An Integrated Mixed-Integer-Linear-Programming Model for Long-term Production Scheduling Optimization of Sublevel Caving Mines

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ABSTRACT

Production scheduling is one of the key components to ensuring mine viability because the mining industry faces declining ore grades and marginal reserves. Among underground mining methods, sublevel caving (SLC) is a commonly used method allowing earlier production than sublevel stoping with less-required upfront development than block caving. To achieve an optimized life-of-mine production schedule, mathematical modeling has been approved as a deterministic approach to address the complexity of a mining operation. This paper presents mixed-integer linear programming (MILP) framework for the long-term production scheduling of SLC. The model determines which machine placements to start being mined in each period over the horizon to maximize the NPV. Furthermore, the model satisfies constraints like development activities, mining, and processing capacities, continuous mining of machine placements, restrictions on the allowable number of active machine placements, grade blending, and vertical and horizontal sequencing. The formulations are coded and developed in Jupyter Notebook, and the Python interface of IBM ILOG CPLEX Optimization Studio 20.1.0 is used to solve the model.

1. Introduction

The mining industry's attention is on seeking a more economical way of extraction or increasing the efficiency of the current underground mining methods [11]. Therefore, caving methods such as Sublevel caving (SLC) which can be applied in the hard rock mass have gained more popularity because of their high potential production rates and low operating costs [4]. As a caving mining method, SLC relies on the gravity flow of materials, which tends to be random and highly irregular. Moreover, the chaotic material flow which leads to the pulsation of ore and waste inflows at the drawpoint makes it more complicated to model the production scheduling for an SLC mine [13]. Therefore, providing an exhaustive mathematical model that can integrate all activities, including development, mining, haulage, processing, and stockpiling at the same time, could increase the practicality and strength of the model [2].

Underground mining scheduling is subject to several discrete and continuous decisions which indicate the location, destination, extraction time, and the mining units' extraction portion over the mine's life, along with vertical and horizontal precedence relations [11]. A production schedule aims to define the most profitable extraction sequence of the material that produces the desired market specification while meeting the demanded quantities of run-of-mine ore at each period and

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satisfying a set of physical and operational constraints [8]. Compared to the manual and heuristic methods, the exact algorithms created by mathematical programming models would be a good alternative to provide an operationally optimal multi-time-period schedule in mines [6].

Although linear- and mixed-integer programming models have significant potential for optimizing production scheduling in both open-pit and underground mines, most of these models address the extraction sequence and do not consider the pre-extraction material flow in the formulation. In particular, the development activities before starting ore extraction have not been treated as an integrated part of mining optimization models.

This paper represents a long-term mathematical programming framework based on mixed-integer linear programming (MILP) formulation for the SLC method. The machine placements (MPs) are the decision units in the model. The MILP model maximizes the mining project's net present value (NPV). The model's primary goal is to integrate the schedules of all development activities, including the capital, ventilation, orepass, and operational development, into the mining and processing schedules. The model further controls the number of active machine placements, extraction portion in each period, continuous mining, and the extraction duration within the life of the mine.

In the following sections, a summary of the relevant works is provided in the area of SLC production scheduling optimization, followed by an overview of the conceptual mining strategy employed in this paper. Section 4 represents the general definition of the MILP model which includes all sets, parameters, decision variables, and formulations. Section 5 provides the implemented MILP model results, while Section 6 outlines the conclusions and further works to improve the model application and performance.

2. Relevant Literature Review

Planning in the SLC mines was traditionally based on planners' experience and estimations from earlier projects. However, in recent years, there have been some efforts to automate and optimize production scheduling through developments in operation research and an increase in computational power. Khazaei and Pourrahimian [5] presented a comprehensive review and summarized the researchers' attempts in the SLC production scheduling optimization field which mainly focused on the extraction sequencing of mining units.

As an early work, Almgren [1] set up a long-term production scheduling of the Kiruna mines. The presented model used the Lagrangian relaxation as a practical technique to reduce the number of binary variables, but it forced each block to be mined completely in each period which does not make sense on an operational scale. Additionally, this research showed that predicting the flow of blasted ore is almost impossible, which can increase the uncertainty of the production planning process. Topal [15] formulated a combined model that integrates both long- and short-term of Kiruna's mines. The attention was to solve one-year subproblems with a monthly resolution in a five-year schedule resolution. The model included a reduced number of binary and continuous variables that tracked the extraction portion of the different ore types in each production block in each period. However, the proposed model was not solvable in its monolithic form. Dagdelen [3] presented the sequencing constraint that minimized the deviation from planned quantities of three ore types of the Kiruna mines. Since the number of integer variables exceeded the maximum capacity of the solver, the one-year subproblems were proposed to achieve production plans for a seven-year time horizon. But to obtain a seven-year production schedule, the model had to be run seven times.

Kuchta et al. [7] worked on the tractability of their MILP model using two aggregation techniques in a five-year time horizon for three ore grades. First, 12 production blocks were merged into a single machine placement. Then, for each machine placement, the earliest and the latest start dates were calculated according to load-haul-dump machine (LHD) availability, sequencing, and demanded quantity constraints. Regarding sequencing and LHD availability constraints, as well as the precedence relationships between machine placements, the earliest start date of each machine placement is indicated. Moreover, the latest start date of each machine placement is measured by demanded quantity constraints and bounds on an acceptable amount of deviation between demand and production, preventing the underlying machine placements from being locked in. However, they reduced the number of binary variables and increased the tractability of the model but using more robust methods to determine the latest start date for each machine placement was still required.

Newman [9] proposed a heuristic-based algorithm that can solve the Kiruna mine production scheduling problem at the machine placement level and the production block level. They solved an aggregated model to determine a set of reasonable starting times for each machine placement, restricting the model to a subset of start times. After solving the aggregated model, some extra constraints are added to the original model to tighten the search space. Despite that this heuristic procedure remarkably mitigated the solution time and deviations from planned production quantities over those obtained from the model with the only long-term resolution, it can only be applied for two-year time horizons or fewer.

Newman [10] presented a mixed-integer program to schedule long- and short-term production at LKAB's Kiruna mine. The model uses an optimization-based decomposition heuristic algorithm to generate solutions with deviations that comprised about 3–6% of total demand in about a third of an hour. The formulation integrated short- and long-term production scheduling decisions to closely align production and demand quantities for all ore types and periods. The heuristic is divided into two phases, one is to solve five subproblems of the original model, and the other is to use the provided information from the previous phase to solve a modified version of the original model. The first three subproblems penalize deviations only for each of the three ore types. The next two subproblems penalize either overage or underage production, resulting in adding several constraints to the original model. All subproblem solutions are feasible for the original model. The solution time of the model will be the maximum among the five solution times since all subproblems are run in parallel. However, independent solving of the subproblems does not guarantee an optimal solution.

Shenavar et al. [14] formulated a long-term production scheduling for a 2D representation of the Golbini bauxite mine in Iran, a real sublevel caving mine. They proposed a four-step procedure to generate the optimal production schedule. The first step is generating the economic block based on the geological model. Then, the unnecessary blocks are removed from the economic block model by implementing the floating stope boundary optimizer. In the next step, an ILP model was conducted to determine the optimal mining sequence to achieve the maximum NPV. Finally, the development activities start with respect to the mining sequence to the extent required annually. Regardless of how this model could reduce the number of decision variables, a 3D production schedule is needed for more precise analysis, and the development activities are required to be integrated with production scheduling.

3. Problem Definition

Scheduling activities in an SLC mine is a very complex task. Confined blasting conditions, chaotic material flow, and continuous mixing of ore and waste make the loading process more complex at drawpoints. Furthermore, the precedence relations and the restrictions for the number of active production faces due to the geotechnical constraints are some other reasons for this complexity. For clarification, all development and mining activities and several assumptions used in the proposed MILP formulations are provided.

Figure 1 illustrates the schematic layout used to develop the model. The vein-like deposit with stable surrounding rock that caves in a controlled manner after drilling and blasting operations contributes to the use of the SLC mining method. The mine, on the largest scale, is divided into production areas, each containing its group of orepasses and ventilation systems. Miners first drill vertical shafts, haulage level drift, and access route driven along the strike of the orebody and provide access to all production areas. The development plan for each specific production area consists of capital development (man-way-raise), orepass development, and operational development (perimeter drifts and production drifts). A man-way-raise is extracted due to transferring workers and ventilation installations to supply fresh air in each sublevel. In addition, two orepasses that extend from the main haulage level (the lowest level) to the uppermost level ($l=1$) are drilled using Alimak raising technique for transferring material from each active sublevel to the haulage level. Finally, operational developments, including perimeter and production drifts, are mined to create horizontal sublevels for mining operations, including drilling, blasting, and loading extracted material from drawpoints to the orepasses. A perimeter drift in a sublevel is driven along to the strike of the orebody and is offset from the ore–waste contact on the footwall side. The perimeter drift is developed to access the production drifts for ore transportation, services, and ventilation. The production drifts are spaced between 12 and 30 m on horizontal centers that are staggered between the levels to provide optimal coverage for drilling and allow for the downward flow of caved material. Fan-shaped rings are drilled from the production drifts. Placing explosives in the holes and blasting the rings in sequence, destroying the ceiling on the blasted sublevel is performed to recover the ore. Miners recover the ore on each sublevel, starting with the overlying sublevels and proceeding downwards. Depending on the regularity of the orebody, train systems or tuck haulages are utilized to transport ore from the orepasses to the crusher stations. After crushing, the ore is hoisted from the haulage level to the surface with a skip system.

Figure 2 shows the considered mining and development activities in the model. The development activities are divided into two categories: one is the activities that provide access to all production areas, and the other is the ones driven in each specific production area for ore extraction in sublevels. Activities like a vertical shaft, haulage drift, access route developments, and installed ventilation facilities are in the first category. In contrast, man-way-raise, and its ventilation requirements, in addition to orepass and operational developments, fall into the second category. SLC is generally used as the primary mining method in mechanized mines with independent unit operations. All unit operations including drilling, charging, blasting, loading, and transportation are performed separately, resulting in a standardized procedure and safe operation [12].

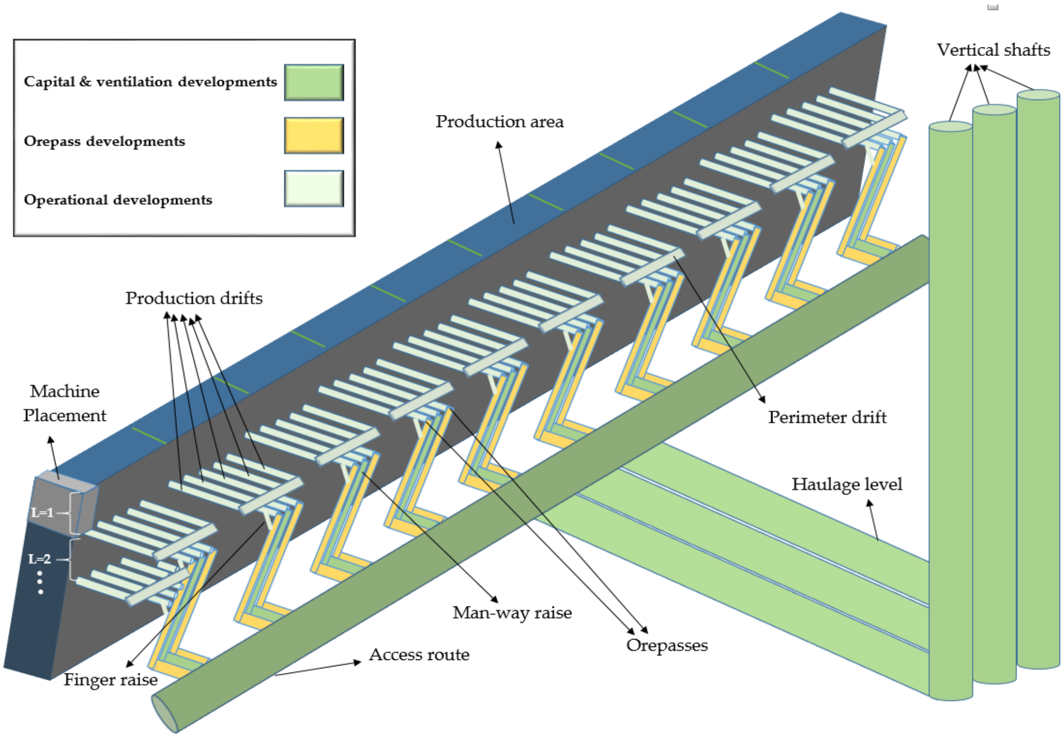

Figure 1. Schematic layout of Sublevel caving developments.

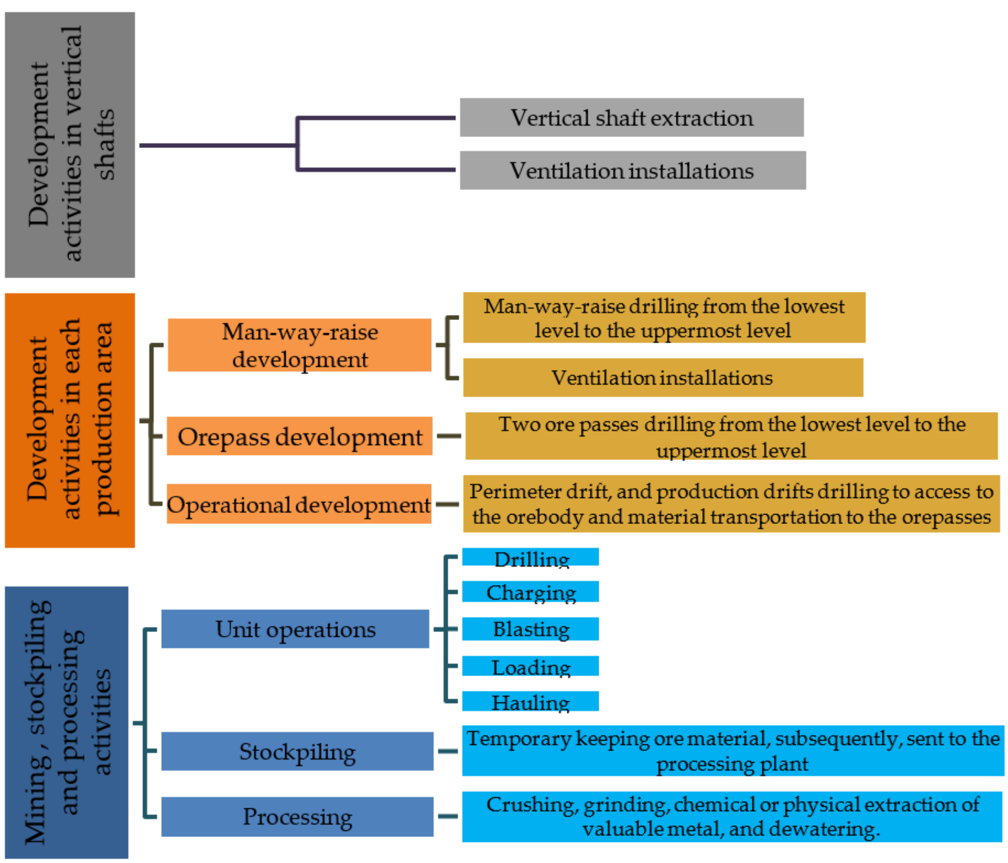

Figure 2. Mining and development activities in the SLC mine.

4. Mathematical Model

Underground mining scheduling is subject to discrete decisions to extract stopes of ore, along with vertical and horizontal precedence relations between them. Since linear programming (LP) models cannot capture the discrete decisions required for mine scheduling, MILP models are generally the appropriate mathematical programming alternative that provides an exactly provable optimum solution for complex scheduling problems [5]. This paper represents a mathematical programming framework based on MILP formulation for the SLC method. The MILP model maximizes the mining project's NPV and determines the capital, ventilation, orepass, operational development schedules, and mining and processing schedules. The model further incorporates operational levels control, machine placement extraction duration control, and continuous mining of each machine placement.

SLC mining production relies on the gravity flow of materials, which tends to be random and highly irregular. Moreover, the chaotic material flow which leads to the pulsation of ore and waste inflow at the drawpoint makes it more complicated to model the production scheduling for a sublevel caving mine [12]. Therefore, providing an exhaustive mathematical model that can integrate all activities, including development, mining, haulage, and processing at the same time, could increase the practicality and strength of the model.

On the largest scale, the mine organizes in several operation areas ranging from 400 to 500 meters in length and consists of about ten horizontal sublevels and a group of vertical orepasses. The orepasses provide access to sublevels, which are horizontal cuts that are positioned progressively. The basic mining entity within the model is machine placement. Machine placement is where one LHD operates which contains between 1 and 3 million tons of ore and waste rock and belongs to a unique production area. Here, the machine placements are the decision units. The output of the production schedule would be the periods in which each of the machine placements located in a sublevel within a production area is extracted and sent to the processing plant. The model given in this paper consists of long-term decisions and restrictions at the machine placement level as the basic mining unit. Continuous-valued variables track the development activities in each level and the material mined in each machine placement at each period, while binary variables control the precedence of development activities and whether machine placement *m* starts being mined at period *t*.

The following sets, parameters, and decision variables are defined for the SLC production scheduling formulation with a set of V production areas, L levels, and M machine placements planned to be mined in T periods.

4.1. Sets

S_{T_m}	Set of periods that a machine placement m can start to be mined
S_V	Set of production areas.
S_{M_v}	Set of machine placements in the production area v .
S_{M_l}	Set of machine placements in all production areas on the level l .
$S_{M_{v,l}}$	Set of machine placements in the production area v on the level l .
$S_{M_m}^V$	Set of machine placements whose start period is restricted vertically by machine placement m .

$S_{M_m}^{H}$ Set of machine placements whose start period is forced by adjacency to machine placement m.

4.2. Decision Variables

$x_m^t \in [0,1]$ Continuous variable representing the portion of the machine placement to be mined at period t.

$x_{m,p}^t \in [0,1]$ Continuous variable representing the portion of machine placement m sending directly to the processing plant at period t.

bs_m^t Binary integer variable equals one if mining of machine placement m is started at period t; otherwise, it is zero.

as_m^t Binary integer variable equals one if machine placement m is active at period t; otherwise, it is zero.

$b_{sd_l}^{\ t} \in \{0,1\}$ Binary integer variable controlling the precedence of vertical shafts developments. It is equal to one if vertical shaft development on level l, sd_l, has started by or in period t; otherwise, it is zero.

$b_{vds_l}^{\ t} \in \{0,1\}$ Binary integer variable controlling the precedence of ventilation development in vertical shafts. It is equal to one if ventilation development on level l, vds_l, has started by or in period t; otherwise, it is zero.

$b_{mwr_{v,l}}^{\ t} \in \{0,1\}$ Binary integer variable controlling the precedence of man-way-raise development in each production area. It is equal to one if man-way-raise development in production area v on level l, $mwr_{v,l}$, has started by or in period t; otherwise, it is zero.

$b_{vd_{v,l}}^{\ t} \in \{0,1\}$ Binary integer variable controlling the precedence of ventilation development in man-way-raise in each production area. It is equal to one if ventilation development in production area v on level l, $vd_{v,l}$, has started by or at period t; otherwise, it is zero.

$b_{opd_{v,l}}^{\ t} \in \{0,1\}$ Binary integer variable controlling the precedence of orepass development in each production area. It is equal to one if orepass development in production area v on level l, $opd_{v,l}$, has started by or at period t; otherwise, it is zero.

$b_{od_{v,l}}^{\ t} \in \{0,1\}$ Binary integer variable controlling the precedence of operational developments in each production area. It is equal to one if operational development in production area v on level l, $od_{v,l}$, has started by or at period t; otherwise, it is zero.

$d_{sd_l}^{\ t} \in [0,1]$ Continuous variable represents the vertical shaft development activities to be completed on level l at period t.

$d_{vds_l}^{\ t} \in [0,1]$ Continuous variable represents the ventilation development activities in vertical shafts to be completed on level l at period t.

$d_{mwr_{v,l}}^{\ t} \in [0,1]$ Continuous variable represents the man-way-raise development activities to be completed in production area v on level l at period t.

$d_{vd}{}_{v,l}^{t} \in [0,1]$	Continuous variable represents the ventilation development activities in the man-way-raise to be completed in production area v on level l at period t.
$d_{opd}{}_{v,l}^{t} \in [0,1]$	Continuous variable represents the orepass development activities to be completed in production area v on level l at period t.
$d_{od}{}_{v,l}^{t} \in [0,1]$	Continuous variable represents the operational development activities to be completed in production area v on level l at period t.

4.3. Parameters

$R_{m,p}^{t}$	Revenue generated by selling the final commodity of the ore sent from machine placement m to the processing plant at the period t.
CSD_{l}^{t}	Total cost of vertical shafts development on level l at period t.
$CVDS_{l}^{t}$	Total cost of ventilation development in vertical shafts on level l at period t.
$CMWR_{v,l}^{t}$	Total cost of man-way-raise development in production area v on level l at period t.
$CVD_{v,l}^{t}$	Total cost of ventilation development in man-way-raise in production area v on level l at period t.
$COPD_{v,l}^{t}$	Total cost of orepass development in production area v on level l at period t.
$COD_{v,l}^{t}$	Total cost of operational development in production area v on level l at period t.
csd_{l}^{t}	Variable cost per length of vertical shafts development on level l at period t.
$cvds_{l}^{t}$	Variable cost per length of ventilation development in vertical shafts on level l at period t.
$cmwr_{v,l}^{t}$	Variable cost per length man-way-raise development in production area v on level l at period t.
$cvd_{v,l}^{t}$	Variable cost per length of ventilation development in man-way-raise in production area v on level l at period t.
$copd_{v,l}^{t}$	Variable cost per length of orepass development in production area v on level l at period t.
$cod_{v,l}^{t}$	Variable cost per length of operational development in production area v on level l at period t.
o_m	Ore tonnage in machine placement m.
g_m	Average grade of ore in machine placement m.
dil_m	Mining dilution of machine placement m.

r_m	Mining recovery of machine placement m .
rp^t	Processing recovery: the portion of mineral recovered in machine placement m at period t .
sp^t	Selling price in present value terms obtainable per unit of the mineral commodity at period t .
sc^t	Selling cost in present value terms obtainable per unit of mineral commodity.
ec_m^t	Mining and processing cost per ton of ore extracted from machine placement m at period t .
p^t	Penalty cost per ton associated with tonnage deviations at period t .
l_a	The uppermost level in each production area assumed the same for all of them $(l=1)$.
l_v	The lowest level in each production area assumed the same for all of them $(l=L)$.
$\sigma_m^{t',t}$	Equals one if machine placement m started to be mined at period t' and is being mined at period t ; otherwise, it is zero.
dl_{sd_l}	Vertical shaft development length on the level l .
dl_{vds_l}	The length of ventilation development in the vertical shaft on the level l .
$dl_{mwr_{v,l}}$	The length of the man-way-raise development in the production area v on the level l .
$dl_{vd_{v,l}}$	The length of ventilation development in man-way-raise in production area v on the level l .
$dl_{opd_{v,l}}$	Orepass development length in production area v on the level l .
$dl_{od_{v,l}}$	Operational development length in production area v on the level l .
$\underline{Dev_{sd}^t}$	Lower bound on vertical shaft development at period t .
$\overline{Dev_{sd}^t}$	Upper bound on vertical shaft development at period t .
$\underline{Dev_{vds}^t}$	Lower bound on ventilation development in vertical shafts at period t .
$\overline{Dev_{vds}^t}$	Upper bound on ventilation development in vertical shafts at period t .
$\underline{Dev_{mwr}^t}$	Lower bound on man-way-raise development for all production areas at period t .
$\overline{Dev_{mwr}^t}$	Upper bound on man-way-raise development for all production areas at period t .

$\underline{Dev_{vdv}^{t}}$ Lower bound on ventilation development in man-way-raise for all production areas at period t.

$\overline{Dev_{vdv}^{t}}$ Upper bound on ventilation development in man-way-raise for all production areas at period t.

$\underline{Dev_{opd}^{t}}$ Lower bound on orepass development for all production areas at period t.

$\overline{Dev_{opd}^{t}}$ Upper bound on orepass development for all production areas at period t.

$\underline{Dev_{od}^{t}}$ Lower bound on operational development for all production areas at period t.

$\overline{Dev_{od}^{t}}$ Upper bound on operational development for all production areas at period t.

$\underline{Ton^{t}}$ Lower bound on mining capacity at period t.

$\overline{Ton^{t}}$ Upper bound on mining capacity at period t.

$\underline{Ton_{p}^{t}}$ Lower bound on ore processing capacity at period t.

$\overline{Ton_{p}^{t}}$ Upper bound on ore processing capacity at period t.

Ton_{d}^{t} Tonnage demanded at period t.

$\underline{g_{p}^{t}}$ Lower bound on an acceptable average grade by processing plant at period t.

$\overline{g_{p}^{t}}$ Upper bound on an acceptable average grade by processing plant at period t.

4.4. Variables Calculations

Some economic parameters are measured by other parameters in the formulation. Eq. (1) shows the revenue that can be obtained when the ore available in a machine placement is sent directly to the processing plant. Vertical shaft development cost, ventilation development cost in vertical shaft, man-way-raise development cost, ventilation development cost in man-way-raise, orepass development cost, and operational development cost are calculated by Eq. (2) to Eq. (7), respectively. Total ore production of mine is calculated in Eq. (8). The amount of ore material sent directly from mine to the processing plant is measured in Eq. (9). The metal content that is sent directly from mine to the processing plant is controlled in Eq. (10).

$$R_{m,p}^{t} = (o_{m} \times g_{m} \times r_{m} \times rp^{t} \times (sp^{t} - sc^{t})) - (o_{m} \times (1 + dil_{m}) \times ec_{m}^{t}) \tag{1}$$

$$CSD_{l}^{t} = csd_{l}^{t} \times dl_{sd_{l}} \tag{2}$$

$$CVDS_{l}^{t} = cvds_{l}^{t} \times dl_{vds_{l}} \tag{3}$$

$$CMWR_{v,l}^{t} = cmrw_{v,l}^{t} \times dl_{mwr_{v,l}} \tag{4}$$

$$CVD_{v,l}^{t} = cvd_{v,l}^{t} \times dl_{vd_{v,l}} \tag{5}$$

$$COPD_{v,l}^{t} = copd_{v,l}^{t} \times dl_{opd_{v,l}} \tag{6}$$

$$COD_{v,l}^{t} = cod_{v,l}^{t} \times dl_{od_{v,l}} \tag{7}$$

$$PRD^{t} = \sum_{m=1}^{M} \left[(o_m \times (1 + dil_m) \times x_m^t \right] \tag{8}$$

$$PRP^{t} = \sum_{m=1}^{M} (o_m \times (1 + dil_m) \times x_{m,p}^t) \tag{9}$$

$$GPRP^{t} = \sum_{m=1}^{M} (g_m \times o_m \times (1 + dil_m) \times x_{m,p}^t) \tag{10}$$

4.5. Objective Function

The optimization model maximizes the NPV of caving operations. The NPV of the whole operation is determined by calculating the profit from mining, and processing of the material extracted from machine placements, as well as considering capital development (vertical shafts, and man-way-raises), ventilation development (installation), orepass development, and operational development (perimeter drifts, and production drifts) over the life of mine.

The objective function formulated in Eq. (11) contains three main parts. First, the NPV of each machine placement extracted over mine life is calculated. The next is all development components, including the capital, ventilation, orepass, and operational developments that are added to the formula.

$$Max \left[\sum_{t=1}^{T} \sum_{m=1}^{M} \frac{(R_{m,p}^{t} \times x_{m,p}^{t})}{(1+i)^t} \right] - \left[\sum_{t=1}^{T} \sum_{l=1}^{L} \frac{CSD_l^t \times d_{sd_l}^{\ t} + CVDS_l^t \times d_{vds_l}^{\ t}}{(1+i)^t} + \sum_{t=1}^{T} \sum_{v=1}^{V} \sum_{l=1}^{L} \frac{CMWR_{v,l}^t \times d_{mwr_{v,l}}^{\ t} + CVD_{v,l}^t \times d_{vd_{v,l}}^{\ t} + COPD_{v,l}^t \times d_{opd_{v,l}}^{\ t} + COD_{v,l}^t \times d_{od_{v,l}}^{\ t}}{(1+i)^t} \right] \tag{11}$$

4.6. Capital Development Constraints:

Capital development is divided into two phases.

- *Phase 1* is to drill vertical shafts as the primary spaces to install ventilation infrastructures, hoisting requirements, and entry-exit system for workforces.
- *Phase 2* is man-way-raises drilled in each production area to construct ventilation facilities, and, at the same time, the only way to transport the workers and operators to the sublevels.

Eqs. (12) to (16) model the vertical shafts as the main access to the mine. Eq. (12) defines the vertical shaft development capacity. The inequality ensures that the total length of capital development required in each period is within the stated lower and upper limits of the total available equipment capacity for developing the mine. Eqs. (13) to (15) control the precedence relation between the sections of vertical shaft developments and the ventilation developments on each level. Eq. (16) ensures that the ventilation development on the level l, ${d_{vds}}_l^t$, starts after completing the vertical shaft developments on that level, $\sum_{t' \le t} {d_{sd}}_l^{t'}$.

$$\underline{Dev_{sd}^t} \le \sum_{l=1}^{L} (dl_{sd_l} \times {d_{sd}}_l^t) \le \overline{Dev_{sd}^t} \qquad \forall t \in \{1,2,...,T\} \tag{12}$$

$$ {b_{sd}}_{l+1}^t - \sum_{t' \le t} {d_{sd}}_l^{t'} \le 0 \qquad \forall l \in \{1,2,...,L-1\}, t \in \{1,2,...,T\} \tag{13}$$

$$\sum_{t' \le t} {d_{sd}}_l^{t'} - {b_{sd}}_l^t \le 0 \qquad \forall l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{14}$$

$$ {b_{sd}}_l^t - {b_{sd}}_l^{t+1} \le 0 \qquad \forall l \in \{1,2,...,L\}, t \in \{1,2,...,T-1\} \tag{15}$$

$$ {b_{vds}}_l^t - \sum_{t' \le t} {d_{sd}}_l^{t'} \le 0 \qquad \forall l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{16}$$

Eqs. (17) to (21) model the man-way-raises drilled in a production area to provide transportation for workers and operators to each sublevel. Eq. (17) defines the man-way-raise development capacity constraint for the mine. The inequality ensures that the total length of man-way-raise development required in a period is within the stated lower and upper limits of the total available equipment capacity. Eqs. (18) to (20) control the precedence relation between the sections of man-way-raise developments on each level within a production area. Eq. (21) prevents any man-way-raise ventilation development on the level l, ${d_{vd}}_{v,l}^t$, before completing the man-way-raise developments on that level in the production area v, $\sum_{t' \le t} {d_{mwr}}_{v,l}^{t'}$.

$$\underline{Dev_{mwr}^t} \le \sum_{v=1}^{V} \sum_{l=1}^{L} (dl_{mwr_{v,l}} \times {d_{mwr}}_{v,l}^t) \le \overline{Dev_{mwr}^t} \qquad \forall t \in \{1,2,...,T\} \tag{17}$$

$$ {b_{mwr}}_{v,l}^t - \sum_{t' \le t} {d_{mwr}}_{v,l+1}^{t'} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L-1\}, t \in \{1,2,...,T\} \tag{18}$$

$$\sum_{t' \le t} {d_{mwr}}_{v,l}^{t'} - {b_{mwr}}_{v,l}^t \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{19}$$

$$b_{mwr\,v,l}^{\;t} - b_{mwr\,v,l}^{\;t+1} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T-1\} \tag{20}$$

$$b_{vd\,v,l}^{\;t} - \sum_{t' \le t} d_{mwr\,v,l}^{\;t'} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{21}$$

4.7. **Ventilation Development Constraint:**

Ventilation development follows the order of mining the capital development. Therefore, like capital development, ventilation development consists of two phases.

- *Phase 1* is to install ventilation facilities in vertical shafts.
- *Phase 2* is ventilation facility installation in man-way-raises drilled in each production area.

Eqs. (22) to (26) define the ventilation developments installed in vertical shafts. Eq. (22) defines the ventilation development capacity constraint for the mine. The inequality ensures that the total length of ventilation development required in each period is within the stated lower and upper limits of the total available equipment capacity for the installation of ventilation facilities. Eqs. (23) to (25) control the precedence relation between the sections of ventilation developments and the ventilation developments on each level. Eq. (26) ensures that no man-way-raise development, $b_{mwr\,v,l_v}^{\;t}$, can start before completing the ventilation development in the vertical shaft on level l_v, $\sum_{t' \le t} d_{vds\,l_v}^{\;t'}$.

$$\underline{Dev_{vds}^{t}} \le \sum_{l=1}^{L} (dl_{vds_l} \times d_{vds\,l}^{\;t}) \le \overline{Dev_{vds}^{t}} \qquad \forall t \in \{1,2,...,T\} \tag{22}$$

$$b_{vds\,l+1}^{\;t} - \sum_{t' \le t} d_{vds\,l}^{\;t'} \le 0 \qquad \forall l \in \{1,2,...,L-1\}, t \in \{1,2,...,T\} \tag{23}$$

$$\sum_{t' \le t} d_{vds\,l}^{\;t'} - b_{vds\,l}^{\;t} \le 0 \qquad \forall l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{24}$$

$$b_{vds\,l}^{\;t} - b_{vds\,l}^{\;t+1} \le 0 \qquad \forall l \in \{1,2,...,L\}, t \in \{1,2,...,T-1\} \tag{25}$$

$$b_{mwr\,v,l_v}^{\;t} - \sum_{t' \le t} d_{vds\,l_v}^{\;t'} \le 0 \qquad \forall v \in \{1,2,...,V\}, t \in \{1,2,...,T\} \tag{26}$$

Eqs. (27) to (31) formulate the man-way-raise ventilation development requirements for each production area. Eq. (27) checks the man-way-raise development capacity for the mine. The inequality ensures that the total length of man-way-raise ventilation development in each period is in the stated range of the total available equipment capacity. Eqs. (28) to (30) dictate the precedence relationships between the stages of ventilation facility installation in man-way-raise developments on each level within a production area. Eq. (31) preclude any orepass development, which is commenced on the lowest level in the production area v, $b_{opd\,v,l_v}^{\,t}$, before completing man-way-raise ventilation developments on the uppermost level in that production area, $\sum_{t'\leq t} d_{vd\,v,l_a}^{\,t'}$.

$$\underline{Dev_{vdv}^{t}} \leq \sum_{v=1}^{V}\sum_{l=1}^{L}(dl_{vd_{v,l}} \times d_{vd\,v,l}^{\,t}) \leq \overline{Dev_{vdv}^{t}} \qquad \forall t \in \{1,2,...,T\} \tag{27}$$

$$b_{vd\,v,l}^{\,t} - \sum_{t'\leq t} d_{vd\,v,l+1}^{\,t'} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L-1\}, t \in \{1,2,...,T\} \tag{28}$$

$$\sum_{t'\leq t} d_{vd\,v,l}^{\,t'} - b_{vd\,v,l}^{\,t} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{29}$$

$$b_{vd\,v,l}^{\,t} - b_{vd\,v,l}^{\,t+1} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T-1\} \tag{30}$$

$$b_{opd\,v,l_v}^{\,t} - \sum_{t'\leq t} d_{vd\,v,l_a}^{\,t'} \leq 0 \qquad \forall v \in \{1,2,...,V\}, t \in \{1,2,...,T\} \tag{31}$$

4.8. **Orepass Development Constraints:**

In each production area, the orepass development starts after finishing the man-way-raise development and its ventilation requirements in that production area. Eq. (32) dictates the total orepass development length between the lower and upper bounds of the total available equipment capacity. Eqs. (33) to (35) control the precedence relationships between the stages of orepass developments on each level within a production area. Eq. (36) enforces the operational development on the uppermost level in each production area, $b_{od\,v,l_a}^{\,t}$, to complete orepass developments on level l_a, $\sum_{t'\leq t} d_{opd\,v,l_a}^{\,t'}$.

$$\underline{Dev_{opd}^{t}} \leq \sum_{v=1}^{V}\sum_{l=1}^{L}(dl_{opd_{v,l}} \times d_{opd\,v,l}^{\,t}) \leq \overline{Dev_{opd}^{t}} \qquad \forall t \in \{1,2,...,T\} \tag{32}$$

$$b_{opd\,v,l}^{\,t} - \sum_{t'\leq t} d_{opd\,v,l+1}^{\,t'} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L-1\}, t \in \{1,2,...,T\} \tag{33}$$

$$\sum_{t'\leq t} d_{opd\,v,l}^{\,t'} - b_{opd\,v,l}^{\,t} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{34}$$

$$b_{opd\,v,l}^{\,t} - b_{opd\,v,l}^{\,t+1} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T-1\} \tag{35}$$

$$b_{od_{v,l_a}}^{\;t} - \sum_{t' \le t} d_{opd_{v,l_a}}^{\;t'} \le 0 \qquad \forall v \in \{1,2,...,V\}, t \in \{1,2,...,T\} \qquad (36)$$

4.9. **Operational Development Constraints:**

If a machine placement is scheduled to be mined in a period, a set of operational development must be ready ahead or in that period. Eq. (37) defines the operational development capacity constraints for the mine. This equation ensures that the total length of operational developments required in each period is within the defined lower and upper limits of each production area's total available equipment capacity. Eqs. (38) to (40) control the lateral precedence relation of the operational development required for mining each machine placement. Eq. (41) ensures that the operational developments in production area v, and on level l, $\sum_{t' \le t} d_{od_{v,l}}^{\;t'}$, must be completed before any of the machine placements in that production area, and on that level($m \in S_{M_{v,l}}$) starts being mined, bs_m^t.

$$\underline{Dev_{od}^t} \le \sum_{v=1}^{V}\sum_{l=1}^{L}(dl_{od_{v,l}} \times d_{od_{v,l}}^{\;t}) \le \overline{Dev_{od}^t} \qquad \forall t \in \{1,2,...,T\} \qquad (37)$$

$$b_{od_{v,l+1}}^{\;t} - \sum_{t' \le t} d_{od_{v,l}}^{\;t'} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L-1\}, t \in \{1,2,...,T\} \qquad (38)$$

$$\sum_{t' \le t} d_{od_{v,l}}^{\;t'} - b_{od_{v,l}}^{\;t} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \qquad (39)$$

$$b_{od_{v,l}}^{\;t} - b_{od_{v,l}}^{\;t+1} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T-1\} \qquad (40)$$

$$bs_m^t - \sum_{t' \le t} d_{od_{v,l}}^{\;t'} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T\}, m \in S_{M_{v,l}} \qquad (41)$$

4.10. **Mining and Processing Capacity:**

Eq. (42) enforces the mining capacity, using continuous decision variable x_m^t, between the acceptable lower and upper limits of the total available equipment capacity for each period. Eq. (43) controls the quantity of mill feed using the continuous decision variable $x_{m,p}^t$ which is included in the dependent variable of PRP^t. This constraint ensures that the processing capacity does not exceed the predefined limits.

$$\underline{Ton^t} \le PRD^t \le \overline{Ton^t} \qquad \forall t \in \{1,2,...,T\} \qquad (42)$$

$$\underline{Ton_p^t} \le PRP^t \le \overline{Ton_p^t} \qquad \forall t \in \{1,2,...,T\} \qquad (43)$$

4.11. Active Machine Placements Constraints:

Crew and LHD availability limit the number of active machine placements in each period. LHD restrictions in each level, each production area, and each period within all production areas are controlled by Eq. (44) to Eq. (46), respectively.

$$\sum_{m \in S_{M_l}} as_m^t \le LHD_l \qquad \forall l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{44}$$

$$\sum_{m \in S_{M_v}} as_m^t \le LHD_v \qquad \forall v \in \{1,2,...,V\}, t \in \{1,2,...,T\} \tag{45}$$

$$\sum_{m=1}^{M} as_m^t \le LHD_t \qquad \forall t \in \{1,2,...,T\} \tag{46}$$

4.12. Continuous Mining Constraints:

Each machine placement must be continuously extracted after opening until closing. Eq. (47) forces variable as_m^t to be zero if no portion of machine placement m is extracted at period *t*, while Eq. (48) changes the value of as_m^t to 1 when a portion of machine placement m is extracted at period *t*.

Eq. (49) ensures that if extraction from machine placement m is started during or after period two, at least a portion of the machine placement is extracted until all of the material within that machine placement has been extracted; otherwise the machine placement must be closed. Eq. (50) is used for period one. It ensures that if extraction from machine placement m is started in period one, the related variable as_m^t for the machine placement is equal to 1.

$$as_m^t \le M \times x_m^t \qquad \forall t \in \{1,2,...,T\}, m \in \{1,2,...,M\} \tag{47}$$

$$x_m^t \le as_m^t \qquad \forall t \in \{1,2,...,T\}, m \in \{1,2,...,M\} \tag{48}$$

$$as_m^t - as_m^{t-1} \le bs_m^t \qquad \forall m \in \{1,2,...,M\}, t \in \{2,...,T\} \tag{49}$$

$$as_m^1 - bs_m^1 \le 0.5 \qquad \forall m \in \{1,2,...,M\} \tag{50}$$

4.13. Grade Blending Constraints:

Eq. (51) ensures that the material extracted meets the ore quality specification of the processing plant. These inequalities are defined based on predefined cut-off grades in the processing plant. The dilution factor is considered using dil_m for each machine placement.

$$\underline{g_p^t} \le \left[\frac{GPRP^t}{PRP^t}\right] \le \overline{g_p^t} \qquad \forall t \in \{1,2,...,T\} \tag{51}$$

4.14. Vertical and Horizontal Sequencing Constraints:

When extracting the ore body using the SLC method, some strict operational rules need to be satisfied to ensure the method's success. Only after the machine placement on the upper level has been retreated a safe distance can the machine placement on the level below be commenced. Within the level, the machine placements need to be mined in a planned sequence to minimize the impact of stress and explosive damage on neighbouring machine placements.

Figure 3 illustrates six machine placements. Considering the operational rules, vertical sequencing enforces MP5 to start being mined only after MP2 is brought back to a safe distance. Furthermore, horizontal sequencing dictates that MP2 starts being mined once after MP1 retreats past enough its neighbour on the same level. Eqs. (52) and (53) enforce vertical and horizontal sequencing between machine placements modelled with the long-term resolution, respectively. It should be mentioned that with the appropriate sets of machine placements in $S^{V}_{M_m}$ and $S^{H}_{M_m}$, these constraints enforce the discussed operational rules. Moreover, at the right side of Equations (52) and (53), the required extraction percentage for the predecessor machine placements is determined. It means that if the δ_m percent of the predecessor machine placement tonnage is extracted by the period t, the machine placement can start being mined at this period (The δ_m is assumed constant for all machine placements, e.g. $\delta_m = 50\%$).

$$bs^{t}_{m} - \sum_{t'=1}^{t} x^{t'}_{m'} \le \delta_m \qquad \forall m \in \{1,2,...,M\}, t \in \{1,2,...,T\}, m' \in S^{V}_{M_m} \tag{52}$$

$$bs^{t}_{m} - \sum_{t'=1}^{t} x^{t'}_{m'} \le \delta_m \qquad \forall m \in \{1,2,...,M\}, t \in \{1,2,...,T\}, m' \in S^{H}_{M_m} \tag{53}$$

4.15. Variable Control Constraints:

Eq. (54) enforces the extracted material from each machine placement to be sent to the processing plant in each period. Eq. (55) to Eq. (56) ensure that the total fraction of material that is mined and sent to the processing plant are less than one over the scheduling periods. Eq. (57) to Eq. (62) ensure that all developments (capital, ventilation, orepass, and operational) are going to be completed once over the life of the mine. Eq. (63) prevents a machine placement from being mined more than once. Eq. (64) ensures that two deviation variables are non-negative. Eq. (65) ensures that all continuous decision variables in the model are between zero and one. Eq. (66) guarantees all binary variables are non-negative and integers.

$$x_{m,p}^{t} = x_{m}^{t} \qquad m \in \{1,2,...,M\}, \forall t \in \{1,2,...,T\} \tag{54}$$

$$\sum_{t=1}^{T} x_{m}^{t} \leq 1 \qquad \forall m \in \{1,2,...,M\} \tag{55}$$

$$\sum_{t=1}^{T} x_{m,p}^{t} \leq 1 \qquad \forall m \in \{1,2,...,M\} \tag{56}$$

$$\sum_{t=1}^{T} d_{sd_l}^{\ t} \leq 1 \qquad \forall l \in \{1,2,...,L\} \tag{57}$$

$$\sum_{t=1}^{T} d_{vds_l}^{\ t} \leq 1 \qquad \forall l \in \{1,2,...,L\} \tag{58}$$

$$\sum_{t=1}^{T} d_{mwr_{v,l}}^{\ t} \leq 1 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\} \tag{59}$$

$$\sum_{t=1}^{T} d_{vd_{v,l}}^{\ t} \leq 1 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\} \tag{60}$$

$$\sum_{t=1}^{T} d_{opd_{v,l}}^{\ t} \leq 1 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\} \tag{61}$$

$$\sum_{t=1}^{T} d_{od_{v,l}}^{\ t} \leq 1 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\} \tag{62}$$

$$\sum_{t \in T_m} bs_{m}^{t} \leq 1 \qquad \forall m \in \{1,2,...,M\} \tag{63}$$

$$\overline{z}_t, \underline{z}_t \geq 0 \qquad \forall t \in \{1,2,...,T\} \tag{64}$$

$$x_{m}^{t}, x_{m,p}^{t}, d_{sd_l}^{\ t}, d_{vds_l}^{\ t}, d_{mwr_{v,l}}^{\ t}, d_{vd_{v,l}}^{\ t}, d_{opd_{v,l}}^{\ t}, d_{od_{v,l}}^{\ t} \in [0,1] \tag{65}$$

$$bs_{m}^{t}, as_{m}^{t}, b_{sd_l}^{\ t}, b_{vds_l}^{\ t}, b_{mwr_{v,l}}^{\ t}, b_{vd_{v,l}}^{\ t}, b_{opd_{v,l}}^{\ t}, b_{od_{v,l}}^{\ t} \in \{0,1\} \tag{66}$$

5. Implementation of the MILP Model on An Illustrative Example

The performance of the proposed models was analyzed based on NPV, mining production, and the practicality of the generated schedules. The model aims to maximize the NPV at a discount rate of 15%, while assuring that all constraints are satisfied during the life of the mine.

The proposed formulation is implemented on a small-scale illustrative case. The vein-like orebody lying 300 meters below the surface consists of 18 machine placements with the size of 25×25×50.

Mining occurs at a depth ranging from 300-375 meters on three different levels. Each level has a height of 25 meters. The SLC mine is organized as follows: three production areas, each with the length of 50 meters, consisting of about 3 horizontal sublevels, a group of two vertical orepasses, and one man-way-raise. The man-way-raise provides access to sublevels, which are horizontal cuts positioned progressively deeper in the earth's surface. In each sublevel, the LHDs load and haul the blasted ore from the production face to the ore passes, where the ore is transported directly to the main haulage level.

In this paper, several major assumptions are used in the MILP formulations. The assumptions are based on the framework for applying operations research methods in SLC mining method.

1) The vertical shafts are considered the primary point from where all materials (ore and waste) exit the mine.
2) Materials from the vertical shaft are managed on a first-in-first-out basis with no mixing.
3) Run-of-mine ore can be sent directly to the processing plant or the ore stockpile which are, both located on the surface.
4) The purpose of the surface-located ore stockpile is to store ore that exceeds the current processing plant capacity and is reclaimed in the future.
5) For each level, the vertical shaft development starts from the bottom of the top level, $l-1$, and continues to the top of the next level, $l+1$. This approach applies to all levels except for $l=1$, and the lowest level. The vertical shaft development in $l=1$ is from the ground surface to the top of the second level. The last level's vertical shaft development includes vertical shaft in that level, as well as haulage drift, and access route developments.
6) The ventilation installations on each level follow vertical shaft development on that level. It means that after the vertical shaft development is completed at the level l , the ventilation development will be commenced.
7) To prevent potential damage, orepasses and man-way-raise are not permitted to be mined simultaneously. Therefore, man-way-raise is drilled before orepasses start to be mined in each production area.
8) No operational developments are allowed to be started before finishing man-way-raise and orepasses developments in each production area.
9) A predefined number of LHDs can operate simultaneously within each sublevel, each production area, and even the whole mine. It is mainly to minimize the impact of stress and explosive damage on neighboring machine placements and prevent congestion and potential damage caused by LHDs drive.
10) Once an LHD starts to mine a machine placement, the machine placement must be continuously mined and if the machineplacement is not mined completely, it will be shut downed.
11) Vertical sequencing requires at least 50% of machine placement m be mined before beginning to mine machine placement located underneath machine placement m .
12) Horizontal sequencing requires that adjacent machine placements on the left and right sides of the machine placement m begin to be mined once 50% of machine placement m is mined.

Table 1 shows the machine placements located in each specific production area and level. For instance, machine placements 9 (MP9) and 10 (MP10) belong to production area 2, and level 2.

Table 1. Machine placement's locations in production areas and levels.

	Production area 1		Production area 2		Production area 3	
Level 1	**MP1**	**MP2**	**MP3**	**MP4**	**MP5**	**MP6**
Level 2	**MP7**	**MP8**	**MP9**	**MP10**	**MP11**	**MP12**
Level 3	**MP13**	**MP14**	**MP15**	**MP16**	**MP17**	**MP18**

Figure 4 provides a schematic layout of production areas, levels, machine placements, and development activities through two cross-sections (i) along the strike of the orebody and (ii) along the dip of the orebody.

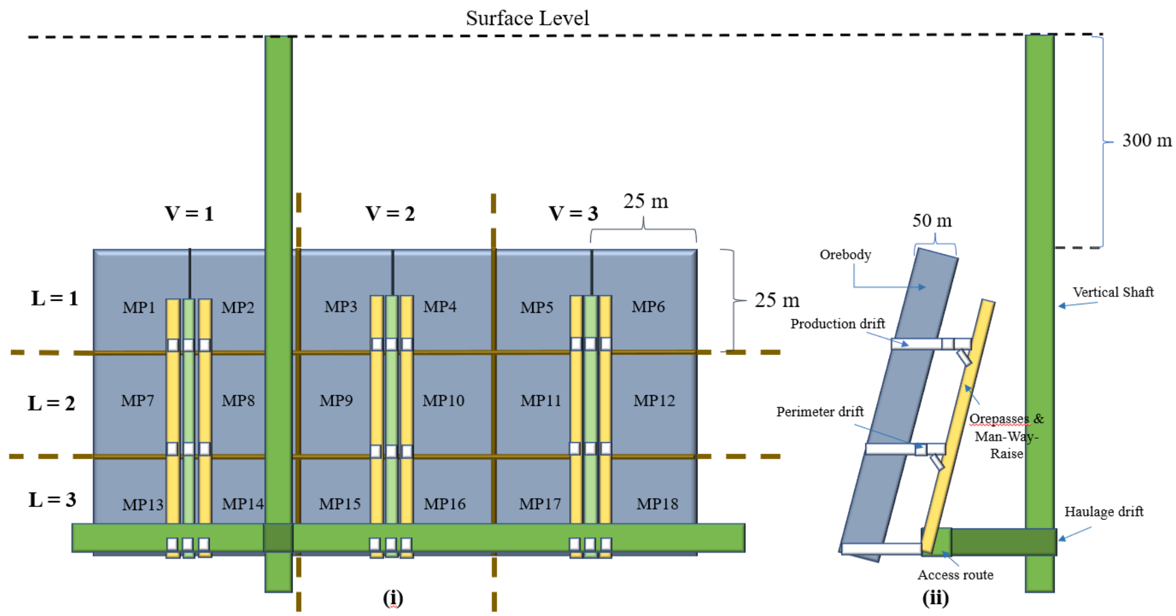

Figure 4. A schematic representation of production areas, levels, machine placements, and development layout: (i) is a cross-section along the strike of the orebody, and (ii) is a cross-section along the dip of the orebody.

Table 2 represents the annual lower-bound (LB), and the upper-bound (UB) limits of all given input data to the model over ten-year time horizon. The annual discount rate is assumed 15% to calculate NPV. The developed model has been coded in Jupyter notebook while taking advantage of CPLEX Python API to solve and optimize the model on a PC Intel Core i7, 2.60 GHz, with 12 GB of RAM, running Windows 10.

Figure 5 to Figure 11 represent the obtained development activities' schedules from the model. Figure 12 illustrates the mine production and processing plant feed during mine life. Figure 13 shows active periods and the extracted portion from each machine placement. Figure 14 and Figure 15 illustrate the starting period of each machine placement and the number of active machine placements in each period.

Table 2. Input data to the model.

Parameters	Unit	t=1 LB	t=1 UB	t=2 LB	t=2 UB	t=3 LB	t=3 UB	t=4 LB	t=4 UB	t=5 LB	t=5 UB	t=6 LB	t=6 UB	t=7 LB	t=7 UB	t=8 LB	t=8 UB	t=9 LB	t=9 UB	t=10 LB	t=10 UB
Vertical shaft development	**meter**	0	500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Vertical shaft ventilation	**meter**	0	500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Man-way-raise development	**meter**	0	0	0	0	0	300	0	300	0	300	0	300	0	300	0	300	0	300	0	300
Man-way-raise ventilation	**meter**	0	0	0	0	0	300	0	300	0	300	0	300	0	300	0	300	0	300	0	300
Orepass development	**meter**	0	0	0	0	0	300	0	300	0	300	0	300	0	300	0	300	0	300	0	300
Operational development	**meter**	0	0	0	0	0	600	0	600	0	600	0	600	0	600	0	600	0	600	0	600
Mining capacities	**K ton**	0	160	0	200	0	230	0	270	0	270	0	270	0	270	0	270	0	270	0	270
Processing plant capacities	**K ton**	0	150	0	180	0	220	0	250	0	250	0	250	0	250	0	250	0	250	0	250
Processing plant head grade	**float**	0	0.7	0	0.7	0	0.7	0	0.7	0	0.7	0	0.7	0	0.7	0	0.7	0	0.7	0	0.7

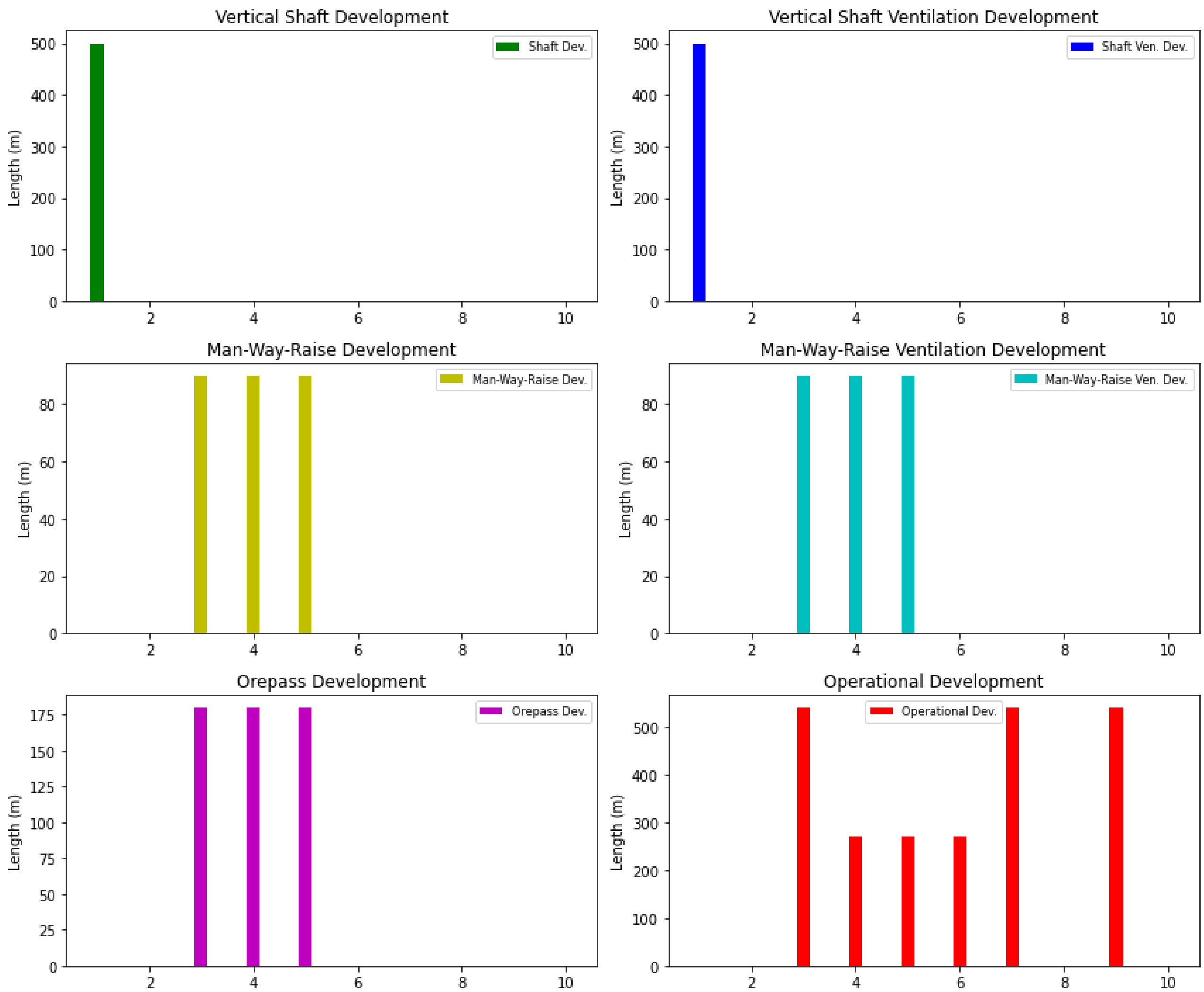

Figure 5. Development activities over the life of mine.

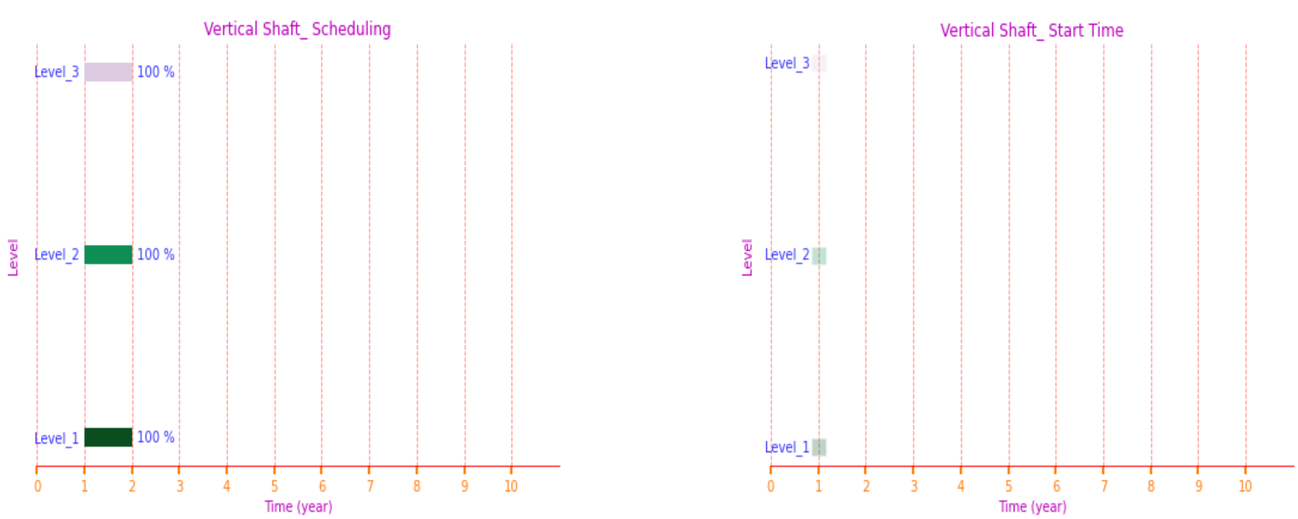

Figure 6. Vertical shaft development scheduling.

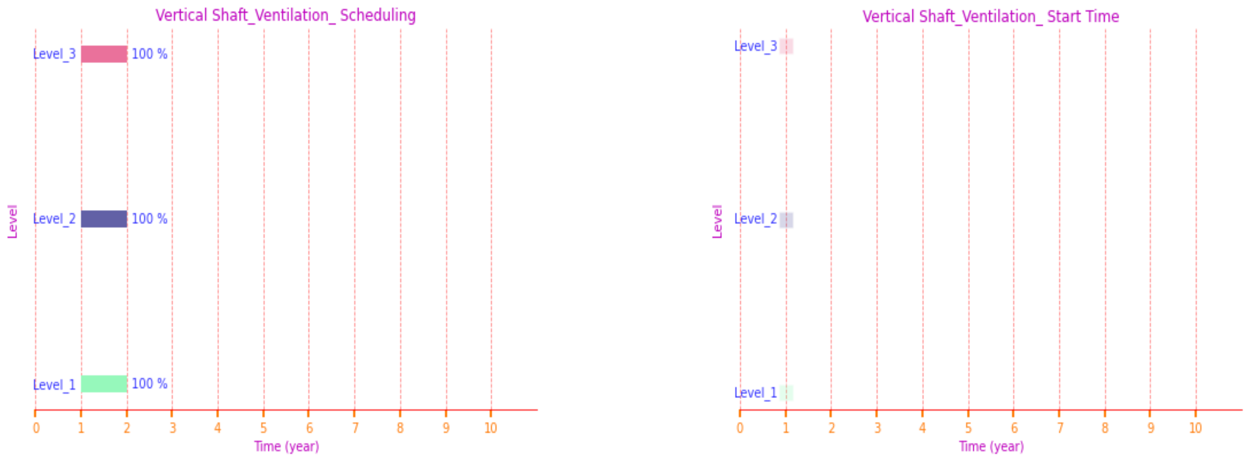

Figure 7. Vertical shaft ventilation scheduling.

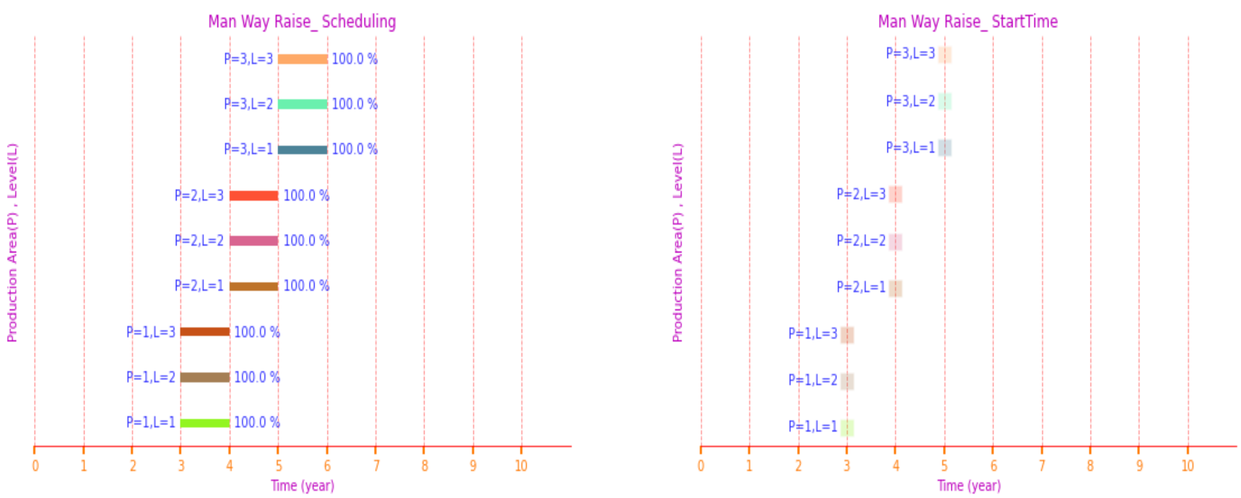

Figure 8. Man-way-raise development scheduling.

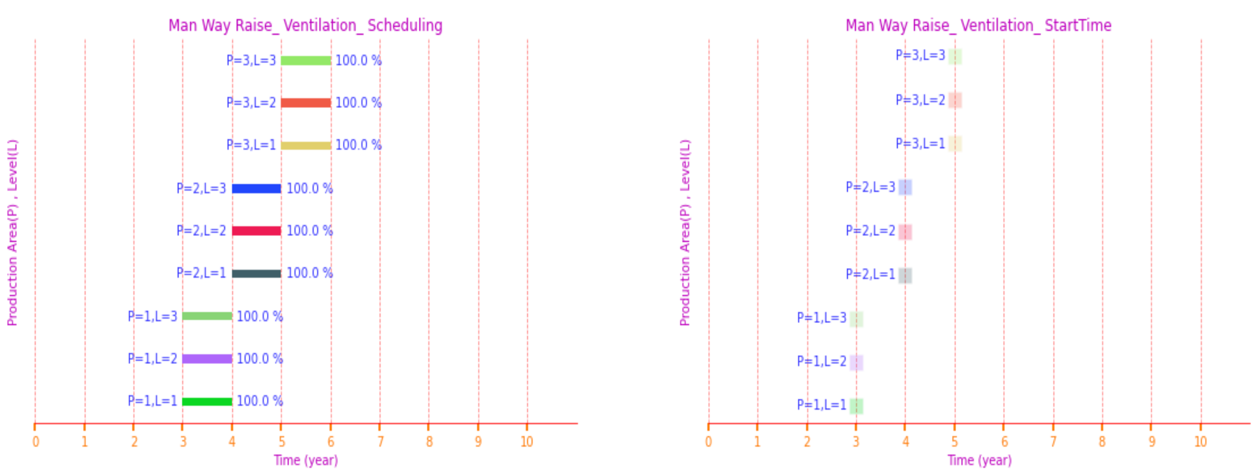

Figure 9. Man-way-raise ventilation scheduling.

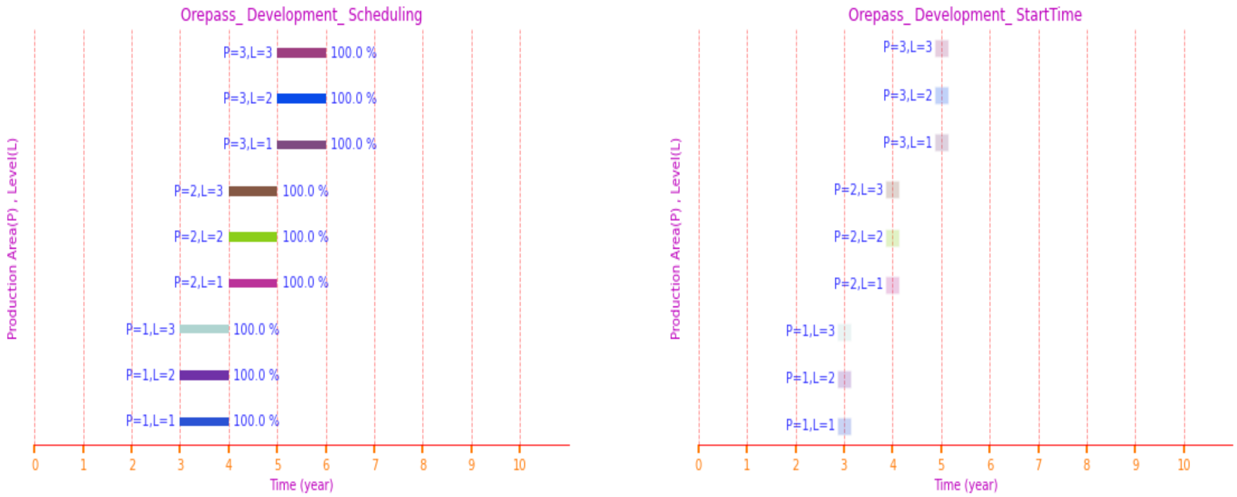

Figure 10. Orepass development scheduling.

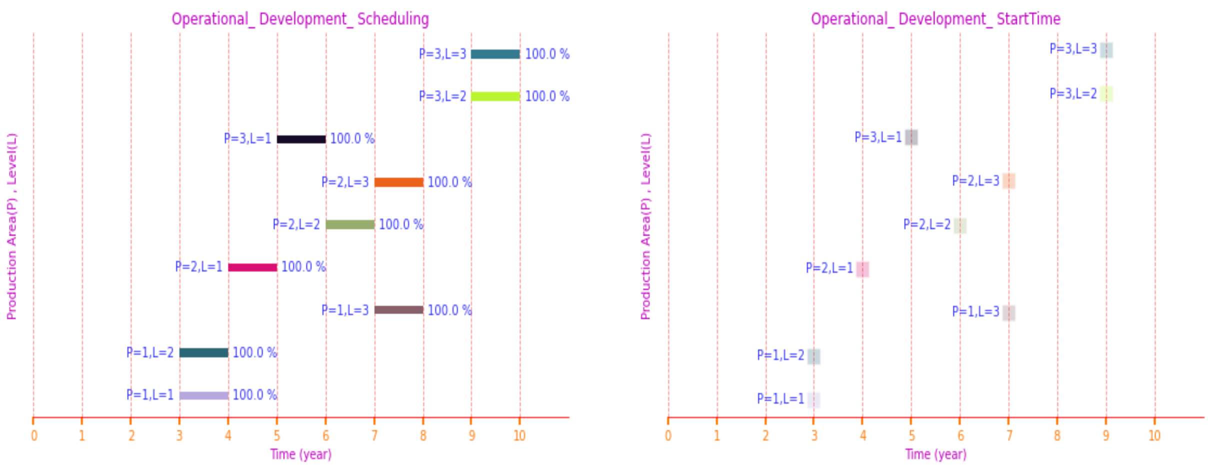

Figure 11. Operational development scheduling.

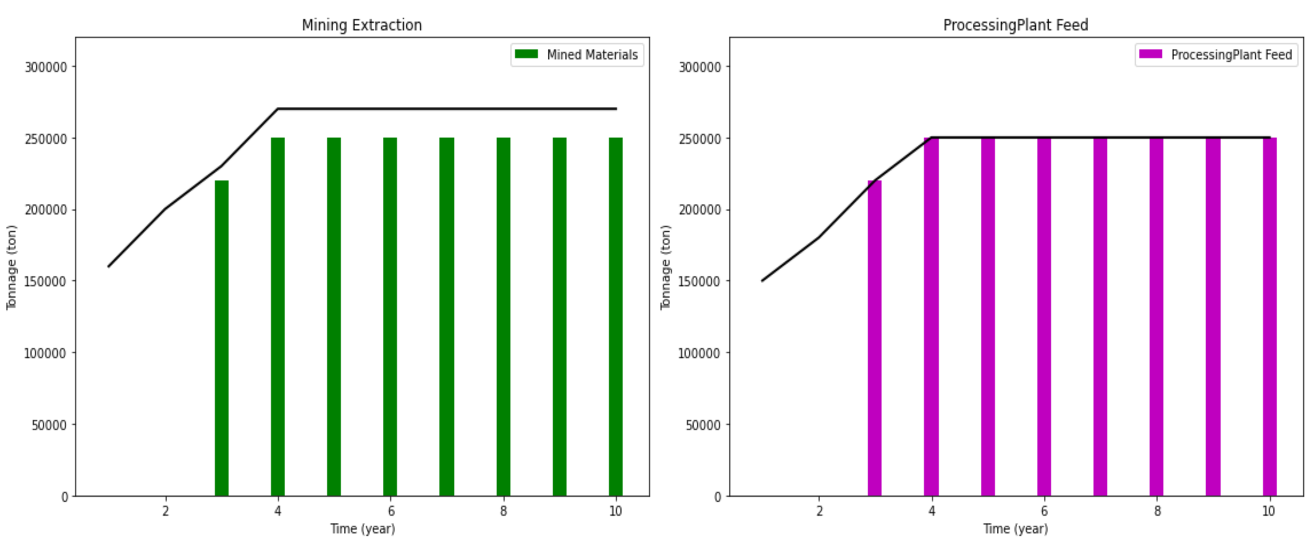

Figure 12. Mining extraction and processing plant output over the life of mine.

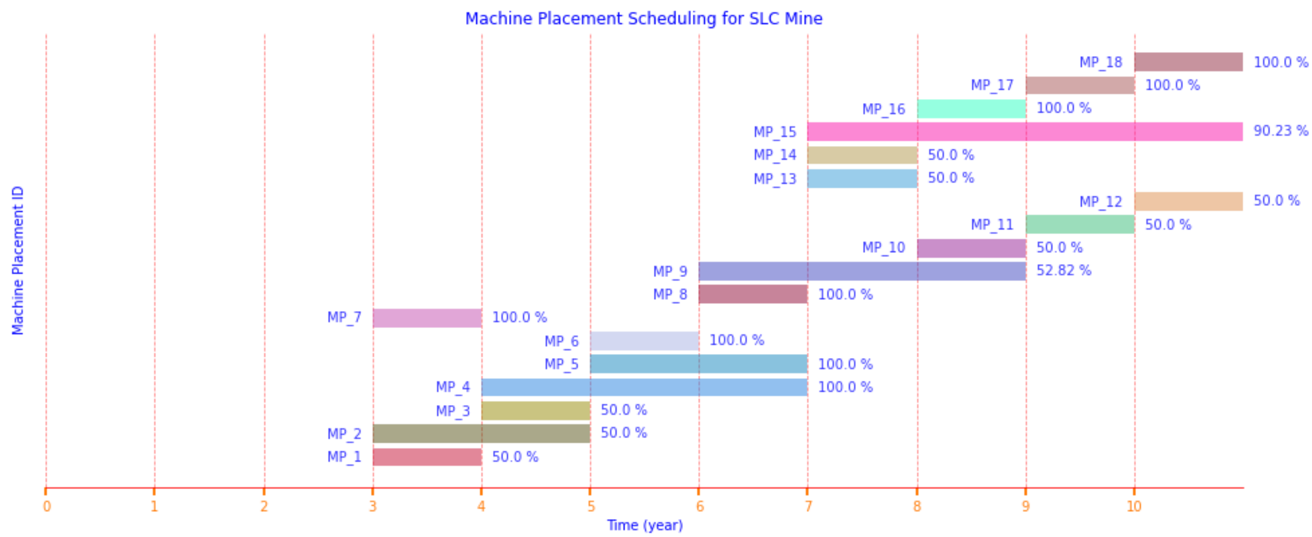

Figure 13. Machine placement scheduling including start time, extraction duration, extraction portion (%), and end time.

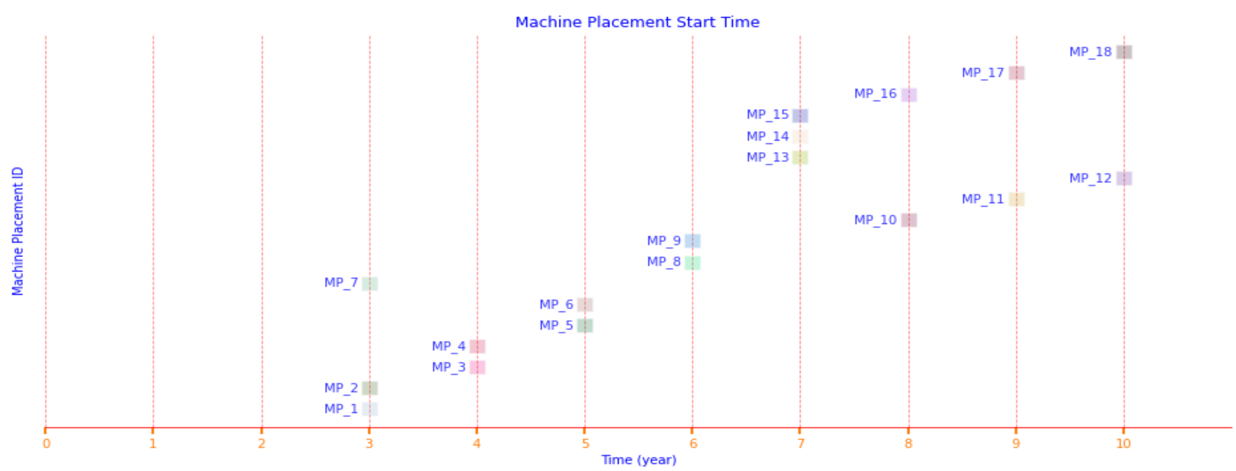

Figure 14. Machine placements start times over the life of mine.

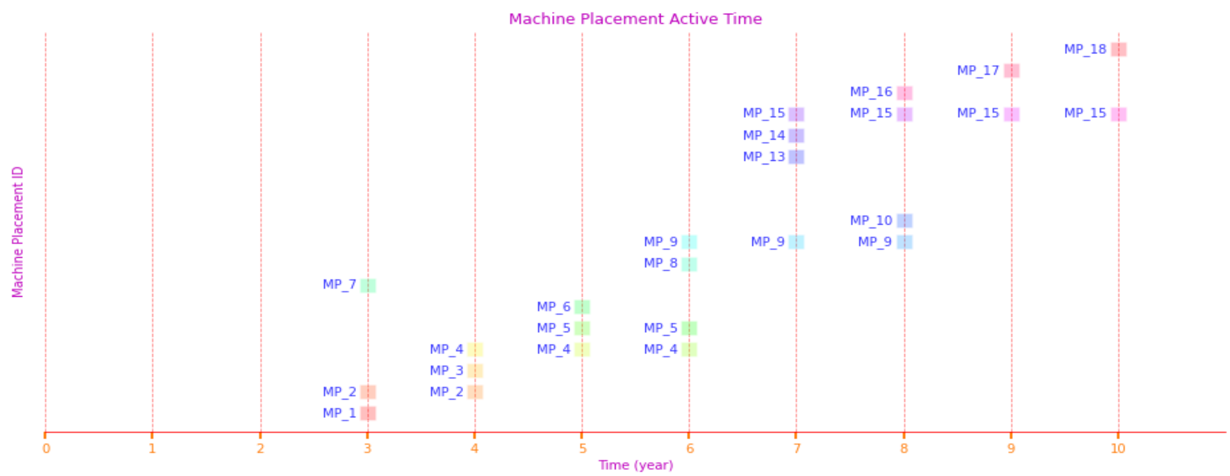

Figure 15. Active machine placements in each period.

6. Conclusions and Future Works

This paper presents an integrated mathematical model to optimize the sublevel production scheduling. MILP formulations for SLC production schedule were developed and implemented in the Jupyter notebook using CPLEX Python API to solve and optimize. The schedule of each machine placement is controlled by the completion of the development activities in the production area and the level that the machine placement is located while considering the vertical and horizontal precedence relationships between machine placements. The mine planner also controls the number of new machine placements that need to be started to be mined in each period to meet the mine production, the number of active machine placements in each period, and the average production grade.

The model also monitors the pre-extraction material flow, development activities, and integrates them into the machine placement schedules. All formulations maximize the NPV and, at the same time, minimize deviations from planned production quantities while adhering to operational constraints.

Future research will focus on modifying the model for handling post-extraction material flow. In particular, the development activities prior to starting ore extraction as well as the use of stockpiling to manage processing plant capacity, and the interplay of material flows from the mine to a stockpile, mine to plant, and a stockpile to plant, need to be treated as an integrated part of mining optimization model. In addition, efficient mathematical techniques will be explored to reduce the number of decision variables and the execution time for large-scale SLC production scheduling.

7. References

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