2006 IFAC International Symposium for Advanced Control of Chemical Processes (ADCHEM), Brazil, April 2-5, 2006.

ASSESSMENT OF ECONOMIC PERFORMANCE OF MODEL PREDICTIVE CONTROL THROUGH VARIANCE/CONSTRAINT TUNING

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Abstract: Multivariate controller performance assessment (MVPA) has been developed over the last several years, but its application in advanced model predictive control (MPC) has been very limited mainly due to issues associated with comparability of variance control objective and that of MPC. MPC has been proven as one of the most effective advanced process control (APC) strategies to deal with multivariable constrained control problems with an ultimate objective towards economic optimization. Any attempt to evaluate MPC performance should therefore consider constraints and economic performance. This work is to establish a link between variance control and MPC in terms of economic performance. We show that the variance based performance assessment may be transfered to economic assessment of MPC. Algorithms for economic performance assessment and tuning are developed through linear matrix inequalities using routine operating process data. The proposed algorithms are illustrated via an industrial MPC application example.

Keywords: performance assessment, model predictive control, economic performance assessment

1. INTRODUCTION

Model predictive control (MPC) has been proven as one of the most effective advanced process control (APC) strategies to deal with multivariable constraint control problems However, less efforts have been made on the performance evaluation of existing MPC applications, especially on economic performance.

Although MVPA has been developed over last several years, its application on MPC evaluation has been very limited mainly due to issues associated with comparability of MVC objective and that of MPC strategy. One of the most important incentives of MPC applications is to deal with multivariable constrained control problems with an ultimate objective on economic optimization. Any attempt to evaluate MPC performance should therefore consider constraints and economic benefits.

In traditional economic benefit analysis, the back off approach has been applied in the benefit analysis of improved process control. The benefit potential is achieved against the base case operation by reducing the variance of quality variables and pushing the average values closer to the optimum point or constraint limit (Muske, 2003). The base case operation should be a period of typical closed-loop operation with the existing control system. Benefit analysis for different base case conditions should be done separately since they may lead to different economic benefit values (Muske, 2003). An appropriate back off away from the constraint limit should be introduced and the optimal operation is too conservative if the constraint limit is never violated. Many different rules have been discussed for allowable constraint violation. A reasonable rule should be adopted in terms of base case condition and desired specifications, e.g., 5 percentage of violation. Once the base case operation and the optimal operation condition are both identified, the economic benefit potential is readily obtained when the economic objective function is explicitly established.

In the latest generation MPC algorithms, a separate steady state optimization is performed at each control cycle in order to drive steady state inputs and outputs to their optimal economic targets For example, industrial model predictive control integrates a linear program (LP) and/or a quadratic program (QP) for economic optimization. Since this LP or QP reflects economic objective explicitly, it can be utilized to evaluate the economic performance of MPC applications. This work is to establish a link between variance control and MPC in terms of economic performance. We show that the variance based performance assessment may be transfered to economic assessment of MPC. Algorithms for economic performance assessment and tuning are developed through linear matrix inequalities using routine operating process data. The proposed algorithms are illustrated via an industrial MPC application example.

The remainder of this paper is organized as follows. In Section 2 several different scenarios are described in the form of constrained quadratic optimization problems. Section 3 presents and explains a systematic approach for the purpose of economic performance assessment. The QP problem is reformulated as LMI in Section 4. An industrial MPC application is evaluated for economic performance in Section 5, followed by concluding remarks in Section 6.

NOTATION

a_{ki}	quadratic coefficient of i^{th} output variable
a_{kj}	quadratic coefficient of j^{th} input variable
b_{ki}	linear coefficient of i^{th} output variable
b_{kj}	linear coefficient of j^{th} input variable
u_{dkj}	target value of j^{th} input variable
u_{kj0}	sampled value of j^{th} input variable
y_{dki}	target value of i^{th} output variable
y_{ki0}	sampled value of i^{th} output variable
K_{ij}	the steady state gain value
N_u	the number of input variable
N_y	the number of output variable
R_{uj}	changing ratio of j^{th} input variable
R_{yi}	changing ratio of i^{th} output variable
U_{holkj}	half of constraint range of j^{th} input variable
U_{qorj0}	quarter of range of j^{th} input variable
U_{Hkj}	high limit of j^{th} input variable
U_{Lkj}	low limit of j^{th} input variable
Y_{holki}	half of constraint range of i^{th} output variable
Y_{stdi0}	standard deviation of i^{th} output variable
Y_{Hki}	high limit of i^{th} output variable
Y_{Lki}	low limit of i^{th} output variable

2. PROBLEM DESCRIPTION

For illustration, assume a multivariable process consists of only two controlled variables with interaction, where y_1 is a quality variable that has direct impact on profit and y_2 is a constrained variable (Figure 1). Because of disturbances, there is variability on both y_1 and y_2. Assuming the optimal operating condition of y_1 is located on its upper limit, it is clear from the figure that the actual average operating condition (dash line) is not at its optimal operating condition, leading to lost profit. The base case operation is defined by its current mean values and variances. A reasonable percentage of constraint violation, 5%, is adopted such that 95% of operation falls within the range of ± 2 times standard deviation. Since the benefit potential is calculated against that of base case operation in all scenarios, in the following we list the problem formulations of optimal operations for different scenarios with quadratic economic objective function in the steady state optimization.

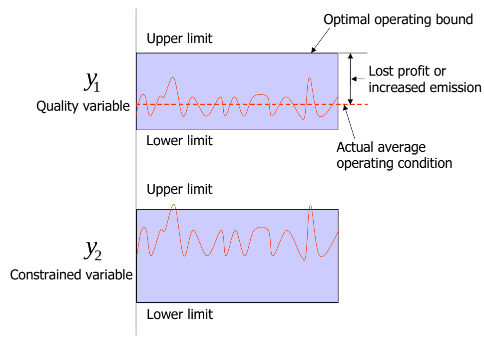

Figure 1. Base case operation

2.1 *Assessment of ideal yield*

In the ideal scenario strict steady state operation is considered and there is no variability on both y_1 and y_2 shown in Figure 1. Under this scenario the operation of y_1 can be pushed closest to its optimal operating point, upper limit in this example. In this case the operating points of y_1 and y_2 are the decision variables, and the corresponding optimization problem can be formulated as follows.

$$\min_{\bar{y}_i,\bar{u}_j} J = \frac{1}{N_L}\sum_{k=1}^{N_L} J_k \tag{1}$$

subject to

$$\begin{aligned} Y_{Lki} \le \bar{y}_i \le Y_{Hki}, \quad i = 1,\cdots,N_y \\ U_{Lkj} \le \bar{u}_j \le U_{Hkj}, \quad j = 1,\cdots,N_u \end{aligned} \tag{2}$$

$$\begin{aligned} &\sum_{j=1}^{N_u}[K_{ij} \times \Delta\bar{u}_j] = \Delta\bar{y}_i, i = 1,\cdots,N_y \\ &\bar{y}_i = \bar{y}_{i0} + \Delta\bar{y}_i, \bar{y}_{i0} = \frac{\sum_{k=1}^{N_L} y_{ki0}}{N_L}, i = 1,\cdots,N_y \\ &\bar{u}_j = \bar{u}_{j0} + \Delta\bar{u}_j, \bar{u}_{i0} = \frac{\sum_{k=1}^{N_L} u_{ki0}}{N_L}, j = 1,\cdots,N_u \end{aligned} \tag{3}$$

where N_L is the sampled data length, and

$$\begin{aligned} J_k = &\sum_{i=1}^{N_y}\left[b_{ki} \times \bar{y}_i + a_{ki}^2(\bar{y}_i - y_{dki})^2\right] + \\ &\sum_{j=1}^{N_u}\left[b_{kj} \times \bar{u}_j + a_{kj}^2(\bar{u}_j - u_{dkj})^2\right], k = 1,2,\cdots,N_L \end{aligned}$$

2.2 Assessment of optimal yield without tuning control

Consider there is no change in the control tuning, i.e., all variability and constraints remain the same as the base case operation. Compared with base case operation, this scenario considers to move the actual average operating point of y_1 to its optimal operating condition as close as possible without tuning control. This is achieved simply by mean shift, and the distance between average operating point and the optimal operating point could be reduced significantly, meaning increased profit. The inequalities in (2) now become

$$\begin{aligned} Y_{Lki} + 2 \times Y_{stdi0} \le \bar{y}_i \le Y_{Hki} - 2 \times Y_{stdi0} \\ U_{Lkj} + 2 \times U_{qorj0} \le \bar{u}_j \le U_{Hkj} - 2 \times U_{qorj0} \\ i = 1,\cdots,N_y, \quad j = 1,\cdots,N_u \end{aligned}$$

2.3 Assessment of improved yield by reducing variability - relation between economic assessment and MVPA

Under this scenario, we consider the tuning of the control such that the variability of y_1 and/or y_2 can be reduced. Reduction of variability can obviously yield opportunity to push operating point closer to the optimum. The potential of variability reduction can be estimated through performance assessment. In general, however, the reduced variability of one variable can transfer to the increased variability of other variables such as constrained variable y_2. Since y_2 has no direct impact on profit, its variability is not of concern as far as it falls within its constraint. Therefore, the variability of quality variable y_1 may be reduced by transferring the variability to the constrained variable y_2. This type of interacting variance reduction is assessed by MVPA (Huang and Shah, 1999). Here, we introduce two variables, the ratios R_y and R_u, which are defined as the ratio of the targeted variance reduction of input/output variables and the existing variance of input/output variables. Likewise, the inequalities in (2) can now be modified as

$$\begin{aligned} &Y_{Lki} + 2 \times Y_{stdi0}(1 + R_{yi}) \le \bar{y}_i \le Y_{Hki} - \\ &\quad 2 \times Y_{stdi0}(1 + R_{yi}), \quad i = 1,\cdots,N_y \\ &U_{Lkj} + 2 \times U_{qorj0}(1 + R_{uj}) \le \bar{u}_j \le U_{Hkj} - \\ &\quad 2 \times U_{qorj0}(1 + R_{uj}), \quad j = 1,\cdots,N_u \end{aligned}$$

R_y and R_u are specified by users but their magnitudes should not be below -1. We call this procedure as benefit potential assessment based on variance reduction. Using MVPA, the potential variability reduction can be readily calculated. However, for MVPA with minimum variance control (MVC) as the benchmark, only variance reduction of output variables can be estimated, and thus it will be limited to the benefit assessment of output variables. For MVPA with LQG as the benchmark (Huang and Shah, 1999), both input and output variables can be considered. As a consequence, we can get an theoretical absolute optimal benefit potential with MVC or LQG as the benchmark.

2.4 Assessment of improved yield by relaxing constraints

If the constraints can be relaxed for all or some variables, the operating condition may be moved further in the direction of increased profit. This move is mainly due to changes of constraints in constraint variables. As a consequence, this will create a new opportunity to transfer more variability from y_1 to y_2 and thus the variability of y_1 could be reduced to increase profit. In this case, we introduce two ratios, S_y and S_u, which are defined as the ratio of targeted (often increased) constraint and the existing constraint. They can be specified by the user, and we call this procedure as benefit potential assessment based on relaxation of constraints. The inequalities in (2) can be modified to accommodate this assessment as

$$\begin{aligned} &Y_{Lki} - S_{yi} \times Y_{holki} + 2 \times Y_{stdi0} \le \bar{y}_i \le Y_{Hki} + \\ &\quad S_{yi} \times Y_{holki} - 2 \times Y_{stdi0}, \quad i = 1,\cdots,N_y \end{aligned}$$

$$U_{Lkj} - S_{uj} \times U_{holkj} + 2 \times U_{qorj0} \leq \bar{u}_j \leq U_{Hkj} + S_{uj} \times U_{holkj} - 2 \times U_{qorj0}, \quad j = 1, \cdots, N_u$$

2.5 Assessment of improved yield by reducing variability and relaxing constraint simultaneously

Since benefit potential could be achieved by either reducing variability or relaxing constraint, they can also be considered simultaneously, hoping to achieve higher yield. In this case, the inequalities in (2.3) and (2.4) can be combined as

$$\begin{aligned} &Y_{Lki} - S_{yi} \times Y_{holki} + 2 \times Y_{stdi0}(1 + R_{yi}) \leq \bar{y}_i \leq \\ &\quad Y_{Hki} + S_{yi} \times Y_{holki} - 2 \times Y_{stdi0}(1 + R_{yi}), \\ &U_{Lkj} - S_{uj} \times U_{holkj} + 2 \times U_{qorj0}(1 + R_{uj}) \leq \bar{u}_j \leq \\ &\quad U_{Hkj} + S_{uj} \times U_{holkj} - 2 \times U_{qorj0}(1 + R_{uj}), \\ &\qquad i = 1, \cdots, N_y, \quad j = 1, \cdots, N_u \end{aligned}$$

2.6 Variability tuning for desired yield

In the benefit potential assessment, the ratios R_y and R_u are specified *a priori* by users. If, instead, we use them as decision variables, the optimal R_y and R_u can be found from optimization accordingly. For notation purpose, we use r_y and r_u in place of R_y and R_u , respectively. A targeted ratio, R_V, is defined as the ratio between the targeted benefit and ideal benefit, where R_V should be within 0 and 1. Given R_V, the ratios r_y and r_u may be calculated but the solutions are not unique. However, to minimize tuning effort, we would want r_y and r_u as smaller as possible. The minimum r_y and r_u may be found through the optimization of the following problem:

$$\min_{\bar{y}_i, \bar{u}_j, r_{yi}, r_{uj}, r} -r \tag{4}$$

subject to

$$\begin{aligned} &Y_{Lki} + 2 \times Y_{stdi0}(1 + r_{yi}) \leq \bar{y}_i \leq Y_{Hki} - \\ &\quad 2 \times Y_{stdi0}(1 + r_{yi}), \quad i = 1, \cdots, N_y \\ &U_{Lkj} + 2 \times U_{qorj0}(1 + r_{uj}) \leq \bar{u}_j \leq U_{Hkj} - \\ &\quad 2 \times U_{qorj0}(1 + r_{uj}), \quad j = 1, \cdots, N_u \\ &-1 < r_{yi}, -1 < r_{uj}, r_{yi} > r, r_{uj} > r \\ &J_k = R_V \times J_{k0}, \quad Equalities \quad in \quad (3) \end{aligned}$$

2.7 Constraint tuning for desired yield

If the variability could not be reduced further, we may achieve desired benefit potential by tuning the constraints. Similarly, a desired ratio, R_C, is defined as the targeted benefit against that of ideal yield. We would also want the change of the constraints, s_y and s_u (a counterpart of S_y and S_u defined before), to be as small as possible. The minimum s_y and s_u can be solved through

$$\min_{\bar{y}_i, \bar{u}_j, s_{yi}, s_{uj}, s} s \tag{5}$$

subject to

$$\begin{aligned} &Y_{Lki} - s_{yi} \times Y_{holki} + 2 \times Y_{stdi0} \leq \bar{y}_i \leq Y_{Hki} + \\ &\quad s_{yi} \times Y_{holki} - 2 \times Y_{stdi0}, \quad i = 1, \cdots, N_y \\ &U_{Lkj} - s_{uj} \times U_{holkj} + 2 \times U_{qorj0} \leq \bar{u}_j \leq U_{Hkj} + \\ &\quad s_{uj} \times U_{holkj} - 2 \times U_{qorj0}, \quad j = 1, \cdots, N_u \\ &s_{yi} < s, s_{uj} < s \\ &J_k = R_C \times J_{k0}, \quad Equalities \quad in \quad (3) \end{aligned}$$

3. SYNTHESIS APPROACH

Economic evaluation of MPC applications includes economic performance assessment, sensitivity analysis and tuning guidelines, which are considered in this section.

3.1 Economic performance assessment

In the assessment of ideal yield, the optimal operating condition is expected to be pushed closest to the constraint for the quality variable without back off. However, in the assessment of optimal yield without tuning control, the benefit potential is obtained by only shifting the mean values of quality variables in the direction of increasing benefit potential without reducing variability and hence the back off depends on the present level of disturbances. By comparing these two scenarios, an economic performance index without tuning can be defined as

$$\eta_E = \frac{\Delta J_E}{\Delta J_I}$$

where ΔJ_E is the optimal yield without tuning control and ΔJ_I is the ideal yield. η_E is the benefit potential ratio that can be realized by just pushing the mean values without reducing the variability, while $1 - \eta_E$ is the benefit potential ratio that is due to no variability. It is noted that $0 \leq \eta_E \leq 1$. If $\eta_E = 0$, no benefit could be obtained without reducing the variability. If $\eta_E = 1$, there is no disturbance and hence no back off is required under the current control strategy. By introducing MVC or LQG benchmark, MVPA gives a theoretical absolute variance lower bound; thus a theoretical economic performance index can be calculated as

$$\eta_T = \frac{\Delta J_T}{\Delta J_I}$$

where ΔJ_T is the theoretical benefit potential upper bound that could be achieved by MVC or LQG plus steady state optimization. ΔJ_T is in part due to the mean value shift and in part due to the variability reduction. It can be seen that $0 \le \eta_T \le 1$. By comparing with η_E, the following inequality holds,

$$0 \le \eta_E \le \eta_T \le 1$$

Therefore, if no variability can be reduced, η_E (or ΔJ_E) can be adopted in the economic performance assessment. $\eta_E = 0$ (or $\Delta J_E = 0$) shows that no benefit potential can be further obtained without tuning the controller. On the other hand, if the benchmark of MVPA is available, η_T (or ΔJ_T) can be utilized instead, which gives an absolute upper bound on the economic benefit potential that could be realized theoretically. The positive value of η_T (or ΔJ_T) does not mean that this benefit potential is practically achievable since MVC itself is usually not applied in practice, but the benefit potential with positive value of η_E (or ΔJ_E) is really achievable.

3.2 Sensitivity analysis

Sensitivity analysis is applied to investigate the impact of variability change or constraint change on the benefit potential. The result shows the importance of different variables based on their contributions to the benefit potential. The size and direction of the change of variability or constraint can be specified by the user. Some constrained variables may have no impact on the benefit potential, and some variables may have great impact on the benefit potential that should be paid special attention on during the operation. It is thus worthwhile to reduce the variability or relax the constraint of these variables such that more benefit potential can be achieved.

3.3 Tuning guideline

As analyzed in the economic performance, ΔJ_I or ΔJ_T can be regarded as an upper bound on the benefit potential against which other scenarios can be compared. The desired potential benefit can never be greater than this upper bound by tuning variability only. Once the desired variability ratio R_V or the desired constraint ratio R_C is specified, the corresponding optimization problem will result in the tuning guideline for variability reduction or constraint relaxation. The tuning guideline tells directly which variables should be tuned and by how much in order to achieve the desired benefit potential.

4. LMI FORMULATION

The quadratic objective function in (1) alone can be transformed into

$$\min_{\bar{y}_i, \bar{u}_j} \gamma \tag{6}$$

subject to

$$\sum_{k=1}^{N_L} \left\{ \sum_{i=1}^{N_y} \left[b_{ki} \times \bar{y}_i + a_{ki}^2 (\bar{y}_i - y_{dki})^2 \right] + \sum_{j=1}^{N_u} \left[b_{kj} \times \bar{u}_j + a_{kj}^2 (\bar{u}_j - u_{dkj})^2 \right] \right\} < \gamma \tag{7}$$

According to *Schur* complement, it can be readily formulated as follows and then solved via LMI technique,

$$\begin{pmatrix} \gamma - Y_{lin} - U_{lin} & X_{lin}^T \\ X_{lin} & I \end{pmatrix} \succ 0 \tag{8}$$

where

$$X_{lin} = \left[\left(\sqrt{\sum_{k=1}^{N_L} a_{k1}^2} \right) \bar{y}_1 \cdots \left(\sqrt{\sum_{k=1}^{N_L} a_{kN_y}^2} \right) \bar{y}_{N_y} \left(\sqrt{\sum_{k=1}^{N_L} a_{k1}^2} \right) \bar{u}_1 \cdots \left(\sqrt{\sum_{k=1}^{N_L} a_{kN_u}^2} \right) \bar{u}_{N_u} \right]^T$$

$$Y_{lin} = \sum_{i=1}^{N_y} \left\{ \sum_{k=1}^{N_L} \left(b_{ki} - 2a_{ki}^2 y_{dki} \right) \bar{y}_i \right\} + \sum_{i=1}^{N_y} \left\{ \sum_{k=1}^{N_L} \left(a_{ki}^2 y_{dki}^2 \right) \right\}$$

$$U_{lin} = \sum_{j=1}^{N_u} \left\{ \sum_{k=1}^{N_L} \left(b_{kj} - 2a_{kj}^2 y_{dkj} \right) \bar{u}_j \right\} + \sum_{j=1}^{N_u} \left\{ \sum_{k=1}^{N_L} \left(a_{kj}^2 y_{dkj}^2 \right) \right\}$$

5. CASE STUDY

5.1 Process and controller description

An MPC is applied in the reactor section of the gas oil hydrotreating unit (GOHTU) to maintain gas oil nitrogen/sulphur specifications, maximize catalyst run length, minimize hydrogen/fuel gas consumption and improve operation safety. It has total 41 output variables (y), 15 input variables (u) and 5 disturbance variables (d). The real-time data collected for this analysis include all y, u, d and associated parameters, such as high/low limits, linear coefficients, quadratic coefficients and targeted steady state values. The data collection lasted for approximately 26.5 hours with sampling time 15 second and total 6350 data points were collected.

5.2 Economic performance assessment

The result shows that $\Delta J_I = 196.7428$ and $\Delta J_E = 13.0889$. The performance indices from

MVPA with MVC as the benchmark are given in Figure 2 and accordingly $\Delta J_T = 186.4489$. The economic performance index is calculated as $\eta_E = 6.7\%$, one can thus conclude that the steady-state operation of this MPC has achieved good economic performance given the existing variability within the set of data studied, and the potential for improved benefit is rather small.

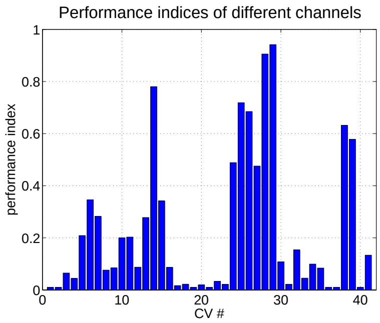

Figure 2. MVPA performance assessment result

5.3 *Sensitivity analysis*

In the variability sensitivity analysis, the variability of chosen variable was reduced 1% to observe its impact on the benefit potential. The result shows that output variables $(y_1, y_{10}, y_{11}, y_{22}, y_{32}, y_{33}, y_{41})$ and input variables (u_9, u_{10}, y_{15}) have effects on the benefit potential and other variables have no effect at all. The impacts of output variable y_1 and input variable u_{15} are much greater than other sensitive variables, which means output variable y_1 and input variable u_{15} should be the first choice to reduce their variability if it is possible. Similarly, the constraint sensitivity analysis shows that output variables $(y_1, y_{10}, y_{11}, y_{18}, y_{19}, y_{22}, y_{32}, y_{33}, y_{38}, y_{39}, y_{41})$ and input variables $(u_1, u_9, u_{10}, u_{15})$ have effects on the benefit potential while the other variables not at all. The sensitivity analysis shows the importance of different variables in the sense of economic performance. For example, the variability sensitivity analysis for output variables is shown in Figure 3.

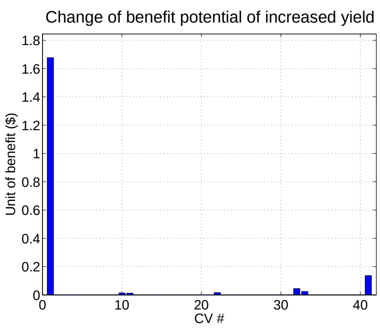

Figure 3. Output variability sensitivity analysis

5.4 *Optimal tuning guidelines*

The desired variability ratio and constraint ratio are both specified as $R_V = R_C = 80\%$ and the desired benefit potential is equal to 157.3942. This target benefit potential may be achieved by either reducing the variability of output variables $(y_1, y_{10}, y_{11}, y_{22}, y_{32}, y_{33}, y_{38}, y_{39}, y_{41})$ and input variables $(u_1, u_2, u_9, u_{10}, u_{15})$ as suggested by variability tuning guideline, or, relaxing the constraint ranges of output variables $(y_1, y_{10}, y_{11}, y_{18}, y_{19}, y_{22}, y_{32}, y_{33}, y_{38}, y_{39}, y_{41})$ and input variables $(u_1, u_9, u_{10}, u_{15})$ as suggested by constraint tuning guideline. The tuning guideline is shown as percentages. The variability tuning guideline for output variables is shown in Figure 4.

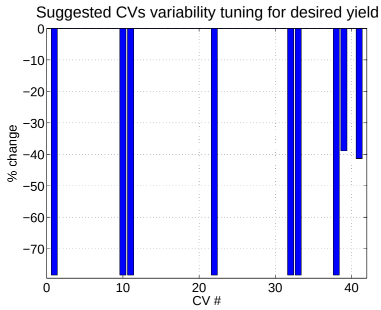

Figure 4. Output variability tuning guideline

6. CONCLUSION

A synthesized approach is proposed for MPC economic performance assessment based on its steady state optimization and variance/constraint tuning. It shows that further benefit potential could be achieved by optimizing its steady state, reducing variability or increasing constraint ranges. The case study demonstrates that it is a powerful tool for the control engineers in the economic performance assessment for existing MPC applications. This tool has been integrated together with MVPA which gives the variability potential improvement. This variability potential could be converted to its economic benefit potential. Synthesis of these two tools will give not only the MPC performance on variance reduction but also its economic benefit potential. They have been integrated into a plant oriented solution for APC performance monitoring.

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