Supporting Information for: Model Specification and Data Aggregation for Emergency Services Facility Location

Draft Date: 6 August 2014

Appendix A: Coverage Calculations

In this appendix, we discuss how we compute the coverage parameter P_{ij} for the deterministic and the probabilistic model, for a Demand Node *i* that is a distance d_{ij} from Station *j*. We drop the *i* and *j* subscripts to simplify the notation, and focus on computation of the coverage parameter *P* for a demand node that is a distance *d* from the ambulance dispatched to respond. In the probabilistic model, the coverage probability *P* equals $\Pr(R(\mathrm{d}) \le t_c)$, where the response time $R(d)$ equals the sum $S + T(d)$ of the "setup time" *S* that occurs before the ambulance starts traveling to the call address, and the travel time $T(d)$ for a distance *d*, and t_c is the coverage time threshold.

We follow the approach in Budge et al. (2010), who found that travel times are well approximated by a log-*t* distribution with distance-dependent median $m(d)$ and coefficient of variation $c(d)$ specified as follows:

$$T(d) = m(d)\exp(c(d)\varepsilon)$$
$$m(d) = \begin{cases} 2\sqrt{d/a} & d \le v_c^2/a \\ v_c/a + d/v_c & d > v_c^2/a \end{cases}$$
$$c(d) = \frac{\sqrt{b_0(b_2+1) + b_1(b_2+1)m(d) + b_2 m(d)^2}}{m(d)}$$
$$\varepsilon \sim \text{Student } t \text{ distributed with } \tau \text{ degrees of freedom}$$

We also model the setup time *S* as log-*t* distributed with median m_S, coefficient of variation c_S, and degrees of freedom τ_S. Thus, the distribution of the response time *R* is the convolution of two log-*t* distributions, which does not have a closed form; one can use either

Monte Carlo simulation or numerical integration (easily programmed in Matlab, for example) to evaluate the cumulative distribution function $F_{R(d)}(x)$, however.

We estimated the parameters of the travel time and setup time distributions using data from high-priority EMS calls in Edmonton in 2008 and obtained maximum likelihood estimates of v_c = 29.65 m/s = 106.7 km/hr., a = 0.0317 m/s^2 = 6.84 km/hr./min., (b_0, b_1, b_2) = (0.3720, 0.9599, 0.0589), and $\tau = 3$, m_S = 84.7 s, c_S = 0.42, and $\tau_S = 5$.

For the deterministic model, we set P equal to 1 for distances $d \le d_c$ and 0 for $d > d_c$. We select the coverage distance threshold d_c as the α-quantile of the distribution for the distance that an ambulance can reach within the coverage time threshold t_c. More precisely, we numerically solve the equation $\Pr(R(d_c) \le t_c) = \alpha$ to obtain d_c, using a standard root-finding procedure in Matlab. In the remainder of this appendix, we provide the complete Matlab code that we used to compute the P_{ij} and d_c:

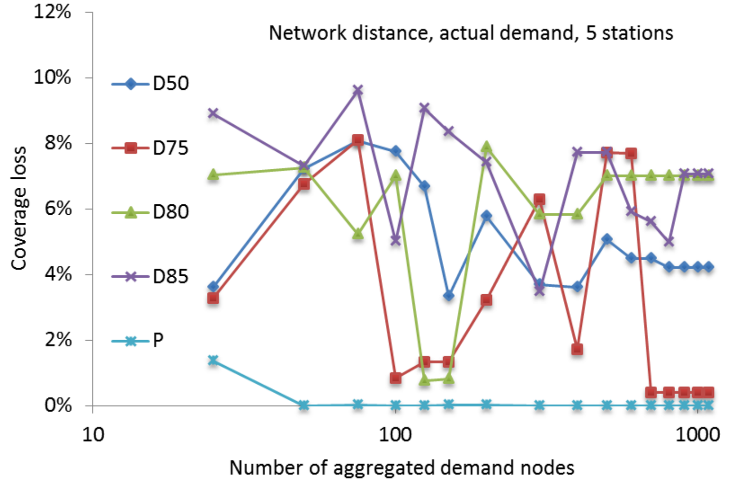
```
function p=responsetimeCDF(x,ms,cs,taus,mt,ct,taut)
p=quadl(@(r) responsetimeintegrand(r,x,ms,cs,taus,mt,ct,taut),0,x);
end

function g=responsetimeintegrand(r,x,ms,cs,taus,mt,ct,taut)
g=setuptimePDF(r,ms,cs,taus).*traveltimeCDF(x-r,mt,ct,taut);
end

function x=responsetimequantile(p,ms,cs,taus,mt,ct,taut)
x=fzero(@(y) responsetimeCDF(y,ms,cs,taus,mt,ct,taut)-p,[eps
1000*(ms+mt)]);
end

function f=setuptimePDF(x,m,c,tau)
f=(1./(c*x)).*tpdf((log(x)-log(m))/c,tau);
end

function p=traveltimeCDF(x,m,c,tau)
p=tcdf((log(x)-log(m))/c,tau);
end
```

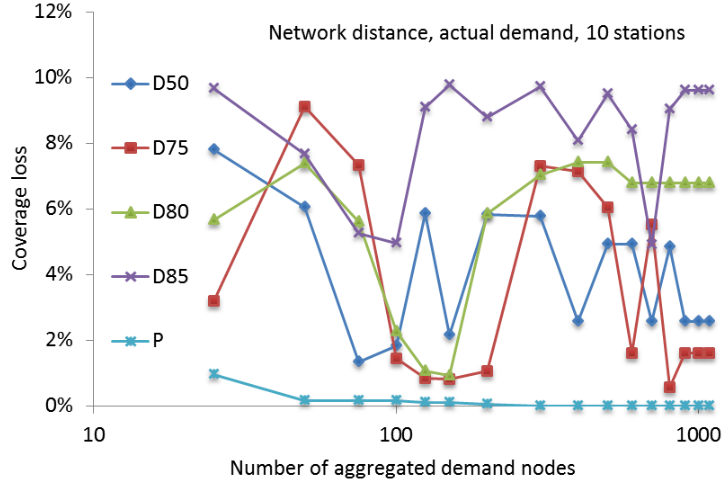
```
function c=traveltimeCV(d,m,b0,b1,b2)
c=sqrt(b0*(b2+1)+b1*(b2+1)*m+b2*m^2)/m;
end

function m=traveltimemedian(d,vc,a)
dc=vc^2/(2*a);
if d<=2*dc,
    m=sqrt(d/a);
else
    m=vc/a+d/vc;
end
```

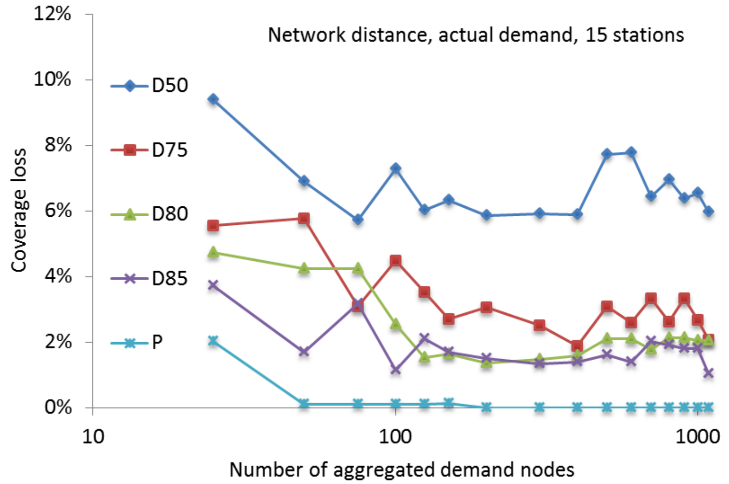
Appendix B: Aggregation Scheme C from Daskin et al. (1989)

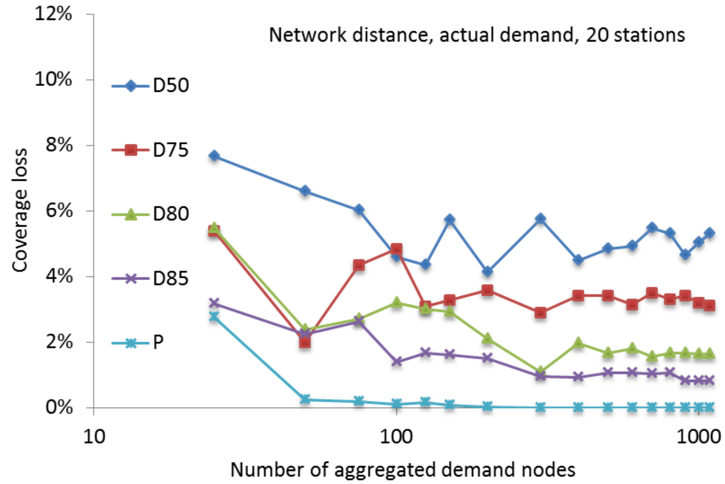
Select aggregate demand nodes as follows. First, select the demand node with the greatest demand. Second, select the demand node that is farthest from the first aggregate node. To select the third and subsequent aggregate nodes, assign each demand node that has not been selected as an aggregate node to its closest aggregate node. Select as the next aggregate node the demand node with the largest product of demand and distance to the aggregate node to which it is assigned. Continue until the required number of aggregate nodes has been selected.

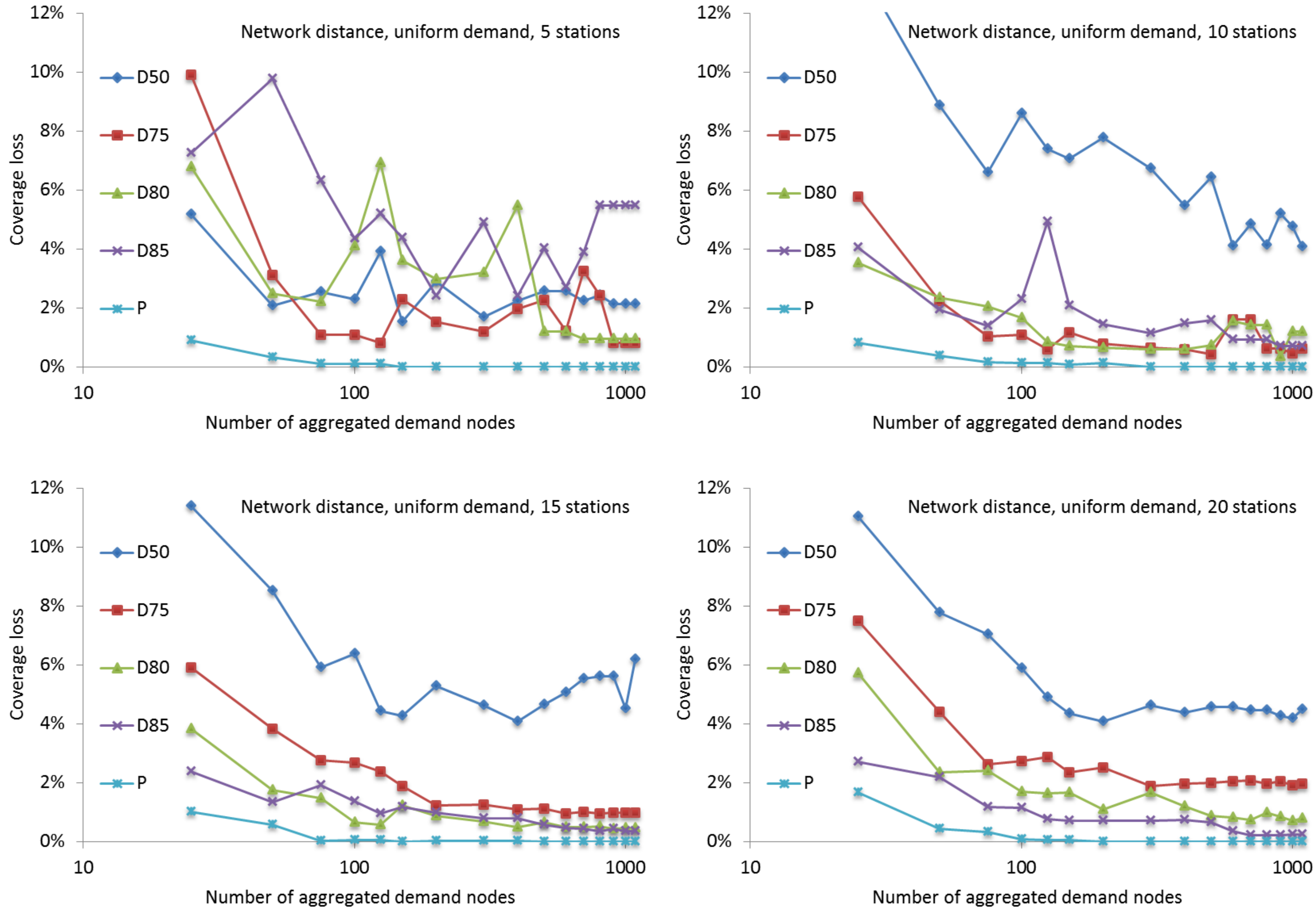
Appendix C: Graphs Relative Coverage Loss versus Aggregation

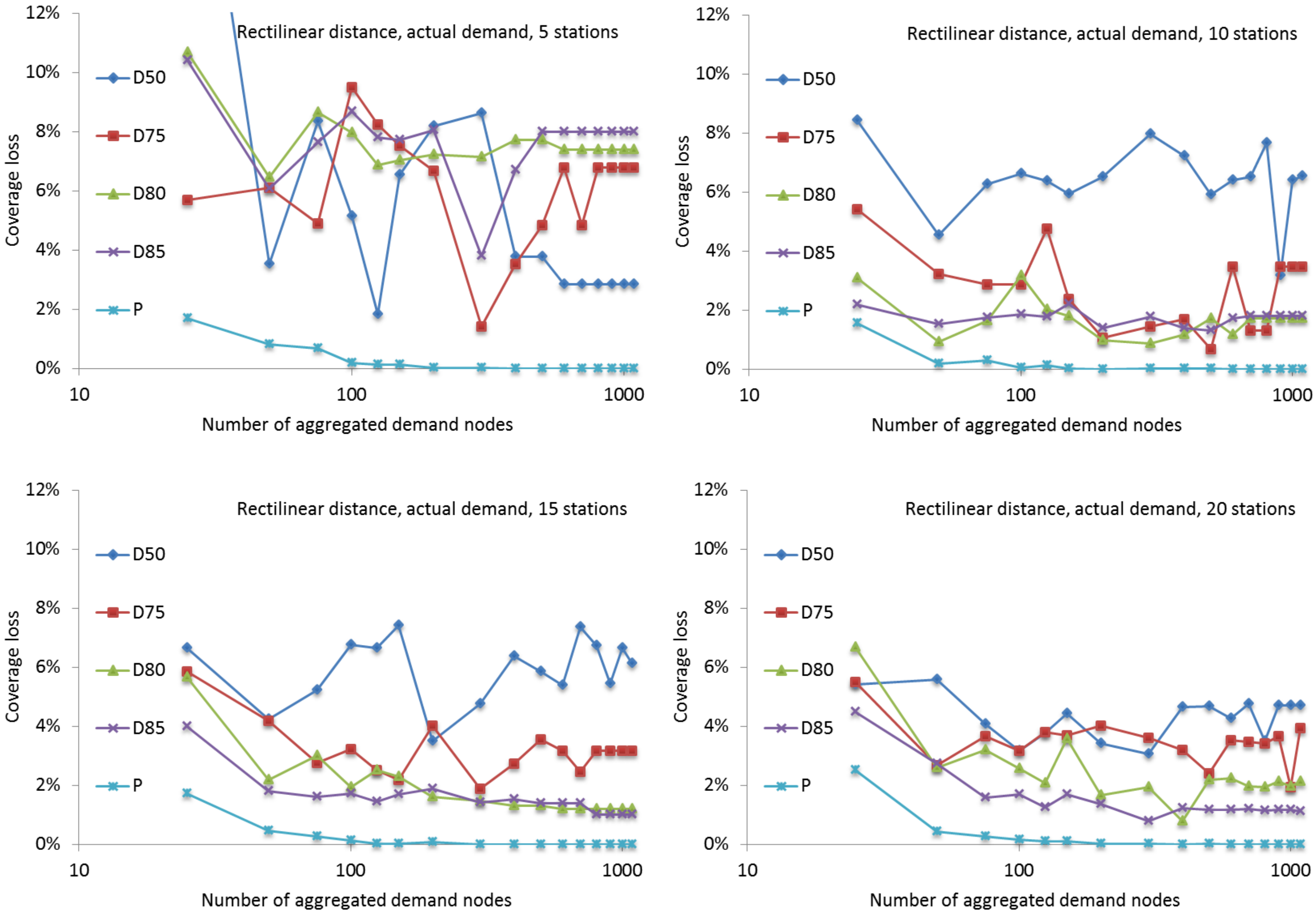


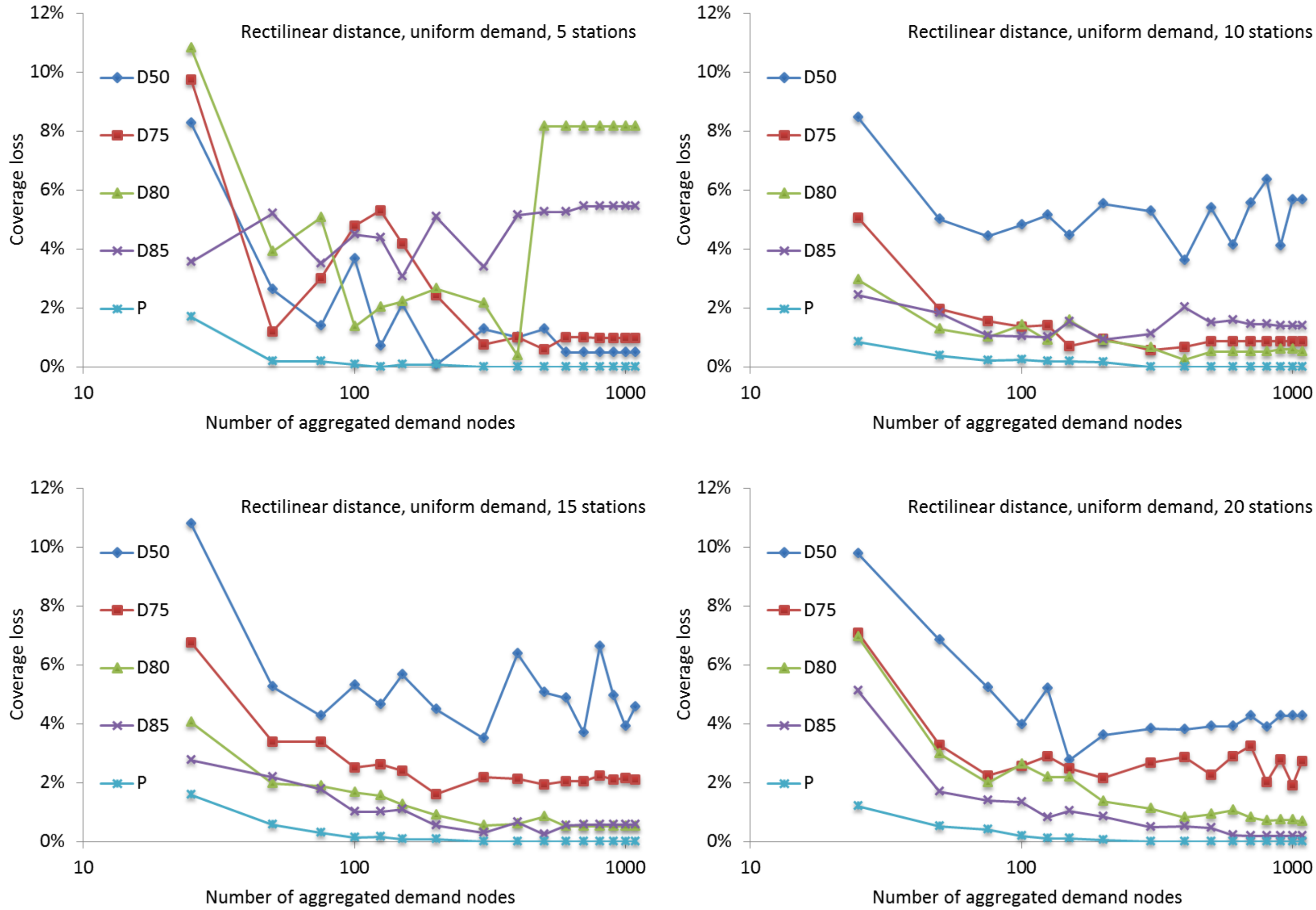


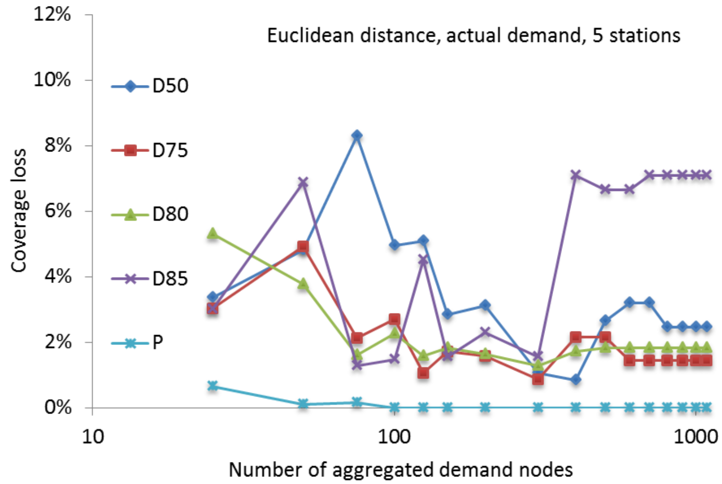


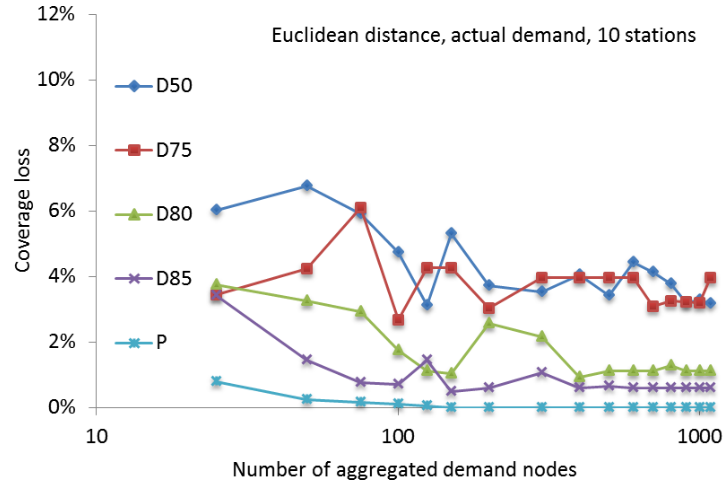


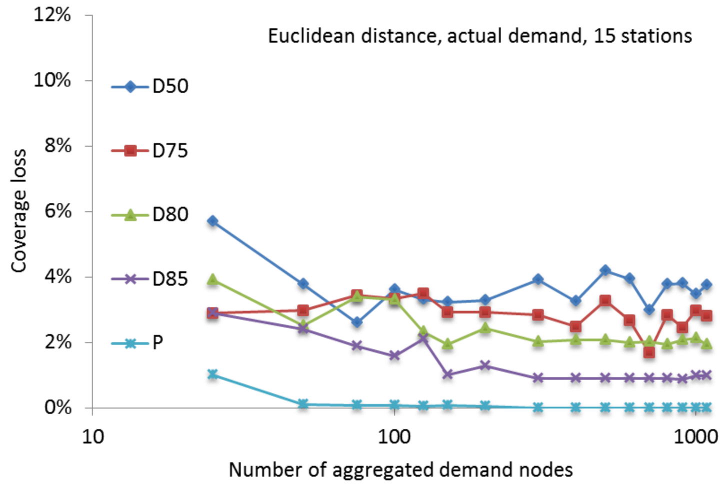

Network distance, uniform demand, 5 stations
Network distance, uniform demand, 10 stations
Network distance, uniform demand, 15 stations
Network distance, uniform demand, 20 stations
Coverage loss
Number of aggregated demand nodes
D50
D75
D80
D85
P
0%
2%
4%
6%
8%
10%
12%
10
100
1000

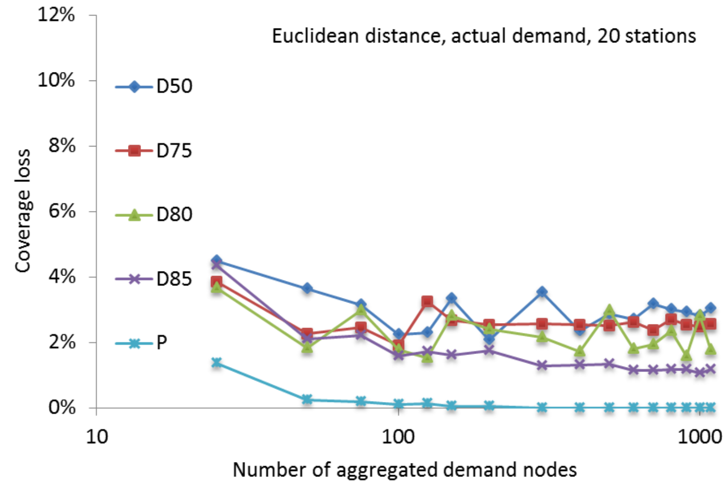

Rectilinear distance, actual demand, 5 stations
Rectilinear distance, actual demand, 10 stations
Rectilinear distance, actual demand, 15 stations
Rectilinear distance, actual demand, 20 stations
Coverage loss
Number of aggregated demand nodes
D50
D75
D80
D85
P
0%
2%
4%
6%
8%
10%
12%
10
100
1000

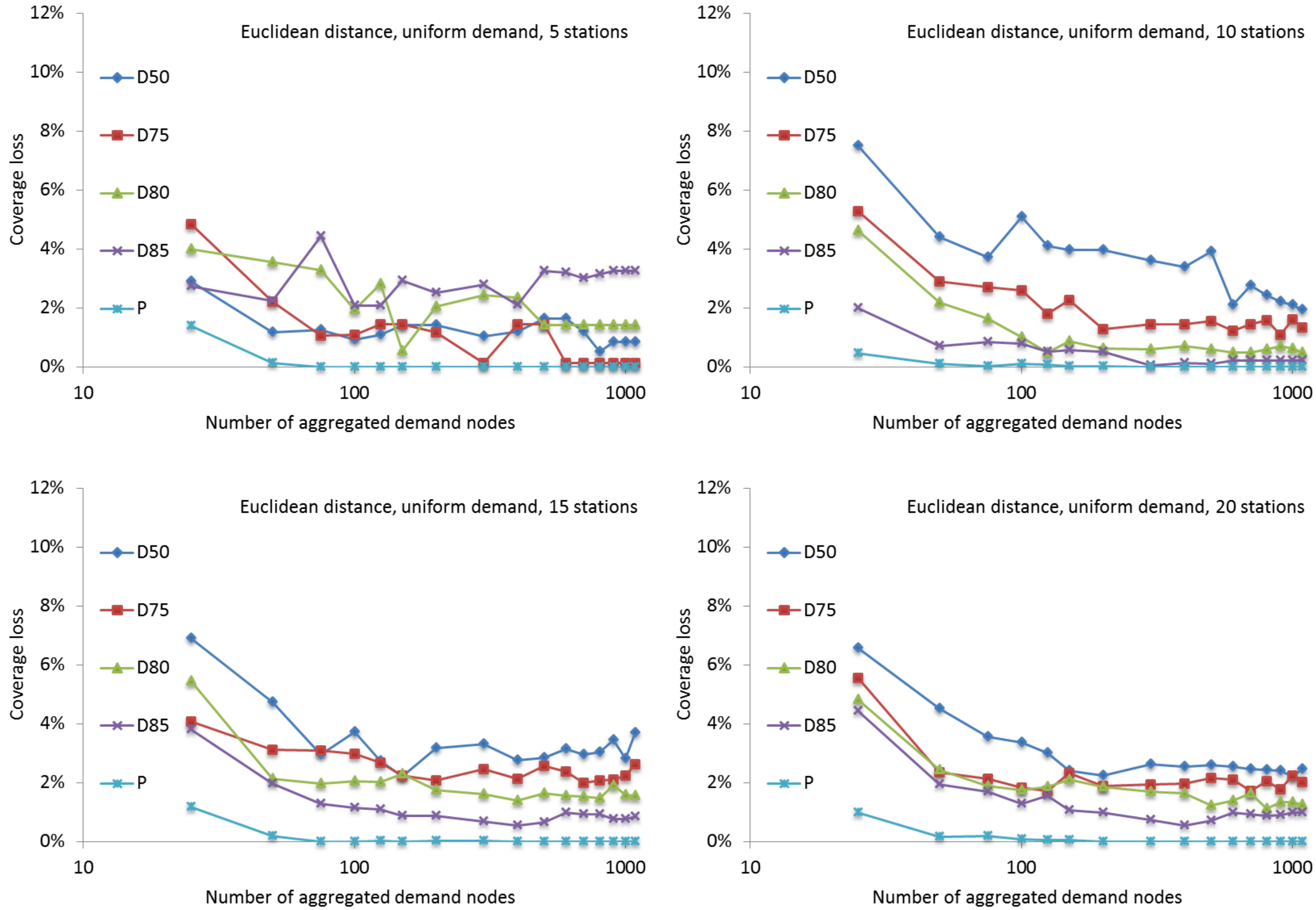

Rectilinear distance, uniform demand, 5 stations
Rectilinear distance, uniform demand, 10 stations
Rectilinear distance, uniform demand, 15 stations
Rectilinear distance, uniform demand, 20 stations
Coverage loss
Number of aggregated demand nodes
D50
D75
D80
D85
P
0%
2%
4%
6%
8%
10%
12%
10
100
1000


Euclidean distance, actual demand, 5 stations
Coverage loss
12%
10%
8%
6%
4%
2%
0%
D50
D75
D80
D85
P
10
100
1000
Number of aggregated demand nodes


Euclidean distance, actual demand, 10 stations
Coverage loss
12%
10%
8%
6%
4%
2%
0%
D50
D75
D80
D85
P
10
100
1000
Number of aggregated demand nodes


Euclidean distance, actual demand, 15 stations
Coverage loss
12%
10%
8%
6%
4%
2%
0%
D50
D75
D80
D85
P
10
100
1000
Number of aggregated demand nodes


Euclidean distance, actual demand, 20 stations
Coverage loss
12%
10%
8%
6%
4%
2%
0%
D50
D75
D80
D85
P
10
100
1000
Number of aggregated demand nodes


12%
10%
8%
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Coverage loss
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D50
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Number of aggregated demand nodes
Euclidean distance, uniform demand, 10 stations
Euclidean distance, uniform demand, 15 stations
Euclidean distance, uniform demand, 20 stations