Determination of Optimal Underground Stope Layout

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ABSTRACT

Stope layout optimization means determining the best dimension and the locations of the stopes and number of stopes. Stope layout in underground mining directly affects other significant aspects of mining such as economic value and mining recovery. Compare to the open-pit mine, limited techniques and algorithms are available to find the optimums in designing underground mine, and most of those fail to provide an exact optimum solution especially in 3D space.

The goal of this research is creating a stope optimizer algorithm to find the best combination of the stops with highest economic value. In fact, this research directly contributes to creating a new heuristic model which can tackle the complexity of stope layout designing and at the same time can achieve to the near-optimal solution in 3D space.

1. Introduction

According to Canada's Economic Action Plan, the energy and mining sector provides over $30 billion a year in revenue to governments. The economics of today's mining industry is such that the major mining companies are increasing the use of massive underground mining methods. They expect that approximately 50 percent of the world's copper production will come from underground mines by 2020. It is a step change for the industry, from the traditional open-pit to a move underground.

Several algorithms have been presented to optimize the stope layout for underground mining in the last 40 years. Most of the earlier works on stope optimization were based on the strong simplifications of the initial problem (Bai et al., 2013). However, some of those indicated the proper algorithms which achieved the important goals such as maximum economic value, maximum mining recovery, minimum ore dilution and minimum ore loss (Dimitrakopoulos et al., 2009).

An ore reserve model, which defined by a set of small regular blocks, is basic input for stope optimization (David, 1988). In order to find optimum stope layout, geotechnical and operational and economical considerations such as characteristics of the ore body, accessing to stopes, mining equipment size, pillar size to be looked at (Bai et al., 2013; Sandanayake et al., 2015a).

Generally, the all presented 2D or 3D algorithms to define the optimal stope layout classify to two groups of mathematical (exact) and heuristic. Mathematical algorithms are supported by mathematical proof; however, the heuristic algorithms are based on constraints and limitations to find an approximate solution. For instance, introduced algorithms by Riddle (1977), Ovanic et al. (1995), and Bai et al. (2013) are mathematical, and the remains follow the heuristic model (Sandanayake, 2014).

2. Literature Review

The generated algorithms to find the optimum stope layout in underground mining are reviewed in the following section and their applications, methodologies, capabilities and restrictions and their similarities and contrasts are discussed. In addition, the different objective functions and constraints, which are considered in those algorithms, are presented.

The existing algorithms for underground stope layout optimization are classified in two groups: field-oriented and level-oriented. In the field-oriented algorithms, the economic value of each block considered as a constant value and determination of underground mining limit takes place on the entire mining area before dividing the mining area to levels or panels. In contrast, the level-oriented algorithms are applied on a level or panel (Jalali, 2006).

Riddle (1977) presented the first algorithm, called "Dynamic Programming Algorithm" to find optimum stope layout in block-caving mining method. This method solves the 3D problems by using multi-section 2D; north-south sections and east-west sections. Although his algorithm is able to optimize the sections, it fails to find the true optimum stope in three dimensions because it does not consider all necessary constraints.

Dynamic Programming Algorithm assumes a 2D section of blocks with *i* rows and j columns and it is formulated as equation (1).

$$P_{ijo} = M_{i,j} + \text{Max}\left\{P_{i+r,j-1}\right\} \tag{1}$$

Where, $M_{i,j}$ is the cumulative net value of blocks, r is the range indicating adjacent blocks, and P_{ijo} is the profit achieved by mining through the block (row "i" of drawpoint "j" and starting at any level of drawpoint "o ")

At the first step, P_{ij1} is calculated for all blocks. Then, column 1 is eliminated and all P_{ij2} is calculated and it continues to the last column. Result in the last column is equal to the cumulative net value of blocks. Then the calculations for other rows are done, and at the last step, maximum profit is determined (Ataee-Pour, 2000).

Deraisme et al. (1984) used the Downstream Geostatistical Approach to determine optimal stope. This model is the 2D sectional numerical models of the deposit. Mathematical morphology helps this model to consider the stope geometry constraints. This approach has been recommended when because of underground mining constraints restrictions, the linear and nonlinear geostatistics are not able to estimate the mineable reserves.

Generally, the Downstream Geostatistical Approach is based on a combination of conditional simulation with underground mining simulation to compare: selectivity; productivity; and profitability in cut-and-fill and block caving methods. The approach steps included the constructing a numerical model of the deposit at first and then defining the outlines of the mineable ore (Ataee-Pour, 2000).

Cheimanoff et al. (1989) described a heuristic approach with binary-tree division technique, called "Octree Division Approach", to move from geological resources to mineable reserves based on the mining constraints and provides a 3D solution to find optimum stope. In fact, this model is based on removing the non-desired mining blocks to define the minimum stope size.

This model covers two main constraints. First, the geometric constraints which are based on the ore-body geotechnical behavior as well as mining equipment. Second, the economic constraints which are based on the cut-off grade and the mining costs such as access cost and services cost (Ataee-Pour, 2000).

Ovanic et al. (1995) developed “Branch and Bound Technique” to optimize outline of the stope based on the optimizing of starting and ending points within each row of blocks. To find the optimum starting and ending points, they used two piecewise linear functions for each row. In addition, they considered a mixed integer approach, called "Type-Two Special Ordered Sets", to optimize stope boundary. In fact, two separate "Type-Two Special Ordered Sets" are defined as stope boundaries (starting and ending points) variable. As a result, the objective function is determined as the difference between the cumulative values obtained for the stope boundaries. Equation (2) presents the objective function of the method.

$$MaximiseSV = \sum_{i=0}^{n} a_i L_i - \sum_{i=0}^{n} a_i T_i \tag{2}$$

Where, SV is the difference between the cumulative values of starting and ending points, a_i is the cumulative block economic value, L_i is the starting point variable (bounded between 0 and 1), and T_i is the starting point variable (bounded between 0 and 1).

In addition, the constraints are based on the geometric limitations, which impact on the minimum and a maximum size of stopes.

In contrast with previous algorithms, having only regular or uniform shapes blocks and having only whole blocks are not required in “Branch and Bound Technique” algorithm. In other words, blocks shape or size does not effect on the optimization, because the block cumulative value function has been developed in this model. These points are really beneficial in case of existing the geological interpretations in the block model.

Alford (1996) described the "Floating Stope Algorithm", which is similar to the "Moving Cone" method in open-pit limit optimization, to set up the optimal stope boundary. The positive points of the "Floating Stope Algorithm" are simplicity and generality. Simplicity comes from generating a three-dimensional assessment optimization and sensitivity analysis point of view, and generality comes from using not for only certain mining method point of view. Having the heuristic approach and lacks rigorous mathematical is one the disadvantages of this algorithm. In addition, the “Floating Stope Algorithm” doesn’t guarantee to find the true optimum stope.

Regarding the definition of "Floating Stope" term, this technique is based on moving a floating stope shape with minimum stope dimensions, through blocks to locate the stope position. The process of floating the stope shape can be based on the best grade stope shapes or based on the possible stope positions. However, the problem is the possibility of overlapping of the stopes in the final result (Sandanayake et al., 2015a).

The objectives function in this algorithm can be maximizing ore tonnes or minimizing waste, maximizing grade, maximizing the profit. Also, the main constraint is the geometry of the stope. Finally, the volume of the stope and the profit were determined as outputs (Ataee-Pour, 2000).

Ataee-Pour (2000) presented a heuristic algorithm and called it "Maximum Value Neighbourhood" (MVN). This algorithm works on a fixed economic block model of an ore-body to provide a 3D analysis of optimization of the stope boundaries. He defined the neighborhood concept based on the number of mining blocks equivalent to the minimum stope size.

The MVN algorithm has taken benefits from its generality, which allows it to be applied for any underground mining method and its simplicity in both concept and computer implementation.

MVN algorithm locates the best neighborhood of a block to find the best combination of blocks to create the maximum profit, while certain mining and geotechnical constraints are considered. The mining constraints in this algorithm are based on the restrictions which are determined by minimum stope dimensions in three principal directions. In addition, the size of the equipment and necessary spaces for the drilling, blasting, loading, and traffic of personnel are important in defining the minimum stope dimensions. Also, several physical parameters such as geotechnical properties of the ore-body and the surrounding rock, dip, depth, thickness of the ore-body which can affect the proposed underground mining methods are set up as constraints in this algorithm. However, he has not paid attention to the maximum limits of the stope dimensions which from ground control considerations point of view are important. Also, ignoring the shape of the mineable stopes is the problem with this algorithm (Sandanayake et al., 2015a).

FORTRAN programming language has been used to develop his study. The following stages for optimization process have been described by Ataee-Pour (2000).

- The block economic value (BEV) is considered for each block.
- The set of possible neighborhoods is determined for each block.
- The feasibility of each neighborhood (If the neighborhood elements are located inside the block model or not) is evaluated.
- The economic value of each neighborhood is calculated.
- The maximum value neighborhood is determined.
- The stope economic value is updated.

Ataee-Pour (2000) mentioned that the MVN algorithm failed to determine the true optimal stope layout because it used a heuristic approach with lacks rigorous mathematical proof, however, the MVN algorithm guarantees the optimum value neighborhood for each block. Based on his experience the problem was how to combine these optimum neighborhoods value to create the optimum layout.

Topal et al. (2010) presented a new methodology to find optimum stope layout in case of single as well as variable stope sizes in three-dimensions. The proposed methodology in their work consisted of three basic elements which are block converter, stope boundary optimizer, and stope visualizer.

Block converter has been created to convert a block model with multiple block sizes into a block model with only one size of blocks with new values.

Stope boundary optimizer element uses a range of all the possible stope sizes, ore price per tonne, mining and processing costs per tonne, backfill costs per cubic meter as well as a fixed stope start-up costs as inputs and the optimum stope boundaries and layout for ore-body are the outputs.

The procedure of stope boundary optimizer element is as follow:

Step 1: Optimizer starts from the smallest available stope size on every possible location and it continues to evaluate all the possible stopes and their profits.

Step 2: An envelope is created on every individual stope and the value of each envelope is calculated.

Step 3: Based on the stope profit, stopes are selected.

Step 4: The selected stopes are checked with highest average envelope stope profit from Step 2.

The stope visualizer element is a program to create the three-dimensional view of the final stope layout (Topal et al., 2010).

Their algorithm works based on two assumptions. Firstly, all stopes have a fixed start-up time, and the production and backfill time have a linear relation with the stope volume. Secondly, the calculation of NPV is based on the mining of single stope at a given time.

They have used two different strategies to stope boundary optimization. Firstly, the strategy based on highest profit per stope which shows a better overall profit. Secondly, the strategy based on highest profit per time which demonstrates a better NPV. However, the main problem with his algorithm is failing to analyze all alternative solutions in the procedure (Sandanayake et al., 2015a).

Bai et al. (2013) suggested a new 3D method using flow algorithms to design stopes layout. This model is based on a cylindrical coordinate. They believed that there was not a general-purpose optimization algorithm suited for all underground mining methods because of the difference between geotechnical constraints in different mining methods. As a result, they introduced an optimization algorithm which was suitable only for sublevel stoping (long-hole) method.

Their optimization algorithm includes two main objective functions. The first one is stope optimizer which consists of stope optimizing based on the specified raise location and height. The second one is finding the best raise location and height. In addition, the footwall and hanging wall slope, the stope width and height are played the constraints roles. Also, the maximum distance of a block from the raise and the horizontal width required, for a block at that distance are the control parameters for the cylindrical system of coordinates.

In order to the constraints considering, they have defined the arc in the graph in the cylindrical system and then after finding the overall optimal stope, they have converted the solution to the Cartesian system.

Since their algorithm is based on the cylindrical coordinate system with vertical raise, this algorithm is not acceptable in the case of sub-vertical or sub-horizontal deposits which need inclined raise. Furthermore, this approach is based on the small ore-body with single raise parameters and it isn't useful for larger ore-bodies which need many contiguous stopes. Additionally, in their approach, they have used fixed development costs and operational costs although those are related to the raise location and height (Bai et al., 2013).

Sandanayake et al. (2015a) offered a new 3D heuristic algorithm that maximizes the economic value regarding the physical mining and geotechnical constraints. This algorithm assessed the stope layout problem by considering fixed and variable stope sizes with and without pillars. Also, this group claims that the algorithm is flexible enough for varying underground mining situations.

More specifically, at the first stage, this algorithm transferred the block model to the economic block model. After defining minimum and maximum stope sizes in terms of a number of mining blocks, all possibilities of stopes are created. By getting average, the material density and grade and economic value of each stope are calculated. Then the sets of positive value possibilities are defined. Based on the stope possibilities overlaps, availability of development levels and pillars, the stope possibilities can be limited. The detail of this step is as follows:

Generating sets of non-overlapping stopes, which mentions as equation (3):

$$[c_{xyz_s} \geq l_{xyz_{s'}} \cap c_{xyz_{s'}} \geq l_{xyz_s}; \forall s, s' \in \delta_s] \tag{3}$$

- Adding a constraint if a pillar width and level height (for the 2D situation) defined as equation (4):

$$[\left|c_{x_s} - c_{x_{s'}}\right| \cup \left|c_{y_s} - c_{y_{s'}}\right| \geq p; \forall s, s' \in \delta_{sk}] \tag{4}$$

Where, x_s, y_s, z_s are x, y, and z coordinates of stope s, c_{xyz_s} is x, y, and z coordinates of the origin mining block in stope s, l_{xyz_s} is x, y, z coordinates of the terminal (last) mining block in stope s, δ_s is the sets of stopes with positive economic value, δ_{sk} is the sets of stopes with positive economic value with pillar width apart, and p is the pillar width.

Determining economic value of each non-overlapping stope set, by summation of stopes economic value in each set, and selecting the set with maximum economic value as a solution is another step of the algorithm (Sandanayake et al., 2015a).

Since with infinite sets of stopes, achieving an optimal solution is impossible, they have to define an upper bound on the number of possible solutions. In addition, they reduced the algorithm time by using the parallelization (Sandanayake et al., 2015a).

Sandanayake et al. (2015b) continued their work on finding optimum stope layout, and they prepared an algorithm similar to their previous algorithm with the limitation on a number of the sets of non-overlapping stopes, and they examined the algorithm for an actual ore body model.

To validate the proposed algorithm, they did a comparison with MVN algorithm, which has been implemented in commercially available software. Results indicated that the solution generated by this algorithm achieve to the almost %10 higher economic value than the MVN algorithm. However, the solution time for MVN is less (Sandanayake, 2014).

Villalba Matamoros et al. (2017) worked on the minimization of inherent internal dilution and conventional profit maximization as the main approaches in the optimization of stope layout. The economic, geotechnical and operational and ore-body quality and quantity constraints are subjected in their model.

In their research, they have defined internal dilution, which is the waste or low-grade waste located within the ore, and external dilution, which is the waste low-grade waste located on the border between ore and waste. In addition, they have divided the dilution to two groups of primary dilution and secondary dilution. The primary dilution is inherent in mining method and their stope dimensioning. However, the secondary dilution is additional non-ore material from rock or backfill outside the stope boundaries which can be as a result of blast-induced over break, unstable wall rock fall and backfill fall.

To start their work, some information is used as the input parameters. The minimum and maximum dimension of the stopes have been mentioned as allowable limits to cover the stability of open stopes as well as the efficiency of equipment. A 3D block model of the ore-body is another input information. In addition, the assays of the drillholes and geological model are considered to provide a better knowledge about the quality of the ore-body model.

They describe an objective function which maximizes profit and minimizes dilution as equation (5).

$$\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K} xwgRP - \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K} xw(M+C) - \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K} xwg(1-O)P(R-\zeta(f(g)) \quad (5)$$

Where, i, j, k are parameters to show the location of blocks where $i = 1, \ldots, I$, $j=1, \ldots, J$, and $k = 1, \ldots, K$, x is binary variable to show if the block is mined as a part of stope or not, w is the tonnage of blocks, g is the grade of blocks, P is the price of metal by tonnes of metal (\$/oz), M is mining costs per tonne mined (\$/t), C is the processing costs per tonne milled (\$/t), O is binary parameters to clarify if the grade of the block is higher than the cut-off grade or not, ζ is penalty for internal dilution grade recovery to minimize the internal dilution (%), R is recovery of metal, and f is recovery function of blocks with a grade less than cut-off.

The first part of this equation covers the revenue of stopes, the second part includes of mining and processing cost of the stopes, and the last part considers the cost associated with internal dilution which assists to minimize the internal dilution.

This objective function has come with 13 sets of constraints such as constraints to ensure that each block is mined only once, constraints for block precedence, constraints to consider the minimum and maximum of height and width of stopes which should be mined, constraints to define the grade greater than the cut-off.

In the heuristic method presented by Villalba Matamoros et al. (2017), the grade fluctuations in the ore-body can make problem in determining the stope layout correctly. Also, they used uninformed and ultimately costly decisions.

Based on the previous algorithms to determine the optimum stope in an underground mine, there are some important notes which are summarized as follows:

- In all methods, ore block model is adopted as an input. Although, in some of those having only regular and uniform shapes blocks are required, and in some cases, having partial blocks are considered as well.
- Various objective functions are described in the mentioned methods. Maximizing the overall profit or maximizing the NPV are most common objective functions, however, other factors such as minimizing the dilution or maximizing tonnage of ore have been mentioned in some cases.
- Some geotechnical considerations are examined in all algorithms as the constraints to find the optimum slope. However, not all methods are covered all geotechnical constraints. Additionally, in few of previous works, economic constraints, such as mining cost, and operational constraints, such as equipment size, are considered as well.
- Seems dealing with more constraints makes the result closer to the true optimum stope layout.
- Before 2000, few algorithms presented to determine the optimal stope layout. However, some of those did not introduce the 3D models and mathematical solution.
- Simplicity and generality are two characters of some algorithms. Simplicity may cover concepts, assessments and analysis steps in the algorithm. Also, generality is the ability in applying for different mining methods. However, Bai et al. (2013) believed that using one algorithm for all underground mining methods was not a proper decision because of difference in geotechnical constraints in different mining methods.
- All algorithms have covered the minimum sizes of stope, nonetheless, not all of the indicated algorithms have acknowledged the maximum limits of the stope dimensions which from ground control considerations point of view are important.
- Some of the mentioned works are able to calculate only single stope size, but others can evaluate the variable stope sizes as well.
- The shape of the ore body plays an important role in some methods.
- Table 1 is the comparison of all algorithms, which completes the mentioned Table by Ataee-Pour (2000).

At the next section, the new heuristic algorithm which follows the same steps as Sandanayake (2014) research will be introduced. This algorithm can reach to the satisfying result in the short time of running.

3. Proposed Algorithm Methodology

The overall process of the proposed algorithm in this research is generated from five main steps. Fig 1 shows these steps. The process starts by using the economic parameters to create the economic block model. The next step is generating stopes, calculate stopes value and find the positive ones. Then, based on the stopes overlaps, all possibilities of combinations of positive value stopes are found. Also, the value of possible stopes combinations is computed and the optimum stopes combination which is the combination with highest economic value is discovered. Finally, the optimum solution is visualized.

3.1. Generate the Block Economic Model

At the first part, the economic block model is prepared. The blocks information is as the first group of input in the algorithm. The blocks information includes the number, coordinates or indexes, grade, tonnage for each block. The second group of the input is economic parameters which contain the metal price, cost of selling, mining cost, processing costs, and recovery.

Table 1. The comparison of mentioned algorithms

	Algorithm	Model Type	Mining Method	Dim.	Mathematical Formulation	Partial Blocks	True Optimality
1	Dynamic Programming Riddle (1977)	Fixed Blocks	Block-Caving	2D	Yes	No	No
2	Downstream Geostatistical Deraisme et al. (1984)	Cross-Sections	Block-Caving Cut-and-Fill	2D	No	No	No
3	Octree Division Cheimanoff et al. (1989)	Not Applicable	All	3D	No	N/A	No
4	Floating Stope Alford (1996)	Fixed Blocks	All	3D	No	Yes	No
5	Branch and Bound Ovanic et al. (1995)	(Ir)regular Blocks	All	1D	Yes	Yes	Yes
6	MVN Ataee-Pour (2000)	Fixed Blocks	All	3D	No	No	No
7	Methodology by Topal et al. (2010)	Not Applicable	All	3D	No	No	No
8	Network Flow Method Bai et al. (2013)	Cylindrical Coordinate	Sublevel Stoping	3D	Yes	No	No
9	Methodology by Sandanayake (2014)	(Ir)regular Blocks	All	3D	No	No	No
10	Methodology by Villalba Matamoros et al. (2017)	Not Applicable	All	3D	No	No	No

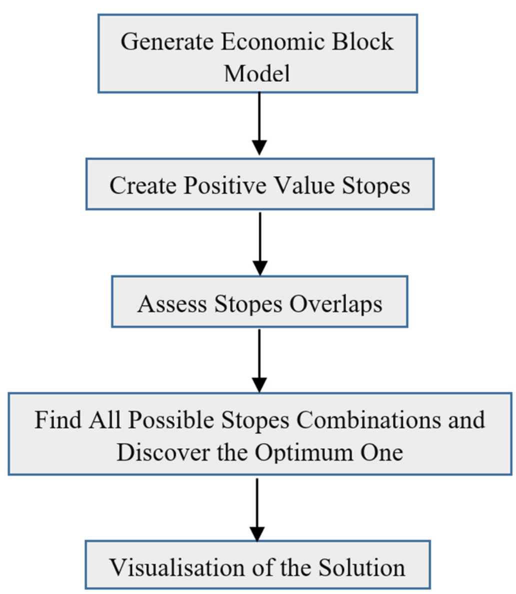

Fig 1. Overall process of the algorithm

To calculate the economic value for each block, the cut-off grade, which is the lowest sufficient grade of the material to send it to the processing, is required. In fact, if the grade of the block is more than the cut-off grade, the economic value of the block is computed by equation (6). However, if the grade of the block is less than the cut-off grade, the equation (7) should be applied and that block is considered as a waste.

$$v = \left[(p - c_s) \times g \times r - (c_m + c_p) \right] \times t \tag{6}$$

$$v = -(c_m \times t) \tag{7}$$

Where v is the block economic value (\$), p is the metal price (\$/tonne or \$/oz), c_s is the cost of selling (\$/tonne), g is the block average grade (oz/tonne or %), r is recovery, c_m is the cost of mining (\$/tonne), c_p is the cost of processing (\$/tonne), and t is the tonnage of the block (tonne).

3.2. Create Positive Values Stopes

At the first step, based on geotechnical and mining constraints, the dimensions of the stopes should be defined. The dimensions of the stopes are based on the number of the blocks at three directions (X, Y, Z axes) which called Lx, Ly, and Lz. Then, the stope with these dimensions is floated along axes to find all stope possibilities. Fig 2 indicates an example of a 2D economic block model with 6 blocks along X-axis and 4 blocks along Y-axis. Also, it shows the starting point for stope floating. In this example, +2 is considered as the value of each block.

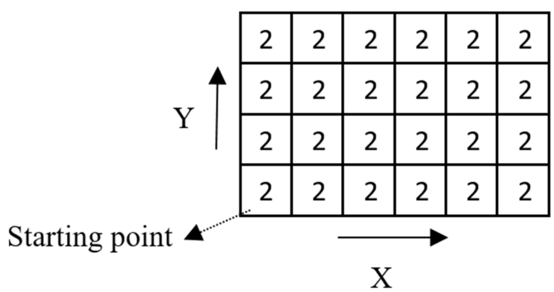

Fig 2. An economic block model (6×4)

Fig 3 illustrates how a 3×3 (Lx=3, Ly=3) stope can float along the axes to create all stope possibilities, which in this case 8 possibilities have been created. Then, the value of each stope should be calculated which is the summation of all blocks value in each stope. For instance, in the current example, the value of 9 blocks are summed to have the value of each stope which equals to +18. Finally, stopes with the positive value are the output of this step.

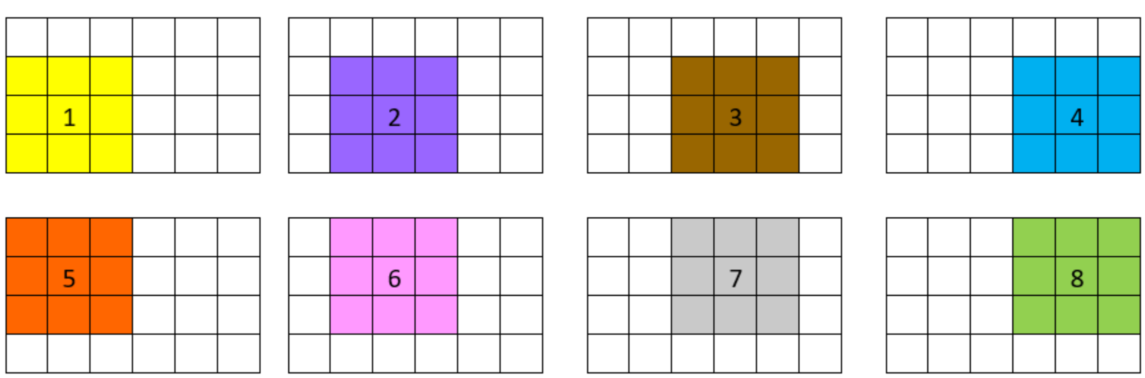

Fig 3. All possible positive stopes

3.3. Assess Stopes Overlaps

This step is about finding possible combinations of stopes with positive economic value. The main consideration in this step is discovering the overlaps between these stopes. In reality, not all stope can combine together because of exciting overlaps between those stopes.

To do this step, an all zero elements matrix with same dimensions of i and j, and equal to a number of positive stopes is created and by looping over all the elements, overlaps can be found. In fact, if two elements (two positive stopes) have one or more common blocks, those are accounted as the stopes with overlap. In the presented algorithm, if two stopes have overlap, element zero is changed to one in the overlap matrix. While, if two stopes do not have any overlaps, element zero is kept in overlaps matrix. As a result, the overlap matrix, a matrix with elements zero and one, is created.

For example, in Fig 4, it is not possible to have stope number 1 and number 7 or number 6 and number 8 at the same time.

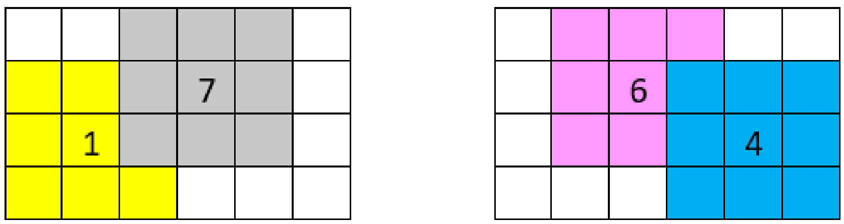

Fig 4. Examples of stope possibilities overlap

3.4. Find All Possible Stopes Combinations and Discover the Optimum One

At this step, the presented algorithm creates the possible stopes combinations. In facts, these combinations can be generated by eliminating the stopes overlaps in each possible combination. For instance, for mentioned case, the result of this step is as Fig 5 with 4 acceptable combinations. After discovering all stope combination, it is time to calculate the economic value of each combination which is a summation of all stopes value in each combination set.

At this step, the values of all combination should be compared and the best one should be discovered. The best combination is the combination with highest economic value. The problem can be formulated as a knapsack problem with conflict graph. In this analogy, the positive stopes are items that can be picked to put in a knapsack, the weights are all equal to one and the positive stope values are the utilities.

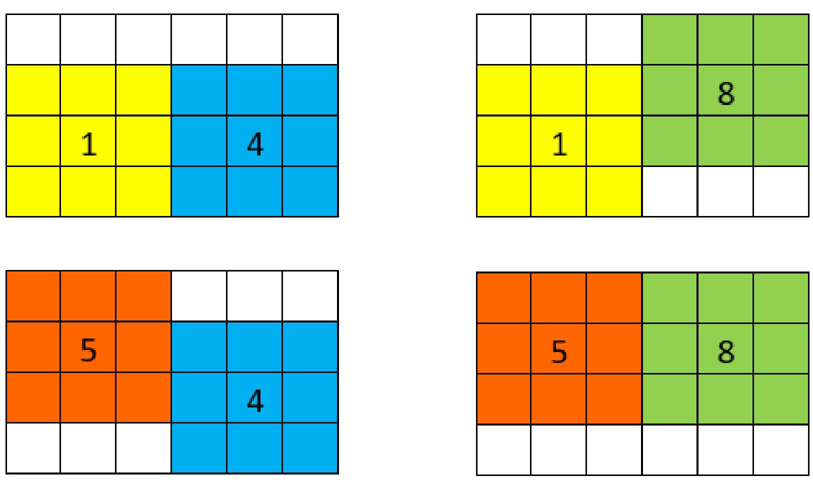

Fig 5. Possible stopes combination sets

Pferschy et al. (2009) used the standard 0-1 knapsack problem and added the weight and the incompatibilities for certain pairs of items as the constraints. They defined that from each conflicting pair the highest value item can be packed into the knapsack. Also, to model the conflicts between the items, they used conflict graph. The graph is represented by a $n \times n$ matrix where the value of *(i,j)* is equal to 1 if the two items cannot be packed together.

Equations (8) presents the formulation of the objective function and equations (9) to (11) indicate the constraints.

Objective function

$$\max \sum_{j=1}^{n} p_j x_j \tag{8}$$

Where n is a number of items, i and j are indicators of the items, p_j is the utility of each item and x_j is the decision variable indicating whether item j is picked in the knapsack. To employ Knapsack problem with conflict graph and find the optimum positive stopes combination, n, i and j, p_j and x_j refer the number of the positive stopes, positive stopes indicators, the value of the positive stopes and the decision variable to show whether stope j is in the optimum combination, respectively. This objective function uses one set of the variable for making a decision about considering each positive stopes in the optimum stopes combination with the highest value or not. In fact, this objective function is to maximize the value of positive stopes combination.

Constraints

$$\sum_{j=1}^{n} w_j x_j \le c \tag{9}$$

$$x_i + x_j \le 1 \forall (i,j) \in E \tag{10}$$

$$x_j \in \{0,1\}\; j = 1,...,\mathrm{n}. \tag{11}$$

Equation (9) indicates one of knapsack constraint, where, c is Knapsack capacity and w_j is the weight of each item. In our case, the capacity of the knapsack is equal to the total number of positive stopes and the weights are equal to one. Equation (10) models the conflict and incompatibility of the positive stopes where E is the set of positive stope indices with overlap. In fact, not two positive stope possibilities with overlap can be in the solution together, however, one of those can be included in the solution. Equation (11) defines the decision variables as the binary variables.

3.5. Visualization of the Solution

The output of this step is the plot of the best stopes combination possibilities which are the highest value combination. Fig 6 shows algorithm process with detail.

4. Implementation of the Algorithm

The algorithm has been tested by applying in the real block model, McLaughlin gold mine, located in CA, USA. This block model contains 68 elevations. To decrease the number of blocks and simplify the problem, only three elevations (22 to 24) have been used. Table 2 indicates the information of the blocks in these elevations. Fig 7 shows the situation of the blocks in these elevations.

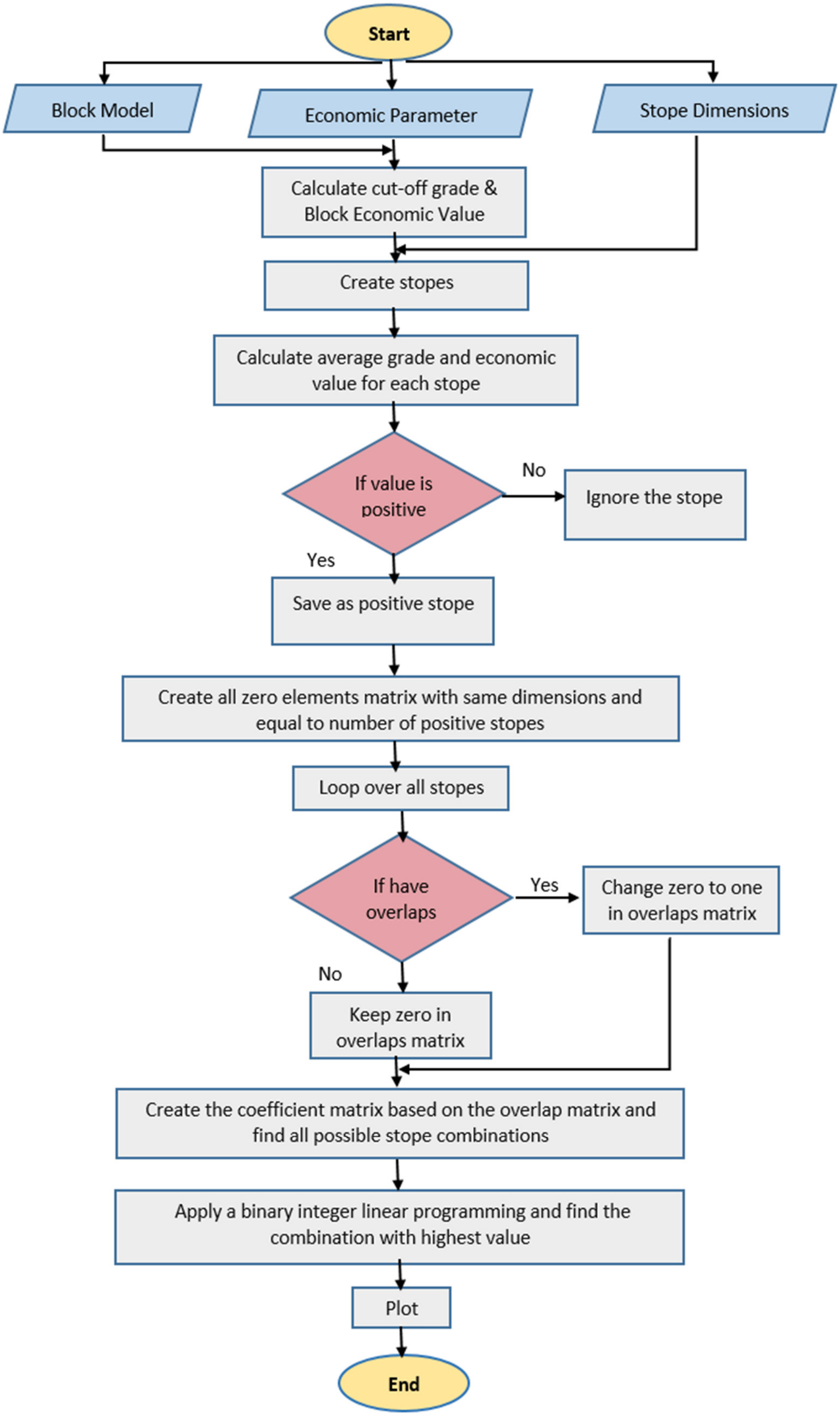

Fig 6: Algorithm to find the optimum stope layout

Table 2. Block model information

Number of blocks	15,577
Blocks size (ft)	25×25×20
Blocks tonnage(tonn)	From 177.08 to 1041.67
Blocks grade (oz/tonn)	From 0 to 1.546
X Coordinate (X index)	6-63
Y Coordinate (Y index)	1-199
Z Coordinate (Z index)	22-24

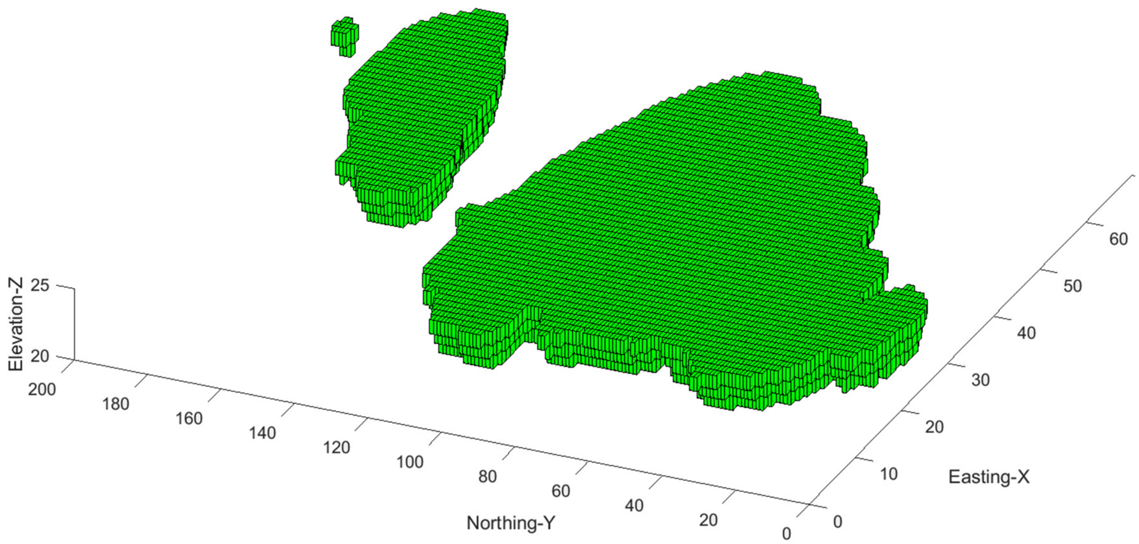

Fig 7. The block model

4.1. Generate the Block Economic Model

To calculate the block economic value, the economic parameters are required, which are listed in Table 3.

Table 3. Economic parameters

Metal price (Au)	$900/oz
Cost of mining	$1.32/tonn
Cost of processing	$19/tonn
Recovery	90%

To determine waste or ore blocks, calculating the cut-off grade is the first step. Then, if the block has a grade more than cut-off grade, equations (6), and if the block grade is less than the cut-off grade, equation (7) is used to calculate an economic value for each block. These calculated values are between $-1,375 and $+1,283,275.

4.2. Create Positive Values Stopes

Based on the rock mechanics considerations, stope dimensions, 3×3×3 is chosen for this study. However, there is the possibility of changing dimensions and scanning the impacts of that.

Creating stopes and separating the positive ones are the next steps. In this block model, 4,818 stopes are generated which 3,212 of those have positive economic value.

It is notable that not all blocks in the initial block model have data. As a result, there are not 27 blocks available to create some stopes. To overcome this matter, the presented algorithm defines stope with more than 20 blocks.

4.3. Assess Stopes Overlaps

In this step, the 3212×3212 overlap matrix is created. This matrix contains 0 value for positive stopes which do not have overlap and 1 value for positive stopes which have overlap. By using computer with processor: Intel(R) Core(TM) i5-4460 CPU @ 3.20GHz and RAM: 6:00 GB, the solution time for this part is 00:04:59.

4.4. Find All Possible Stopes Combinations and Discover the Optimum One

Based on the equations (8) to (11), to employ the presented algorithm to find the optimum positive stopes combination, n, p_j and c should be determined. In this case study, n and c, which is equal to a total number of positive stopes, are 3212 and p_j are the value of those positive stopes.

By running the algorithm, 72,342 positive stopes combinations are created and with zero percent gap, the optimum combination with 368 positive stopes is discovered. The value of this combination is $321,1 M, which is the maximum achievable value of extracting these three levels of McLaughlin mine. Also, the solution time for this part is 00:00:06.

4.5. Visualization of the Solution

Fig 8 indicates the stopes in the final result of the algorithm. The selected stopes in the best stopes combination are demonstrated with the different colors. Also, the blocks in each stope are shown. Some stopes contain less than 27 blocks because as it mentioned before, there is no data available for every block at the initial block model.

5. Summary and Conclusion

This paper presented a method to finding the optimum stopes layout with highest economic value. The presented heuristic algorithm in this method was applied to the gold deposit. It reached to an appropriate result with a minimum percentage of the gap with exact solution in the short time. In fact, the total solution time from beginning to plotting the optimum solution is 00:07:20. Table 4 demonstrates the summary of the solution.

Table 4. Summary of the solution

Number of blocks in the solution	368
The value of the solution	$321,1 M
The solution time	00:07:20

In future, the presented algorithm will be improved in some areas such as production scheduling. Also, I have a plan on upgrading this algorithm by considering more than one mineral type, for example, gold and copper.

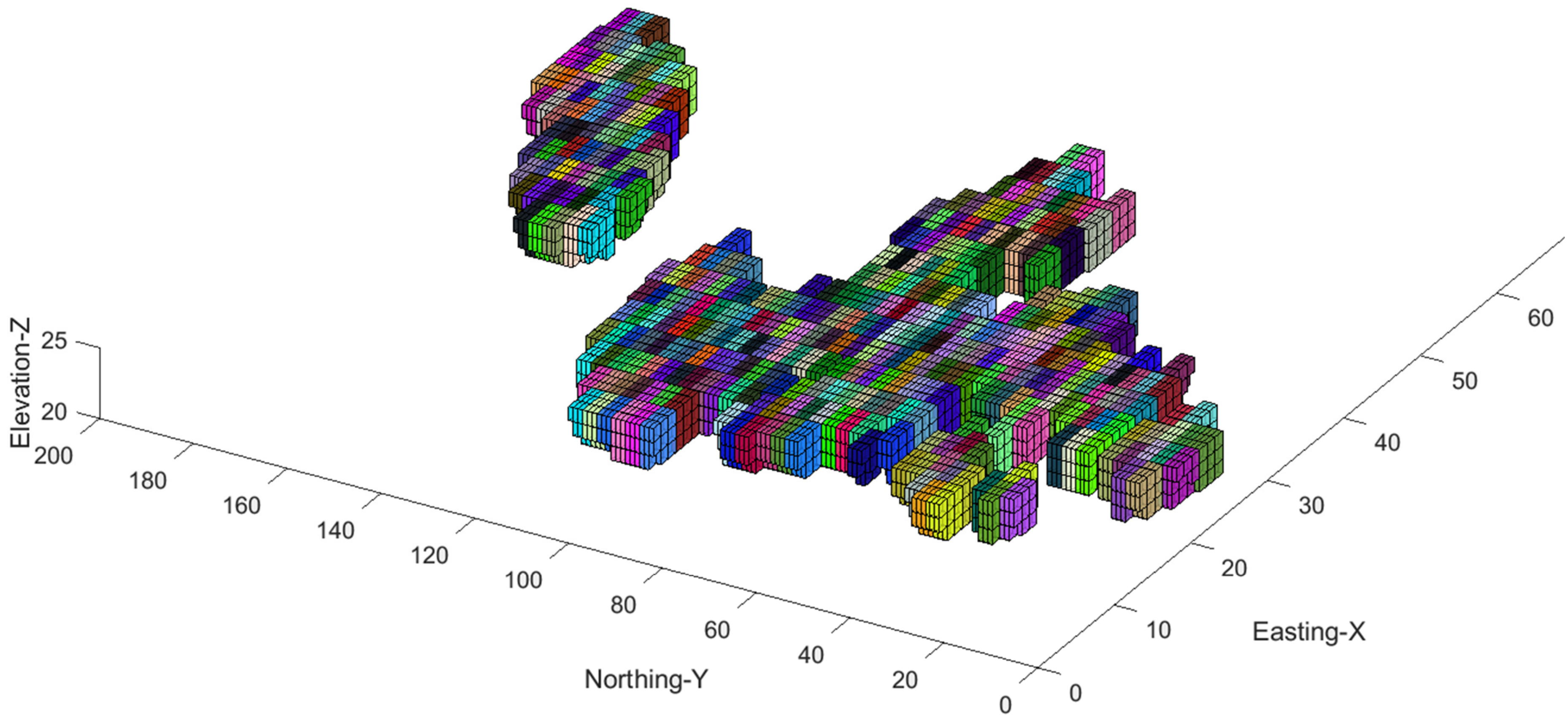

Fig 8. The optimum solution (stopes)

6. References

[1] Alford, C. (1996). Optimization in Underground Mine Design. *International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts*, *220A*

[2] Ataee-Pour, M. (2000). A Heuristic Algorithm to Optimise Stope Boundaries.Thesis, University of Wollongong,

[3] Bai, X., Marcotte, D., and Simon, D. (2013). Underground Slope Optimization with Network Flow Method. *Computers and Geosciences*, 361-371.

[4] Cheimanoff, N. M., Deliac, E. P., and Mallet, J. L. (1989). GEOCAD: An Alternative CAD and Artificial Intelligence Tool That Helps Moving from Geological Resources to Mineable Reserves. *21st Application of Computers and Operations Research in the Mineral Industry:21st International Symposium*, Page 471.

[5] David, m. (1988). *Handbook of Applied Advanced Geostatistical Ore Reserve Estimation.*

[6] Deraisme, J., De Fouquet, C., and Fraisse, H. (1984). Geostatistical Orebody Model for Computer Optimization of Profits from Different Underground Mining Methods. in *Proceedings of the 18th APCOM Symposium*. London, England, pp. pp. 583–590.

[7] Dimitrakopoulos, R. and Grieco, N. (2009). Stope Design and Geological Uncertainty, Qualification of Risk *Journal of Mining Science*, *45* (2),

[8] Jalali, S. E. (2006). Optimization of Underground Mines Limits in Vein Type deposits. Ph.D Thesis Thesis, Amirkabir University of Technology, Iran,

[9] Ovanic, J. and Young, D. S. (1995). Economic Optimisation of Stope Geometry Using Separable Programming with Special Branch and Bound Techniques in *In: Third Canadian Conference on Computer Applications in the Mineral Industry, Rotterdam, Balkema*, pp. 129–135.

[10] Pferschy, U. and Schauer, J. (2009). The Knapsack Problem with Conflict Graph. *Journal of Graph Algorithms and Applications*, *vol. 13, no. 2* 233–249.

[11] Riddle, J. M. (1977). A Dynamic Programming Solution of A Block-Caving Mine Layout. *Application of Computer Methods in the Mineral Industry: Proceedings of the Fourteenth Symposium*, 767–780.

[12] Sandanayake, D. S. S. (2014). Stope Boundary Optimisation in Underground Mining Based on A Heuristic Approach. PH.D Thesis Thesis, Curtin University

[13] Sandanayake, D. S. S., Topal, E., and Asad, M. W. A. (2015a). Designing An Optimal Stope Layout for Underground Mining Based on A Heuristic Algorithm. *International Journal of Mining Science and Technology*, *25* (5), 767-772.

[14] Sandanayake, D. S. S., Topal, E., and Asad, M. W. A. (2015b). A Heuristic Approach to Optimal Design of An Underground Mine Stope Layout. *Applied Soft Computing*, *1568-4946*

[15] Topal, E. and Sens, J. (2010). A New Algorithm for Stope Boundary Optimization. *Journal of Coal Science and Engineering (China)*, *16* (2), 113-119.

[16] Villalba Matamoros, M. E. and Kumral, M. (2017). Heuristic Stope Layout Optimisation Accounting for Variable Stope Dimensions and Dilution Management. *Int. J. Mining and Mineral Engineering*, *Vol. 8, No. 1* 1–18.