large economic benefit potential of the existing control if we are able to reduce their variability. This is because the gas oil nitrogen content (CV1) and sulphur content (CV2) are both quality variables that have great impact on the economic performance.

6.5.2 Variance performance assessment using the LTVD benchmark

From the time series of two data sections, the total variance of data section 1 is significantly larger than that of data section 2. It is considered that the first data section was subject to a major disturbance and the second data section was affected by another but more common disturbance. The second data section is thus considered as more representative one while the first data section needs to satisfy some pre-specified requirement. Note that in this case the regular, the weighted and the generalized LTVD benchmarks are the same. The LTVD benchmarks have not been included in the current version of PATS. We apply the algorithms developed in Chapter 3 to this industrial problem in this subsection. For computational tractability, the 8 controlled variables with the first priority control settings in DCS are selected as an example of applying the LTVD benchmark. Thus we only consider a subset (8CVs) of the original system that has 41 CVs. The aim is to investigate the achievable minimum total variance on the second data section under the condition of meeting the pre-specified requirement on the first data section. From the given data set, the closed-loop time series model was identified for each section. The unitary interactor matrix was calculated as an 8×8 diagonal matrix.

$$\begin{pmatrix} q & 0 & \cdots & 0 & 0 \\ 0 & q & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & q & 0 \\ 0 & 0 & \cdots & 0 & q \end{pmatrix} \tag{6.10}$$

The disturbance affecting the first data section is required to be regulated to follow a pre-specified closed-loop response for each individual output. This closed-loop response of $G_R(i,j)$ takes the form of a first order transfer function and lead-lag system as

$$G_R(i,j) = \frac{\alpha_{ij}}{1-\lambda_i q^{-1}} \quad \text{and} \quad G_R(i,j) = \frac{\alpha_{ij} + \beta_{ij} q^{-1}}{1-\lambda_i q^{-1}} \tag{6.11}$$

respectively, where λ_i is the pre-specified response requirement for the i-th output, α_{ij} and β_{ij} are the ij-th elements of the corresponding unknown decision matrix variables. By using the LTVD benchmark, λ_i for the 8 outputs take the same value, i.e., $\lambda_i = \lambda$, $i = 1, 2, \cdots, 8$. The results are shown in Table 6.1, Figure 6.8 and Figure 6.9, where $\hat{\sigma}^2_{opt}(y_1)$ and $\hat{\sigma}^2_{opt}(y_2)$ are referred to as the optimal total variances of the section 1 and 2

Table 6.1: Performance assessment results based on the LTVD benchmark

G_R	λ	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
First-order	$\hat{\sigma}^2_{opt}(y_1)$	0.1760	0.1761	0.1764	0.1767	0.1771	0.1776	0.1785	0.1806
	$\hat{\sigma}^2_{opt}(y_2)$	0.1804	0.1802	0.1800	0.1796	0.1792	0.1787	0.1780	0.1768
	$\hat{\eta}(y_1)$	0.1424	0.1425	0.1426	0.1428	0.1430	0.1433	0.1439	0.1454
	$\hat{\eta}(y_2)$	0.2907	0.2905	0.2902	0.2899	0.2895	0.2889	0.2879	0.2857
Lead-lag	$\hat{\sigma}^2_{opt}(y_1)$	0.1788	0.1789	0.1791	0.1793	0.1795	0.1800	0.1808	0.1830
	$\hat{\sigma}^2_{opt}(y_2)$	0.1773	0.1772	0.1770	0.1768	0.1766	0.1763	0.1759	0.1753
	$\hat{\eta}(y_1)$	0.1441	0.1442	0.1442	0.1443	0.1445	0.1448	0.1454	0.1470
	$\hat{\eta}(y_2)$	0.2867	0.2866	0.2865	0.2862	0.2860	0.2855	0.2847	0.2829

respectively. It is clear that with the increasing of λ value, the optimal total variance of the first data section is gradually relaxed (Figure 6.8). This simultaneously leaves opportunities for the second data section to improve its variance performance with gradually decreasing optimal total variance. Therefore, The variance reduction on the second data section is achieved at the sacrifice of the variance inflation on the first data section. It is also observed that the optimal total variance of the second data section with lead-lag $G_R(q^{-1})$ is less than the corresponding one with first order $G_R(q^{-1})$, while for the first data section its optimal total variance with lead-lag $G_R(q^{-1})$ is larger than the corresponding one with first order $G_R(q^{-1})$. This is due to the fact that the lead-lag system has one more degrees of freedom than the first order transfer function in the optimization problem. For the variance performance indices, the trends are exactly the same as the trends of the corresponding optimal total variances (Figure 6.9).

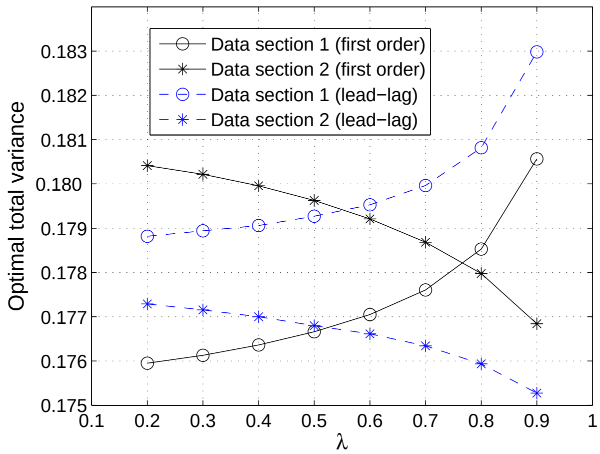

Figure 6.8: The trend of optimal total variance vs user chosen λ value

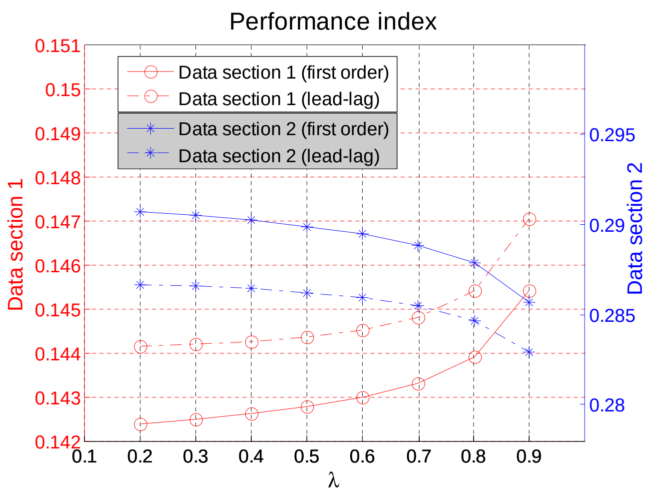

Figure 6.9: The trend of performance index vs user chosen λ value

For the selected 8 controlled variables, the actual total variances from the given data set are 1.6812 for the first data section and 0.8010 for the second data section. By using the MVC benchmark, the minimum total variances were solved as 0.1750 and 0.1723, respectively. The actual total variances and the minimum total variances give rise to the minimum variance performance indices for the given two data sections as $\hat{\eta}_{MVC}(y_1) = 0.1416$ and $\hat{\eta}_{MVC}(y_2) = 0.2769$. By comparison, the optimal total variances from the LTVD benchmark are all greater than their corresponding minimum total variances from the MVC benchmark, and so do their performance indices. Even though the differences are relatively small, the resultant optimal total variances from the LTVD benchmark are practically achievable, while the results from the MVC benchmark are usually not achievable in practice.

6.5.3 Economic performance assessment

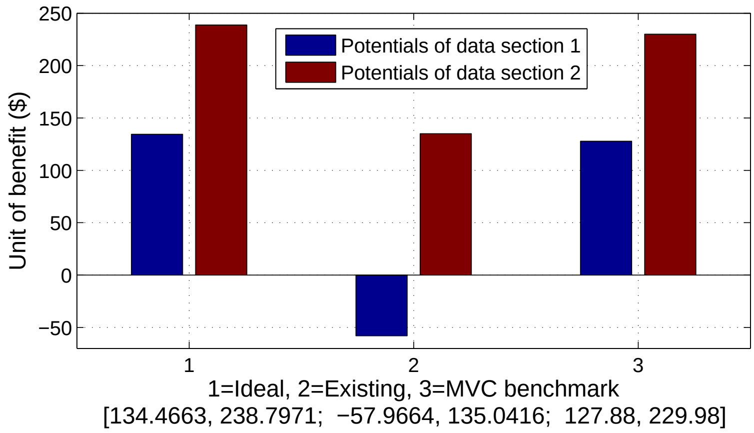

Figure 6.10: Benefit potentials of different scenarios for both data sections

The benefit potentials under different scenarios for both data sections were calculated and illustrated in Figure 6.10, including the results of ideal scenario, existing variability scenario and MVC benchmark by FCOR algorithm.

For the first data section, $\Delta J_{I1} = 134.4663$, $\Delta J_{E1} = -57.9664$ and $\Delta J_{MVC1} = 127.88$. The economic performance index without tuning was calculated as $\eta_{wot1} = -43.11\%$, and the theoretical economic performance indices based on the MVC benchmark by FCOR algorithm was $\eta_{T1} = 95.10\%$. Since the ideal benefit potential ΔJ_{I1} is greater than 0, it implies that the current operating point (mean value) is within the range of the constraint limits. However, the existing benefit potential ΔJ_{E1} is smaller than 0, which means loss potential of benefit. This is mainly due to the fact that the constraint violation percentage of the key quality variable CV1 has exceeded 5%, and in fact it reaches 12.45%. This kind of excessive constraint violation is apparently detrimental since it would directly degrade the product quality. The optimal solution of the existing scenario is to reduce the constraint violation to the tolerance percentage by shifting mean value away from the constraint, and thus causes benefit lost. However, there are still opportunities to gain benefit potential because the optimal theoretical benefit potential (127.88) based on the MVC benchmark is large enough and very close to the ideal one (134.4663). This is achievable by reducing variability if there is sufficient operating range for the manipulated variables. Therefore, with existing constraint limit settings, tuning variability is the key to gain improved benefit for the first data section.

For the second data section corresponding to different operating condition, the optimal benefit potentials were calculated as $\Delta J_{I2} = 238.7971$, $\Delta J_{E2} = 135.0416$, $\Delta J_{MVC2} = 229.98$, and thus the resultant economic performance indices are $\eta_{wot2} = 56.55\%$ and $\eta_{T2} = 96.31\%$. The existing benefit potential value is as large as 135.0416 and shows that the economic performance of this MPC application could be improved given the existing variability and constraint limit settings. This potential for improved benefit is readily achievable by only moving the operating point for the current operation without any other tuning effort. The economic performance index without tuning is also relatively large, indicating that 56.55% of the ideal benefit potential should have been achieved relative to the current operation. According to the variance improvement potential based on the MVC benchmark, the theoretical benefit potential (229.98) is very close to the ideal one (238.7971). This means, even though the ideal benefit potential is never achievable in practice, if the minimum variance control is implementable, then 96.31% of the ideal benefit potential is expected to be achieved. This is resulted partially from mean value shifting (135.0416) and partially from variability reduction (94.9384).

By comparing the results of the two data sections for the two different operating conditions, it can be observed that the benefit potentials from the second data section are much larger than the corresponding ones from the first data section. This implies that the current operating point of the second data section is much farther away from its ideal optimal operating point than that of the first data section. Therefore, even the MVC benchmark shows that the second data section has better performance in the sense of output variance, the economic performance of the first data section is better than the second data section. This example indicates that better variance control does not necessarily imply better economic performance if the operating point is not optimal. As a matter of fact, the extra amount of benefit potential from the second data section has actually been achieved by the first data section.

6.5.4 Optimal tuning guidelines

The first step for solving the optimal tuning problems is to determine the desired target benefit potential on the basis of the optimal benefit potentials calculated in the previous subsection. For the variability tuning problem, the ideal benefit potential, ΔJ_I, is the maximum upper limit that can not be achieved practically because of the inevitable disturbance, and the theoretical benefit potential based on the MVC benchmark, ΔJ_T, is the maximum benefit potential that can be achieved if MVC is implementable and input constraint limits are sufficiently large. If the existing benefit potential, ΔJ_E, is greater than 0, then it should be achievable without any tuning effort. Therefore, it is reasonable to set the target benefit potential as a value which is greater than ΔJ_E and not larger than ΔJ_T. For the constraint tuning problem, however, there is no explicit maximum upper limit for the target benefit potential, ΔJ_I and ΔJ_T can still serve as references in establishing the target benefit potential. In addition, this target should be at least greater than ΔJ_E. As a consequence, the target benefit potential can be set as a value between 0 and 127.88 for the first data section, and a value between 135.0416 and 229.98 for the second data section.

Assume that the desired target benefit potential ratios by tuning variability and by tuning constraint respectively are both specified as $R_V = R_C = 70\%$ for the two data sections. Thus the target benefit potential is equal to 94.13 for the first data section and 167.16 for the second data section. Both values are within their corresponding reasonable ranges.

For the first data section, the solutions show that the target benefit potential 94.13 could be achieved by either tuning variability (Figure 6.11) or tuning constraint (Figure 6.12). The variability tuning guideline suggests to reduce the variability of CV1, CV10, CV11, CV22, CV32, CV33, CV38 by 78.43%, CV39 by 52.26%, CV41 by 57.97% relative to their current standard deviations (Figure 6.11). Alternatively, the constraint tuning guideline suggests to relax the constraints of CV1, CV18, CV19, CV22, CV32, CV33, CV41, MV9, MV10, MV15 by 4.91%, and CV10 by 3.88%, CV11 by 4.63%, CV31 by 0.38%, CV38 by 3.2%,CV39 by 1.81%,MV1 by 1.51% relative to their current constraint limit settings (Figure 6.12).

Similarly, for the second data section, the target benefit potential 167.16 could be also realized in terms of variability or constraint tuning. If the variability tuning is preferred, then the following controlled variables should be selected to reduce their variability relative to their current standard deviations: CV1 (30.91%), CV10 (30.91%), CV11 (30.91%), CV22 (30.91%), CV32 (30.91%), CV33 (30.91%), CV38 (30.91%), CV39 (14.22%), CV41 (3.75%) (Figure 6.13). Note that the percentage values in the brackets are the extent of the corresponding variability reduction. On the other hand, if the constraint limits

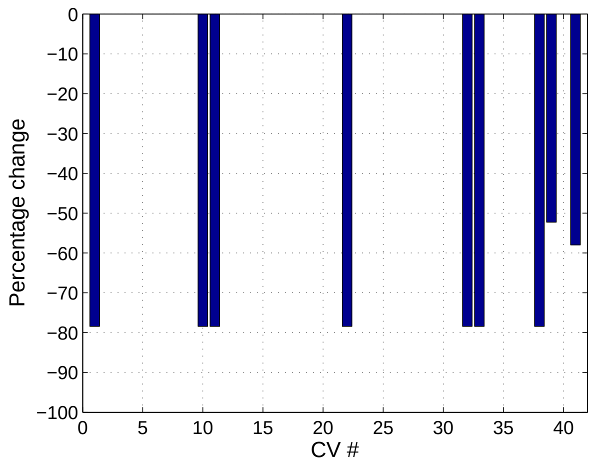

Figure 6.11: Suggested variability tuning guideline for the first data section

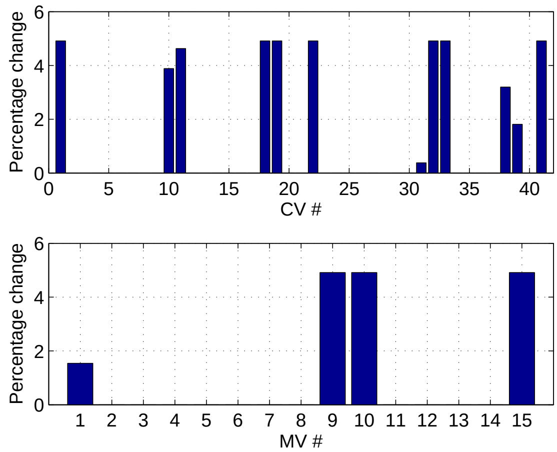

Figure 6.12: Suggested constraint tuning guideline for the first data section

are preferred to be tuned, then the following variables should be taken into account by relaxing their constraint ranges: CV1 (0.92%), CV10 (0.92%), CV11 (0.92%), CV18 (0.02%), CV22 (0.92%), CV32 (0.92%), CV33 (0.92%), CV38 (0.87%), CV39 (0.49%), CV41 (0.92%), MV2 (0.68%), MV9 (0.92%), MV10 (0.92%), MV13 (0.92%) and MV15 (0.92%) (Figure 6.14). Here the bracketed percentages are the extent of the corresponding constraint relaxation.

The variability and constraint tuning guidelines give two optional approaches that would make it possible to tune the control towards the desired benefit potential. Note that the above tuning solutions are only unique in the sense that the maximum tuning percentage change is minimized. In addition, practical process knowledge and experience should

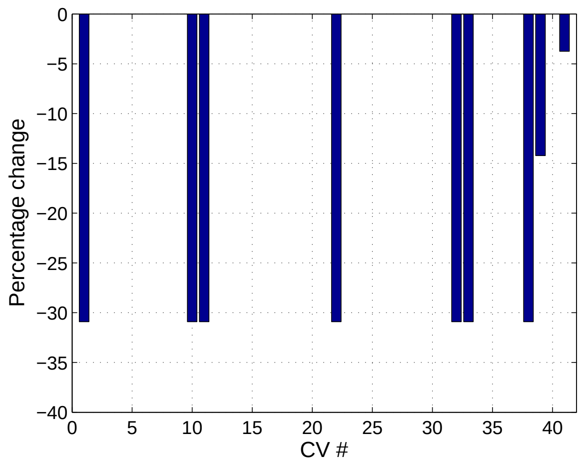

Figure 6.13: Suggested variability tuning guideline for the second data section

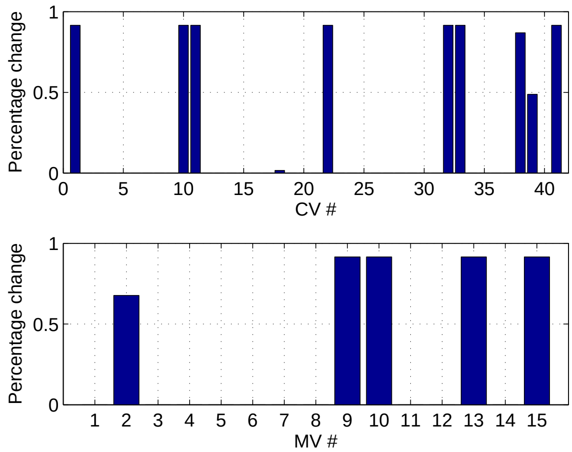

Figure 6.14: Suggested constraint tuning guideline for the second data section

be utilized to decide which one is preferred and whether it is allowed to do such kind of tuning in the real applications. In the case that some variables can not be tuned on either variability or constraint, these variables should be excluded from the corresponding objective functions by setting their coefficients to 0.

6.5.5 Sensitivity analysis

The sensitivity analysis is done by specifying the variability or constraint change percentage of only one variable at a time and all the other variables are kept unchanged. For the sensitivity analysis with respect to variability reduction, the variability of chosen variable is reduced by 1% and then the corresponding increased benefit potential beyond the existing benefit potential is observed and recorded. Similarly, for the sensitivity analysis with respect to constraint relaxation, the constraint range of chosen variable is increased by 1% to see the change of resultant benefit potential.

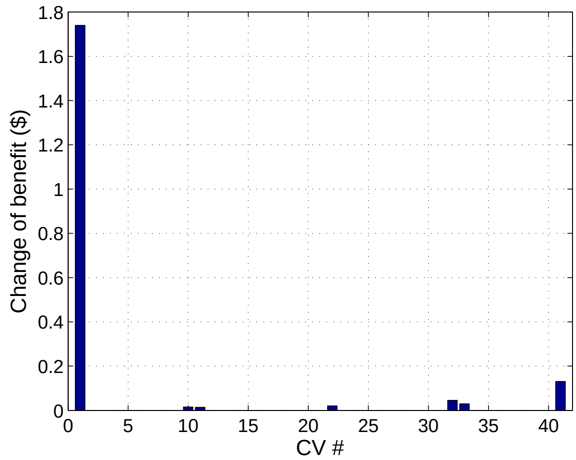

Figure 6.15: Variability sensitivity analysis for the first data section

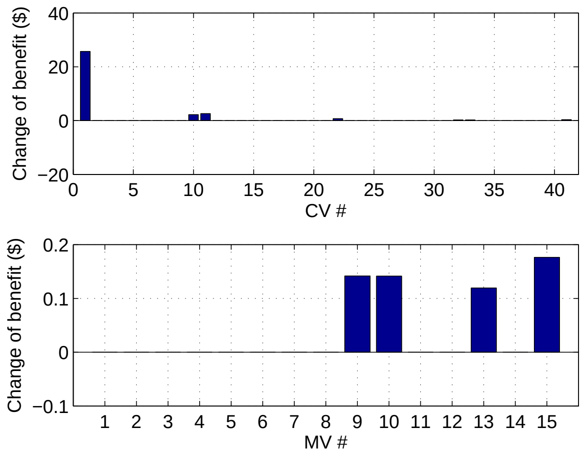

Figure 6.16: Constraint sensitivity analysis for the first data section

For simplicity, only the first data section is illustrated. The variability sensitivity analysis result (Figure 6.15) shows that the benefit potential would be increased by individual variability reduction on CV1 (1.7399), CV10 (0.0151), CV11 (0.0137), CV22 (0.0203), CV32 (0.0450), CV33 (0.0296) and CV41 (0.1305) based on their current standard deviation by 1%. It can be seen that the benefit potential is much more sensitive to reducing variability of CV1 than other variables, CV1 should be the first choice to reduce its variability if it is possible in order to improve the economic performance of this MPC application. Figure 6.16 gives the constraint sensitivity analysis result. The individual 1% constraint relaxation based on the current constraint limit settings of CV1, CV10, CV11, CV22, CV32, CV33, CV41, MV9, MV10, MV13 and MV15 will result in the increase of benefit potential at the amount of 25.6908, 2.2250, 2.5825, 0.7082, 0.2618, 0.2617, 0.3679, 0.1417, 0.1415, 0.1195 and 0.1764, respectively. Therefore, the benefit potential is also most sensitive to the constraint limit change on CV1.

Therefore, from the study of the two given data sections controlled by the investigated MPC application, CV1 is worthwhile to pay more attention than any other variables to both its variability and constraint limit settings. As a consequence, the results of variability and constraint sensitivity analysis show the importance of different variables on their contribution to the increased benefit potential and serve as a reference for MPC tuning on the variability and constraint of each individual variable.

6.6 Conclusion

This chapter focuses on the implementation aspect of APC performance monitoring. The PATS software package has been introduced briefly with its industrial implementation background and framework. A plant-oriented solution for APC performance monitoring was proposed on the basis of the two main application components from the PATS package: MVPA and LMIPA. Their algorithms were summarized as the FCOR algorithm for the variance performance assessment, and benefit potential analysis, optimal tuning guidelines and sensitivity analysis for the economic performance monitoring. The study of a real MPC application shows that a better variance performance does not always imply a better economic performance if the operating points are not optimized. The optimal benefit potentials indicate how much more economic performance can be achieved and the tuning guidelines tell how to achieve the target benefit potential by tuning either variability or constraint. The feasible results from the data analysis of the investigated MPC application denote that this MPC application has potential to have better variance and economic performance.

7 Conclusions and Future Work

7.1 Conclusions

In this thesis, we developed algorithms and tools for monitoring advanced control applications, including performance monitoring of control loops subject to LTV disturbance dynamics, control performance monitoring via structured closed-loop response subject to output variance/covariance upper bound, economic performance analysis and variability/constraint tuning guidelines of MPC applications, and implementation in the process industries. The major contributions are listed below.

- The regular, weighted and generalized LTVD benchmarks are given respectively as three options to solve the performance limit and performance assessment problems of control loops in the processes subject to a special class of LTV disturbances. With different objective functions, the weighted and generalized LTVD benchmarks can always lead to a better trade-off than the regular one in regulating different disturbances in the sense of variance. This applies to both SISO and MIMO systems.
- Necessary and sufficient conditions are derived for the feasibility of full order and reduced order structured closed-loop response subject to output variance/covariance upper bound constraint. It is shown that the full order solution can be obtained in terms of a set of LMIs while the reduced order solution can be generally formulated as a set of BMIs. In particular, when the structured closed-loop response is chosen as

a first order or second order system with time constant specified, the reduced order feasibility problem can also be solved via a set of LMIs.

- A practical solution of the structured closed-loop response is provided as solving an $\mathcal{H}_\infty$ norm minimization problem such that it is as close to the existing closed-loop response as possible in the sense of $\mathcal{H}_\infty$ norm and the output variance/covariance upper bound constraint is satisfied simultaneously.

- A systematic approach is proposed to evaluate the performance of MPC applications with consideration of variability and constraint settings, which is carried out by employing the back off approach and steady state economic optimization of the existing MPC applications. It includes economic performance assessment, sensitivity analysis and variability/constraint tuning guidelines.

- It is shown that the variance based performance assessment may be transferred to economic performance assessment of MPC applications. A theoretical maximum benefit potential is calculated on the basis of the variance based performance improvement potential. The result can be utilized as a reference of target benefit potential for the variability/constraint tuning guidelines.

- A new software package is developed for the process industries and a plant-oriented solution is put forward to monitor the performance of MPC applications. It integrates the real-time data collector, MVPA for variance performance assessment, LMIPA for economic performance analysis and variability/constraint tuning guidelines into the same one platform. This package can be directly applied to process industries.

7.2 Directions for future work

At the end of each individual chapter we summarized the work that we have done. Some relevant issues of this thesis are discussed below that could be conducted in the future.

- When the structured closed-loop response is required to meet some user specified specification and the controller structure is fixed, i.e., PID in most situations, what should be the tuning parameters? This would help tune the controller into the right parameters such that the user specified structured closed-loop response is satisfied.

- The variability/constraint tuning guidelines for the same target benefit potential may not be unique in the MPC performance analysis due to the optional objective functions. In this thesis, we tried to minimize the maximum tuning rate for given target benefit potential. Some other objective functions can also be selected.

- In the variability/constraint sensitivity analysis of MPC applications, the results are simply calculated by a small change on the variability/constraint of one variable only in a sequence. However, the sensitivity of changes on a set of variables is not tried. This may be done by introducing integer programming.

- Even though part of MPC performance monitoring results has been verified on a pilot-scale multi-tank system, the results of MPC performance monitoring in the reactor section of GOHTU should be further verified in the real process.

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