Draw Rate Management in Block Cave Production Scheduling

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ABSTRACT

Planning of caving operations poses complexities in different areas such as safety, ground control, and production scheduling. Draw control is fundamental to the success of block-cave operation. Although some complex theories and mathematical draw control systems have been applied in block-cave mines, most of them did not have an exact production rate curve to manage draw rates of drawpoints and are too complex to provide a solution for real block-caving mines. This paper presents a mixed-integer linear programming (MILP) model to optimize the extraction sequence of drawpoints over multiple time horizons of block cave mines with respect to the draw control systems. Four draw rate strategies are formulated to guarantee practical solutions. Furthermore, dilution and caving are improved indirectly, because the method considers the draw rate strategy. Application and comparison of the four models for production scheduling based on draw control systems are presented using 298 drawpoints over 15 periods.

1. Introduction

Block caving is generally a large-scale production technique applicable to low-grade, massive ore-bodies and the least expensive of all underground mining methods, and can in some cases compete with open-pit mining in cost and production rate. It is a technique which relies on natural processes for its success; therefore gravity is used in conjunction with internal rock stresses to fracture and break the rock mass into pieces that can be handled by miners. This method requires more detailed geotechnical investigations of the ore-body than do other methods in which conventional drilling and blasting are employed as part of the mine production (Rubio, 2006).

Due to an increasing trend in the world to extract minerals and with the mining industry's more marginal resources, it is becoming essential to generate production schedules that will provide optimal operating strategies while meeting practical, technical, and environmental constraints (Burgher and Erickson, 1984; Chanda, 1990; Dagdelen and Johnson, 1986; Pourrahimian, 2013a).

Production scheduling in block caving is generally referred to as draw control. The objectives of draw control are normally separated into short- and long-term scheduling (Diering, 2004a). Draw control in caving operations involves a combination of scheduling and geomechanics (Smith and Rahal, 2001).The use of draw control to mitigate stress damage on the extraction level can be considered as one of the most important aspects of draw management. It is generally accepted that under-draw and over-draw behavior leads to early dilution entry, excessive induced stresses, and loss of planning abilities (Heslop and Laubscher, 1981).

Caveability, in the context of draw control, is primarily concerned with balancing caving rates and production. If draw rates are not controlled, either air gaps or damaging stress concentrations may occur.

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Stress is important because undercut advance rates must be maintained to prevent stress damage to the production level. Draw must also be maintained across the production level to ensure that local stress concentrations and premature dilution entry do not occur (Rahal, 2008a).

In general, draw control is fundamental to success or failure of any block cave operations. If draw from the drawpoints is not controlled, many problems and hazards are faced: unbalanced cave subsidence by poor ground control over time, decreased recovery and productivity, premature waste ingress and recompaction of broken material in the draw columns, infrastructure instability, fragmentation size distribution, more dilution, ore handling difficulties that themselves can lead to tunnel and ore pass collapse or haulage systems failure, flow of muck at the drawpoints, and other safety and financial or safety damages to miners are possible to occur. Consequently, a production planning program that does not incorporate the geotechnical properties of the rock mass within the block-cave mining method will not be used for general purposes of production schedules, since such a plan causes many damages. On the other hand, careful control of the production in long-term planning of a block-cave mine will ensure that the schedule of the drawing process within the cave moderates unwanted problems and preserve mining economics associated with production targets.

Production control and implementation of an effective draw strategy once production commences need a strict schedule planning system. The introduction of a draw control system based on mathematical programming that integrates constraints from other disciplines like geology, mining and metallurgy into the system will become more acceptable as real business planning tools. In this paper, a mathematical formulation is developed to optimize the draw rate as a critical parameter of a block-cave operation to maximize economic goals of companies by regarding the geotechnical production rate curves. This paper considers the draw control system as an operational goal with respect to strategic decisions to achieve an exact solution through an operations research technique.

2. Literature review

Several methods are currently used to generate production schedules in block-cave mines. Previous to modern algorithms and computational developments, block-caving scheduling problems, like other underground mining methods in their large size, seemed intractable when formulated in mathematical form, especially in the case of MILP problems. Mining optimization models available in the literature have been developed for block-cave mines, but they solve only a partial type of planning problem. The literature on draw rate models based on the production rate curve (PRC) for block-caving operations is relatively new. Most of the models use simulation in consideration of PRC to evaluate production schedules. There is not a clear example of block-caving production scheduling using a mathematical approach in the literature that formulates draw control system in the block cave according to PRC. Most of the studies consider only upper and lower boundary for draw rate in their modeling. Riddle (1976), Song (1989), Chanda (1990), Guest et al. (2000), Rahal et al. (2003), Rubio and Diering (2004), and Diering (2004a) presented some preliminary mathematical methods in block caving scheduling optimization. Rahal (2008a) described mixed-integer linear goal programming (MILGP) models. This algorithm assumes that the optimal draw strategy is known. Rahal emphasized that the major outcomes from the research were a preliminary optimized life-of-mine production plan and the identification of areas where additional work can refine the parameters used in the optimization. Weintraub et al. (2008) presented a MIP model to maximize the profit of El Teniente, to arrive at good solutions by developing an aggregation scheme based on cluster analysis. They only focused on size reduction methods and they had no reference to issues of draw control rules. Queyranne et al. (2008) presented a MIP model for block caving that maximizes the NPV and uses the capacity constraints of mine production, maximum opened and active drawpoints, and neighbor drawpoints. The model presents a good method for the optimal solution, but it does not consider the geotechnical and other significance constraints. Smoljanovic et al. (2011) presented an MILP model to optimize the NPV value in a panel cave mine to study the drawpoints' opening sequence. The emphasis is in the precedence, geometrical, and production constraints. He did not consider PRC for draw rate constraint. Parkinson (2012) developed three IP models: Basic, Malkin, and 2Cone, for finding the optimal opening sequence in an automated manner.

Parkinson assumed a constant draw rate for the life of the mine. Epstein et al. (2012) presented and solved an MIP model with an objective function of maximizing the NPV that was successfully used in Chilean copper mines by Codelco for both underground and open-pit extraction. Their model uses the drawpoint as the exploitation unit for underground. 'As in Parkinson, the extraction rate of the Epstein method had a constant value in the mine life. Pourrahimian et al. (2012), Pourrahimian (2013b), and Pourrahimian and Askari-Nasab (2014) made other applications of MILP to develop a practical optimization framework for caving production scheduling. They presented a multi-step method for the long-term production scheduling of block caving to overcome the size problem of mathematical programming models and to generate a robust practical near-optimal schedule. Pourrahimian attempted to find an optimal schedule for the life of the entire mine, solving simultaneously for all periods by consideration all required constraints, but he did not consider geotechnical properties of rock mass through the draw rate constraint. Pourrahimian mentioned that the formulation tries to extract material from drawpoints with a draw rate within the acceptable range without considering a specific shape. Alonso-Ayuso et al. (2014) considered a planning MIP medium range problem for the El Teniente mine in Chile to maximize NPV by introducing explicitly the issue of uncertainty. Khodayari and Pourrahimian (2015a) presented a comprehensive review of operations research in block caving. Rubio and Fuentes (2016) described a simulation methodology to compute production schedule reliability. Maximum tonnage that can be drawn from drawpoints based on the overall drawing strategy assumed as optimization function. Khodayari and Pourrahimian (2016) applied MIQP in the long-term scheduling of block cave mines. They used NPV as the objective function. A constant draw rate was one of the applied constraints in the model.

All of these studies have concentrated on determining the optimum configuration of block-caving operation. The main problems associated with the methods presented above can be summarized as follows; some of them did not incorporate, on a routine basis, operational performance to adjust the medium and the long-term plans because of loss of geotechnical rules in the modeling of actual draw management systems. Constraints must be appropriate with the mining method, objective function, and real geotechnical condition of the rock mass. Maximizing tonnage or mining reserves will not necessarily lead to maximum NPV without aggregation reserve and grade constraint to time dynamic behavior of the fundamental models in linking the mine planning parameters and draw rate curves. During the production, the only control is through the drawpoints. Most of the previous research considers a series of simple definitions of the process of moving and drawing of caved material from drawpoints. The researchers have modeled the draw rate constrains regardless of the production rate curves and only by defining lower and upper bounds. Deciding minimum and maximum draw rates of drawpoints is appropriate according to principles of geotechnical rules in all previous studies. However, during the entire life of any drawpoints, draw rate varies.

The extraction from a drawpoint should start with the acceptable minimum draw rate. Then, the extraction increases during the ram-up period until reaching the acceptable maximum draw rate. In the last periods of the life of the drawpoint the extraction rate decreases. A consequence of disregarding the effect of PRC (ramp-up, high production, and ramp-down) on the extraction of a drawpoint is shown in Fig.1.

With defining the upper and lower boundary for draw rate in a mathematical model, although draw rates are in the defined range, they are arbitrary tonnages over the life of drawpoint. This means extraction may start with a higher draw rate (Fig. 1a and 1d) or it can fluctuate (Fig. 1b and 1c) over the life of drawpoint.

In this paper, a mathematical formulation is developed to optimize the draw rate as a critical parameter of a block-cave operation to maximize economic goals of companies by regarding the geotechnical properties of the rock mass. Although following the PRC in mathematical formulation makes the problem more complex and increases the size of the problem, it is a more realistic method. Fig.2 shows a schematic view of a production rate curve. This profile is usually provided by the geotechnical team to consider many factors such as the engineering and geotechnical properties of the rock mass and safety issues in extraction.

For instance, the draw rates established for Cadia East during the cave initiation (up to 30% of the block height) vary from 115 mm/day to 280 mm/day with an average of 190 mm/day. Higher than 30% to the top of the block, the draw rates vary from 280 mm/day to 400 mm/day with an average of 320 mm/day (Flores, 2014).

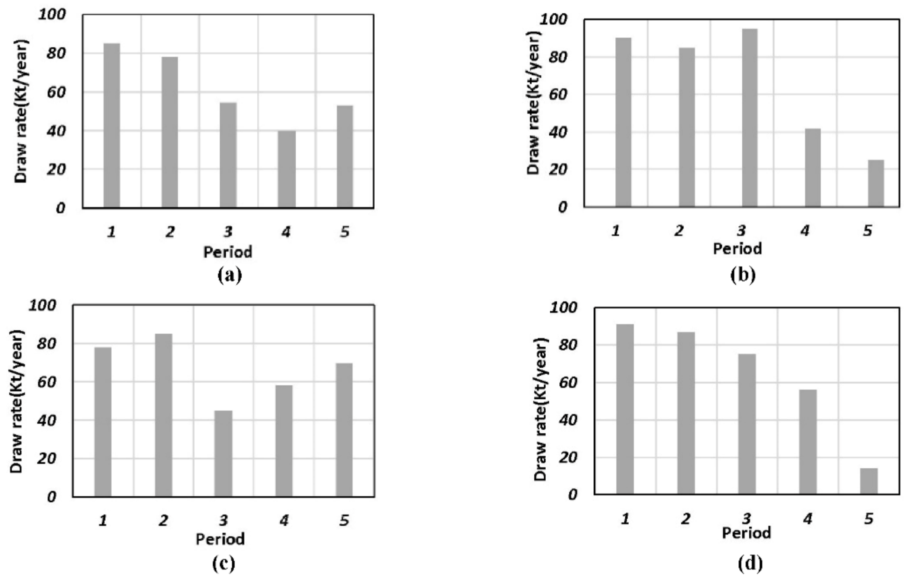


Fig.1. Draw rate variation without considering the production rate curve (min. and max. draw rates are 10 and 100 [kt/year], respectively).

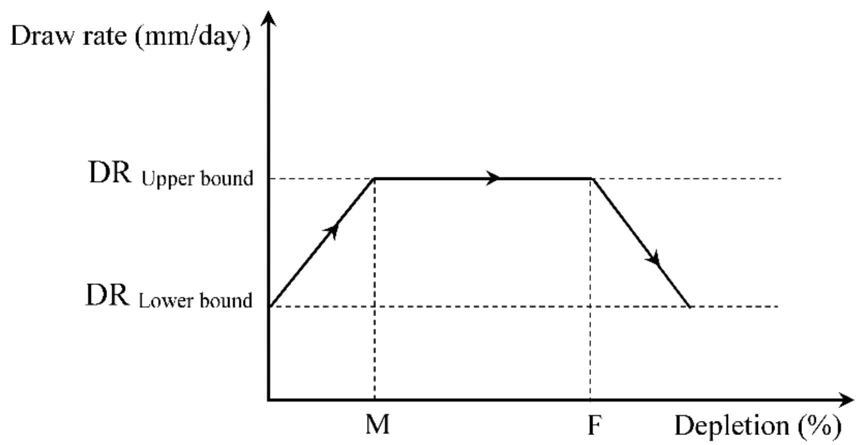


Fig.2. Production rate curve (ramp-up, high-production, ramp-down)

3. Problem Formulation

Mixed-integer linear programming (MILP) formulation presented in this paper maximizes the NPV subject to various operational and geotechnical constraints. These constraints are mining capacity, the maximum number of active drawpoints, the precedence of drawpoints, continuous extraction, the number of new drawpoints, the average grade of production, reserves, activity periods of each drawpoint, and draw rate. Presented draw rate algorithms allow the use of the MILP formulation for large and more complex real block-cave mines planning. The model identifies two types of variables: (i) continuous and (ii) binary. The former is an actual number that represents the percentage of material extraction from the draw column and the later indicates the time when the drawpoint is opened, active, or in any working states during a specified time period. Applications of this variable allows the tractability of "either-or" decisions to the problem. The presented model is coded in MATLAB (2016), and CPLEX/IBM (2015) is used as the optimization engine.

Developing any model requires some decision variables, sets, indices, and parameters that correspond to a scheduling program. The following items are introduced according to the current model:

Indices

$d \in \{1,...,D\}$	Index for drawpoints.
$t \in \{1,...,\mathrm{T}\}$	Index for scheduling periods.
l	Index for a drawpoint belonging to set S^d .

Set

S^d	For each drawpoint, *d*, there is a set S^d defining the predecessor drawpoints that must be started prior to extraction of drawpoint *d*.

Decision variables

$U_{d,t} \in [0,1]$	Continuous decision variable, representing the portion of draw column *d* to be extracted in period *t*.
$A_{d,t} \in \{0,1\}$	Binary decision variable equal to 1 if drawpoint *d* is active in period *t*; otherwise it is 0.
$Z_{d,t} \in \{0,1\}$	Binary decision variable controlling the precedence of extraction of drawpoints. It is equal to 1 if extraction from drawpoint *d* is started in period *t*; otherwise, it is 0.

Parameters

D	Maximum number of drawpoints in the model.
Re_d	Economic value of the draw column associated with drawpoint *d*.
$\mathrm{Re}v_d$	Revenue factor of drawpoint *d*.
DC	Direct mining cost per tonne for drawpoint *d*.
MC	Milling cost per tonne for drawpoint *d*.
OC	Overhead costs.
$DR_{l,d,t}$	Minimum possible draw rate of drawpoint *d* in period *t*.
$DR_{u,d,t}$	Maximum possible draw rate of drawpoint *d* in period *t*.
$N_{Ad,t}$	Maximum allowable number of active drawpoints in period *t*.
$N_{Nl,d,t}$	Lower limit for the number of new drawpoints, the extraction from which can start in period *t*.
$N_{Nu,d,t}$	Upper limit for the number of new drawpoints, the extraction from which can start in period *t*.
Ton_d	Total tonnage of material within the draw column associated with drawpoint *d*.
L	Large number equal to a fraction of $\max\{Ton_d\}$ on minimum draw rate.
i	Discount rate.
M_u	Upper limit of mining capacity in period *t*.
M_l	Lower limit of mining capacity in period *t*.
$Max_{Activity}$	Maximum allowable periods that any drawpoints can be active.
$G_{u,d,t}$	Upper limit of the acceptable average head grade of drawpoint *d* in period *t*.

$G_{l,d,t}$	Lower limit of the acceptable average head grade of drawpoint *d* in period *t*.
$\widetilde{G}_{d,t}$	Average grade of drawpoint *d*.
M	Maximum allowable depletion to reach steady region after ramp-up.
F	Maximum allowable depletion to reach ramp-dawn region after steady.

3.1. Objective function

The objective function of the MILP formulation is to maximize the overall discounted profit, including the cost of the mining operation. The profit from mining a drawpoint depends on the economic value of the draw column and the costs incurred in mining. The revenue per tonne is calculated using the revenue factors per unit of grade material and the mining cost, which includes per tonne overheads and per tonne milling costs (see Equation 1).

$$\mathrm{Re}_d = (\mathrm{Rev}_d \times \widetilde{G}_d) - (DC + MC + OC) \tag{1}$$

The maximization of NPV is closely associated with maximizing ore tonnes, as the ore tonnes generate revenue. The objective function, Equation 2, is composed of the economic value of the draw column and a continuous decision variable $U_{d,t}$ that indicates the portion of a draw column, which is extracted in each period. The most profitable draw columns will be chosen as part of the production to optimize the NPV.

$$Maximize \sum_{d=1}^{D}\sum_{t=1}^{T}\left[\frac{\mathrm{Re}_d}{(1+i)^t}\right]\times U_{d,t} \tag{2}$$

3.2. Constraints

One of the main goals of long-term mine planning would be to integrate internal and external mine planning factors that affect the performance of the mine operation. At the same time, long-term planning is responsible for coordinating strategic goals and operational activities. Obtaining the best solution using an operations research technique forces the mine planner to limit objective function by some constraints, which appear in several different forms: geotechnical, grades, period, advancement direction, and priority of productive units, productivity, production rates, and many others that depend on the mining method.

Knowledge, experiments of mine planners, and corresponding planning horizons have a critical role on the process of assigning constraints to the optimization problems. The following constraints are part of the problem in deriving the formulation:

- **Mining capacity**

$$M_l \le \sum_{d=1}^{D}(Ton_d)\times U_{d,t} \le M_u \tag{3}$$

- **Production grade**

$$\sum_{d=1}^{D}(Ton_d \times (G_{l,d,t} - \widetilde{G}_{d,t})\times U_{d,t} \le 0 \tag{4}$$

$$\sum_{d=1}^{D}(Ton_d \times (\widetilde{G}_{d,t} - G_{u,d,t})\times U_{d,t} \le 0 \tag{5}$$

- **Maximum number of active drawpoints**

$$A_{d,t} \le L.U_{d,t} \tag{6}$$

$$U_{d,t} \leq A_{d,t} \tag{7}$$

$$\sum_{d=1}^{D} A_{d,t} \leq N_{Ad,t} \tag{8}$$

- **Precedence of drawpoints**

$$Z_{d,t} - \sum_{j=1}^{t} Z_{l,j} \leq 0 \tag{9}$$

- **Continuous extraction**

$$\sum_{t=1}^{T} Z_{d,t} = 1 \tag{10}$$

$$A_{d,t} - A_{d,(t-1)} \leq Z_{d,t} \tag{11}$$

$$A_{d,1} = Z_{d,1} \tag{12}$$

- **Number of new drawpoints**

$$N_{Nl,d,t} \leq \sum_{d=1}^{D} Z_{d,t} \leq N_{Nu,d,t} \tag{13}$$

$$\sum_{d=1}^{D} Z_{d,1} \leq N_{Ad,1} \tag{14}$$

- **Reserves**

$$\sum_{t=1}^{T} U_{d,t} \leq 1 \tag{15}$$

- **Maximum activity period**

$$\sum_{t=1}^{T} A_{d,t} \leq Max_{Activity} \tag{16}$$

Equation (3) ensures that the total tonnage of material extracted from drawpoints in each period is within the acceptable range that allows flexibility for potential operational variations. The constraint is controlled by the continuous variable $U_{d,t}$. There is one constraint per period. Equations (4) and (5) force the mining system to achieve the desired grade. The average grade of the element of interest has to be within the acceptable range and between certain values.

According to Equations (6), (7), and (8) in each period, the number of active drawpoints must not exceed the allowable number and has to be constrained according to the size of the ore-body, available infrastructure, and equipment availability. A large number of active drawpoints might lead to serious operational problems. According to the determined advancement direction, for each drawpoint *d,* there is a set S^d, which defines the predecessor drawpoints among adjacent drawpoints that must be started before drawpoint *d* is extracted. To control the precedence of extraction, a binary decision variable $Z_{d,t}$, is employed in the Equation (9).

The constraint introduced by Equations (10) and (11) forces the mining system to extract material from drawpoints continuously after opening until closing. Equation (12) is only used for period one. Based on the footprint geometry, the geotechnical behavior of the rock mass, and the existing infrastructure of the mine, the maximum feasible number of new drawpoints to be opened at any given time within the scheduled horizon must be defined on the basis of Equations (13) and (14). Equation (15) ensures that the sum of fractions of the draw column that are extracted over the scheduling periods in maximum value is one, which means there is selective mining, and thereby all the material in the draw column may not be extracted.

The draw rate needs to be fast enough to avoid recompaction and slow enough to avoid air gaps and dilution. The activity period of the drawpoint, in the context of draw control, is mainly concerned with the assessment of draw rates to adjust extraction tonnage of any drawpoint and prevent any recompaction or dilution in activity life of drawpoints. This constraint causes the maximum activity life of any drawpoint to be limited to a deterministic value, so the draw rate of drawpoint must be large enough to maximize the NPV, yet small enough to prevent over dilution. Equation (16) indirectly affects the draw rate by controlling the number of activity periods of any drawpoint. So if the activity period has a large value, then the draw rate can have smaller values, and increasing the drawpoint activity life increases the probability of recompaction and dilution.

3.2.1. Creating Draw Rate Constraints

In the case of caving, the draw control is concerned with extracting ore in such a way as to achieve production targets while minimizing waste entry, as well as preventing the transfer of stress onto mine workings. It is important to ensure that this draw management system is in place before production begins, to prevent resource loss due to production pressures during cave initiation (Preece and Liebenberg, 2007). Production rates are specified as tonnes per square meter per day per drawpoint. This calculation is often linked to a production rate measured in meters per day (or vertical velocity). Selecting the production rate is one of the primary mine design variables required early in the processing capacity evaluation (Charles et al., 2011).

Achieving maximum profit in minimum time periods by increasing the draw volume from drawpoints is the ideal demand of all mining companies. But the geotechnical limitation of the rock mass does not allow mines to draw material with an extreme velocity. Consequently, according to the market assessment and many limitations related to the extraction of mines, companies have different draw strategies to maximize profit, safety and minimize loss of time, financial cost, and fatalities in the mine. The changes in draw rate are normally classified as a drawpoint opening to ramp-up production, steady state production, and ramp-down to drawpoint closure. In this research, the PRC is classified in four alternative general forms to be modeled and practiced by all mines according to their draw requirements. These four forms are as follows: (i) ramp-up, steady, ramp-down (USD), (ii) ramp-up, steady (US), (iii) steady, ramp-down (SD), and (iv) ramp-up, ramp-down (UD).

In the case of mines that geotechnical and economic issues are considered simultaneously, usage of the first model (USD) is proposed. USD is best because the low draw rate in the early years of the life of drawpoints is caused by the geotechnical characteristics of the surrounding environment aligned with the caved material, and in fact, increasing stress caused by draw and caving process relaxes with a relatively soft tendency. Therefore appropriate management of stress relaxation can be done.

Development of an overlap and disjunctive (OD) system for regulating drawpoint production begins by breaking the production profile into a number of regions where each has a binary indicator variable. When the binary variable for each level takes a value of 0, then the draw constraint for that level is relaxed; otherwise, that region binds production.

Each model according to the number of its profile's regions establishes the same number of new sub-constraints. Any of these regions can be available for a draw but their activity returns to a depletion percentage. For example, in the first model (USD, see Fig. 3), there are three regions and it will have three new sub-constraints. The second region is active if the depletion is between M and F, and thus its binary variable ($A_{d,t}$) takes 1 and other regions' binary variables take 0. The OD system determines which of the regions is active. The main goal of draw rate formulation is to find the draw rate of each drawpoint in each period during the optimization of scheduling based on the defined objective function, constraint, and PRC.

In the first step of the proposed OD system, the line equation for each region is written. For this purpose, according to the Fig. 4, the general equation for the ramp-up region is written based on the depletion and the draw rate (see Equation(17)).

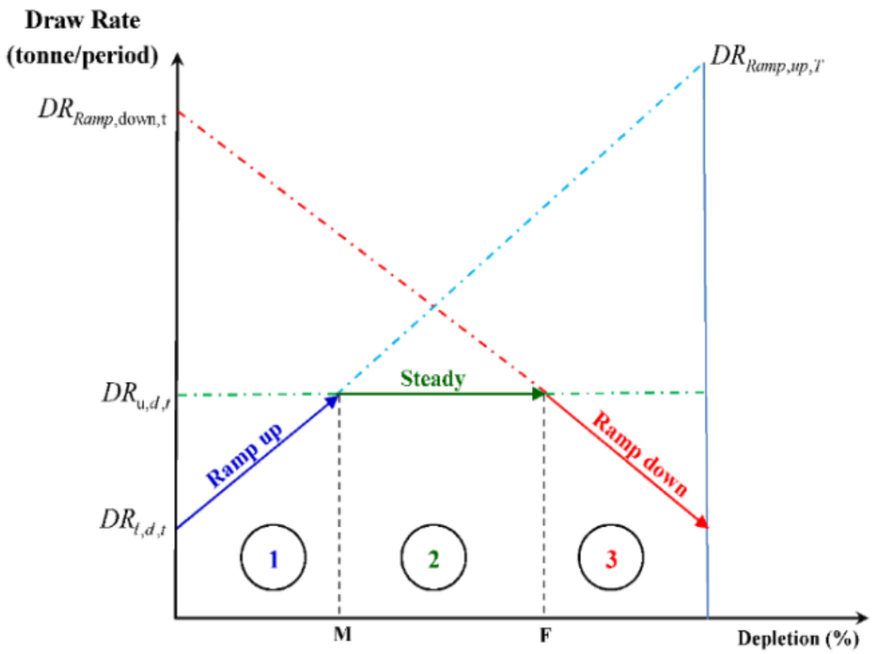


Fig. 3. Ramp-up, steady, and ramp-down situation

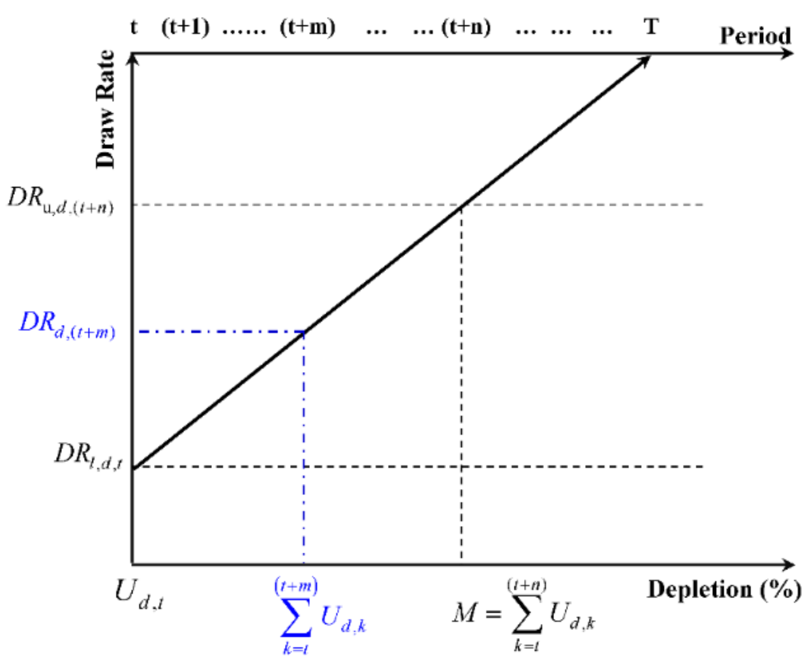


Fig. 4. Basic and ideal line of OD system for ramp-up region

$$(DR_{d,(t+m)} - DR_{l,d,t}) = \frac{(DR_{u,d,t} - DR_{l,d,t})}{M - U_t} \times (U_{d,(t+m)} - U_t) \tag{17}$$

Where M is the maximum allowable depletion to reach $DR_{u,d,t}$; U_t is the percentage of depletion at period 1; $U_{d,(t+m)}$ is the percentage of total depletion until the end of period t. In the simple form, the formula can be written as follows (Equations (18) and (19)):

$$U_{d,(t+m)} = \sum_{t=1}^{t+m} U_{d,t} \tag{18}$$

$$DR_{d,(t+m)} = U_{d,(t+m)} \times Ton_d \tag{19}$$

Equation (20) shows the mathematical structure for the area under the ramp-up region:

$$U_{d,(t+m)} \times Ton_d - \left(\frac{(DR_{Ramp,\mathrm{up},T} - DR_{l,d,t})}{1 - \frac{DR_{l,d,t}}{Ton_d}} \right) \times \sum_{t=1}^{t+m} U_{d,(t+m)} \le DR_{l,d,t} - \left(\frac{(DR_{Ramp,\mathrm{up},T} - DR_{l,d,t})}{1 - \frac{DR_{l,d,t}}{Ton_d}} \right) \times \frac{DR_{l,d,t}}{Ton_d} \tag{20}$$

With a similar process the equations for the steady and ramp-down regions can be written as follows. Equations (21) and (22) are related to the steady and ramp-down regions respectively:

$$U_{Steady,T} \times Ton_d \le DR_{u,d,t} \tag{21}$$

$$U_{d,(t+m)} \times Ton_d + \left(\frac{(DR_{Ramp,down,T} - DR_{l,d,t})}{1 - \frac{DR_{l,d,t}}{Ton_d}} \right) \times \sum_{t=1}^{t+m} U_{d,(t+m)} \le DR_{Ramp,down,T} + \left(\frac{(DR_{Ramp,down,T} - DR_{l,d,t})}{1 - \frac{DR_{l,d,t}}{Ton_d}} \right) \times \frac{DR_{l,d,t}}{Ton_d} \tag{22}$$

Based on the Equations (20), (21) and (22), the problem can be formulated for the required PRC. Fig. 5 illustrates different depletion strategies for a drawpoint.

In the USD situation, extraction begins from drawpoint *d* with a minimum acceptable draw rate in the starting period. Then the draw rate increases gradually (blue) to reach the maximum acceptable draw rate. In the steady region (green), the draw rate follows the maximum acceptable draw rate; therefore the draw rate has a zero slope. By extraction of material from drawpoints and reduction of the amount of remaining materials in the draw columns and reaching to depletion F (%), the draw rate must be reduced with the defined slope. This is the ramp-down region (red). The number of periods in each region depends on M and F values and is obtained as a result of optimization.

Determining M and F is a specific effort and depends on the skill of the geotechnical group of any mines. The number of periods in the steady region decreases by increasing the M value and decreasing the F value. In other words, the USD model is converted to the UD model. A similar definition can be used for other models. However, if the rock mass does not have a good condition, safety problems can happen in the caving progress. If the value of M is too small, it is obvious in the initial periods of extraction with rapid depletion of material that hints of the separation of the huge broken rock from the cave back and, consequently, other phenomena such as air blast and closure of drawpoint are inevitable. On the other hand, if the value of F is too large, rapid rates in the last periods lead to inappropriate control of grade and mixing and reduction in recovery. Therefore, it is clear that the values of M and F have a critical role in the draw control system. Adopting a respectable depletion for these values is crucial to avoid alteration of the models to each other and many undesirable problems.

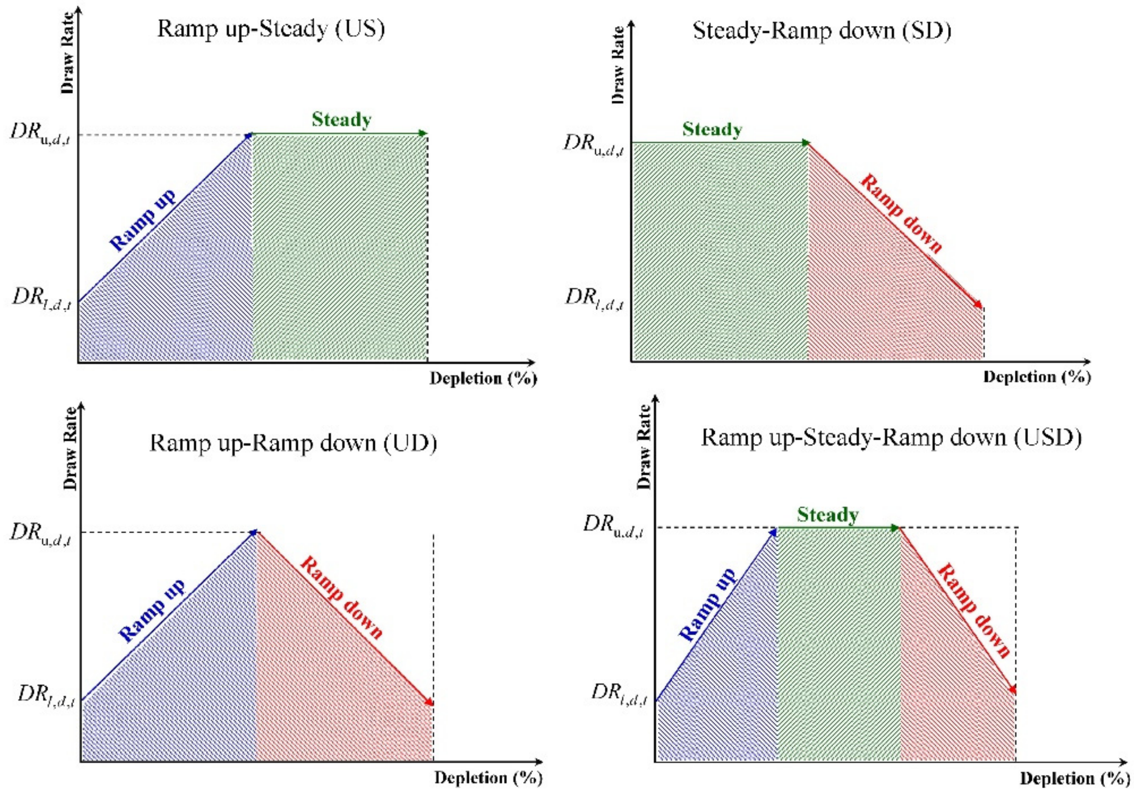


Fig. 5. Different depletion strategies for drawpoint

Finally, in the USD, US, and UD models, in each period that drawpoint depletion is started, the first step of the draw rate must be minimum. Equation (23) forces models to start depletion with a minimum acceptable draw rate.

$$DR_{l,\mathrm{d},t} \times Z_{d,t} - Ton_d \times U_{d,t} \leq 0 \tag{23}$$

4. Solving the optimization problem

The proposed MILP model has been developed in MATLAB (2016), and solved in the IBM ILOG CPLEX (2015) environment. A branch-and-cut algorithm is used to solve the MILP model, assuring an optimal solution if the algorithm is run to completion. Authors have used the gap tolerance (EPGAP) of 4% as an optimization termination criterion. This is a relative tolerance between the gap of the best integer objective and the objective of the remained best node.

5. Case study

In this paper, the main dataset contains 298 drawpoints (see Fig. 6b). The total tonnage of material is almost 36.7 Mt with an average density of 2.7 (t/m^3) and an average grade of 1.12% Cu. Fig. 6a illustrates the grade distribution. The performance of the proposed MILP models is analyzed based on the maximizing net present value at a discount rate of 12%. The draw control system, by enrolling an exact production rate curve, seeks to optimize and present a practical block cave planning. This model assures that all the constraints are satisfied during the mine-life.

The models were tested on a Dell Precision T7600 computer with Intel(R) Xeon(R) at 2.3 GHz, with 32 GB of RAM. In all models, as part of the implementation of the models, the maximum depletion of the draw column from the ramp-up to steady (M) is assumed to be 42%, the maximum depletion of draw column from steady to ramp-down region is assumed to be 90%. The scheduling parameters have been summarized in Table 1.

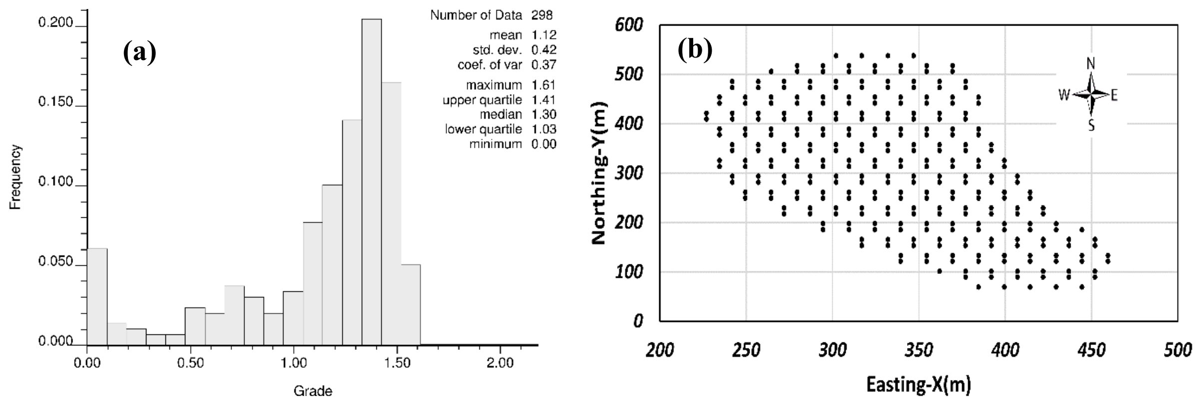


Fig. 6. Case study: (a) grade distribution of %Cu, (b) drawpoints' configuration (dots represent drawpoints)

Table 1. Production scheduling parameters

Parameters	Value	
Number of periods	15	
Maximum number of activity period of drawpoints	5	
Draw rate (kt/year)	Min	11
	Max	40
	M (%)	42
	F (%)	90
Number of active drawpoints	Period one	100
	Other periods	75
Number of new drawpoints	Max	35
	Min	5
Grade (%)	Max	1.5
	Min	1.0

The models are verified over 15 periods in the south to north (SN) advancement directions. This advancement direction was selected using the methodology presented by Khodayari and Pourrahimian (2015b).Khodayari and Pourrahimian (2015b) showed that the cumulative economic value of all drawpoints can be used to determine the best advancement direction to maximize the NPV. Fig. 7 shows the mining advancement direction for 298 drawpoints. As shown, the best direction is from south to north (SN).

The problem was solved in the SN direction for all draw rate models. Table 2 shows the results of all models. The USD model is a comprehensive model because it has all other models' details. The obtained NPV from the USD is $47 M with the optimality gap of 4%. The total running time in this direction is 07:27:26. This time, according to the complex structure of the USD model, is desirable. The total extracted tonnage from all drawpoints is 30.1 Mt. This means that about 82% of all material is extracted in this model.

Fig.8 shows the production and average grade of production in each period for the USD draw rate model. It can be seen that the model attempts to deplete the drawpoints with a higher grade earlier. The average grade of production has a descending trend during the mine life. In period one, because of the minimum draw rate and total allowable active drawpoints, production is less than the maximum mining capacity. However, after period one the draw rate can deplete other draw rates according to PRC, so mining capacity reaches to the maximum boundary. During the last periods, because of reaching the top of the draw column and of the probability of dilution, the average grade of production is less than in previous years. But it also must be noted that dilution is reduced, because of the controlling feature of the production rate curve in the draw control system. The difference between the maximum grades in the first period and minimum one in the last period is less than 0.22%. By this way, the USD model can reduce dilution with respect to the draw control.

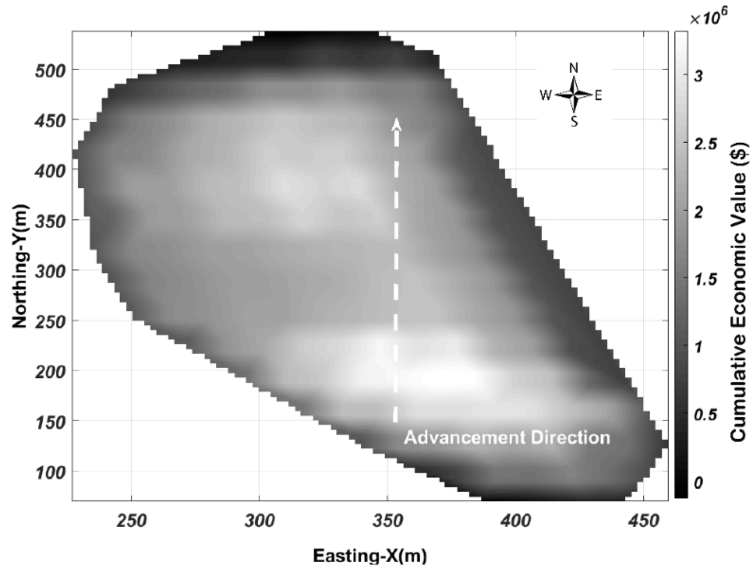


Fig. 7. Mining advancement direction

Table 2. Numerical results of all models

Model	CPU time	Total extraction (Mt)	Extraction (%)	NPV (M$)	Constraint number	Variable number	
						Con.	Bin.
USD	07:27:26	30.1	82	47.00	47257	4470	8940
US	12:20:21	30.3	82.6	47.02	42787	4470	8940
UD	26:15:18	29.3	79.8	45.81	42787	4470	8940
SD	02:25:12	29.4	80.1	47.99	38317	4470	8940

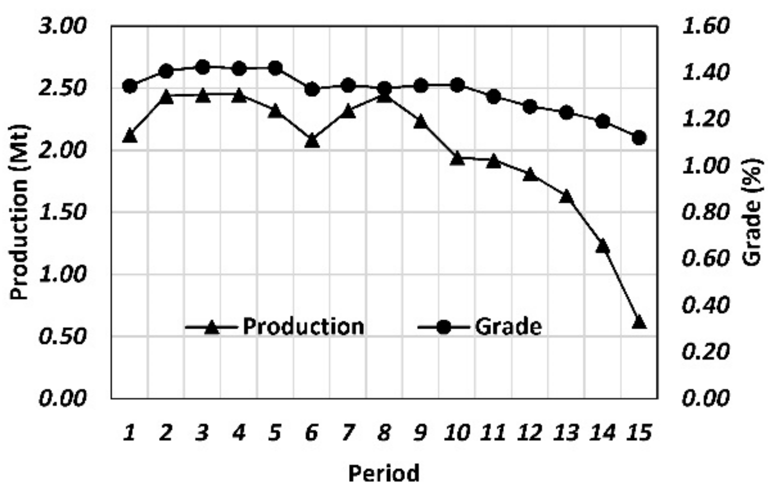


Fig.8. Production tonnage and average grade of the production in USD draw rate model

Fig. 9 shows the maximum number of active drawpoints in each period. The number of active drawpoints in period one is more than in other periods because depletion from all drawpoints must be within the minimum draw rate in the first period according to the minimum draw rate constraint. Hence it needs a greater number of active drawpoints in period one. Afterwards, during the next ten years, the mine works with maximum allowable active drawpoints. From periods 12 to 15, this number gradually reduces.

Fig. 10 illustrates the starting period of the drawpoints during the mine life. The maximum number of drawpoints is opened in period one. The number of started drawpoints in periods 6 and 11 is more than in other periods, except for period 1. This is because of the defined maximum activity period for each drawpoint, which is 5 periods.

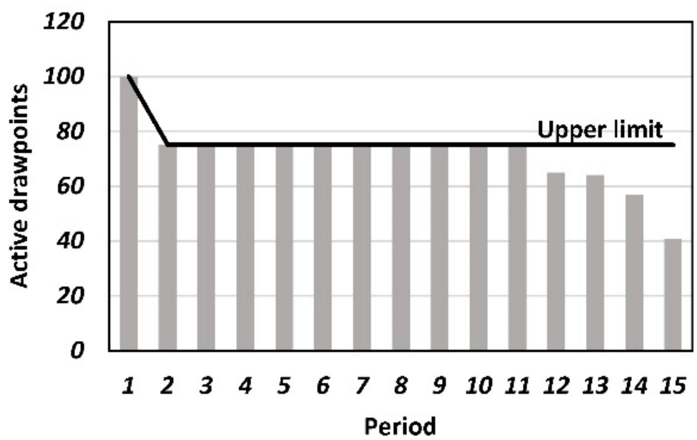


Fig. 9. Number of active drawpoints in each period during the mine life

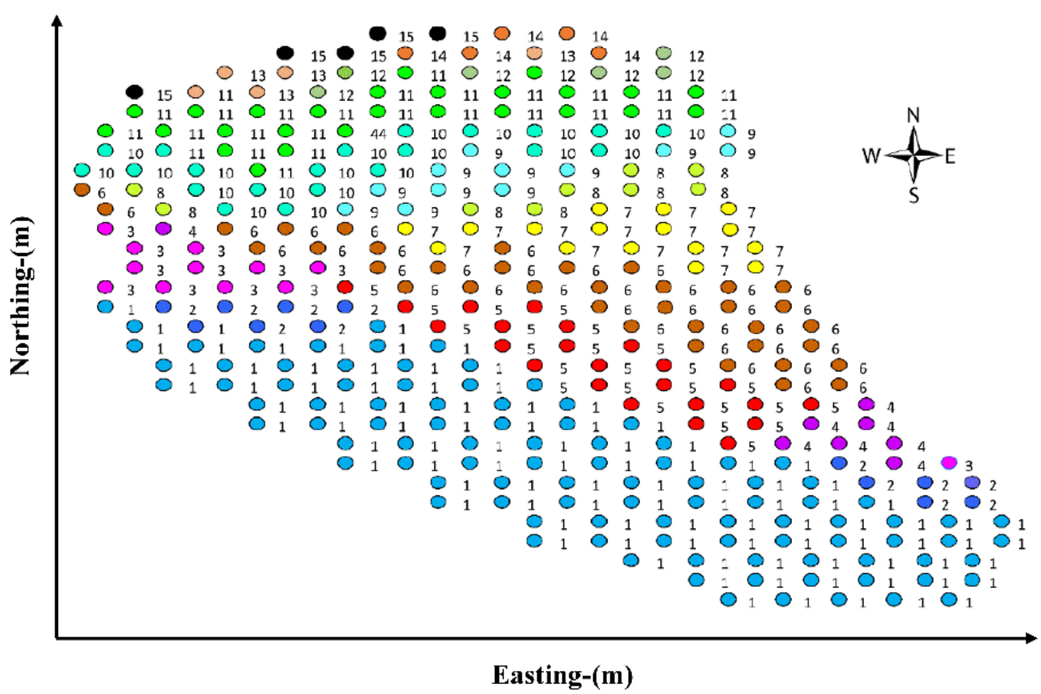
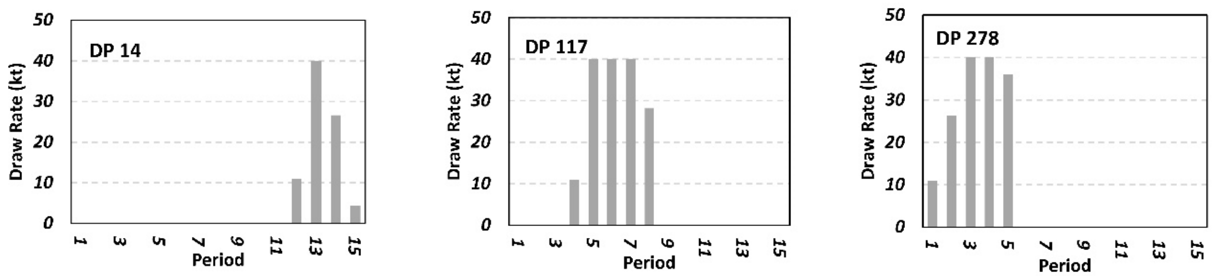


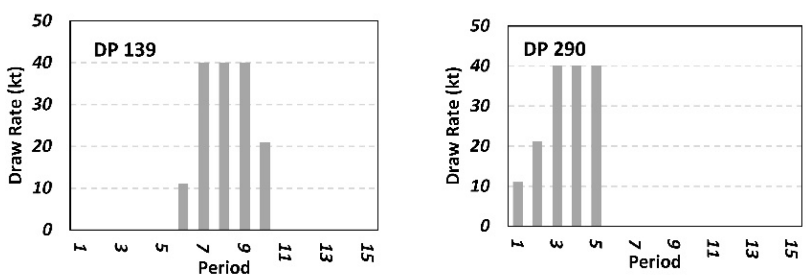
Fig. 10. Starting period of extraction from each drawpoint

According to the defined draw rate strategy (USD), drawpoints cannot be depleted arbitrarily, but it is possible because of the objective function and the constraints, the material with lower economic value remains in the number of drawpoints. Fig. 11 illustrates the draw rate changes for different drawpoints in the NS direction.

It is clear that the defined lower bound and the PRC for selected drawpoints are satisfied. For instance, extraction from DP 278 is started in period 1 with the minimum acceptable draw rate (11 kt), then it increases gradually to reach the maximum acceptable draw rate (40 kt) in period 3. After a steady extraction with the maximum draw rate for two periods, the draw rate drops. It can be seen that the tonnage of extraction from drawpoints varies based on the drawpoints' economic values because the objective function maximizes the NPV and the tonnage of extraction from each draw column is a result of optimization. One of the advantages of the draw control system is controlling the surface displacement by using the draw rate in all drawpoints during the mine life. The results show that all the defined constraints have been satisfied. The draw rate amount for each drawpoint and starting and finishing periods are obtained as a result of optimization. The model extracts the material from each draw column based on the defined draw rate model while maximizing the NPV of the operation.

In addition to the examination of the USD draw rate strategy, other draw rate models (US, SD, and UD) were also investigated with the same advancement direction. It is obvious that the value of the NPV for the SD strategy is higher than others because the tonnage of the material that can be depleted from the active drawpoints in early periods is higher than from other strategies. Table 3 summarizes the result of different strategies.





DP ID	Initial Tonnage (kt)	Extracted Tonnage (kt)	Extracted Tonnage (%)
14	82.00	82.00	100.0
117	169.15	159.15	94.1
139	157.35	151.95	96.6
278	174.83	153.35	87.7
290	169.06	152.15	90.0

Fig. 11. Obtained draw rate as a result of optimization for different drawpoints

Table 3. Numerical results for different draw strategies

Model	Initial Tonnage (Mt) & Direction	Extracted Tonnage (Mt)	NPV (M$)
USD	36.7 - SN	30.1	47.00
US	36.7 - SN	30.3	47.02
SD	36.7 - SN	29.4	47.99
UD	36.7 - SN	29.3	45.81

Fig. 12 shows the cash flow for different strategies. Fig. 13 shows the height of draw columns at the end of some periods for the USD draw rate strategy. The defined advancement direction has also been satisfied, as shown. The height of the drawpoints represents the surface displacement at the end of each period. One of the advantages of the draw control system is that it controls surface displacement by using the draw rate in all drawpoints during the life of the mine.

It is obvious that the cash flow during the first eight years of the mine life for the SD model is greater than that for other models. As shown in Table 3, the SD model has the highest NPV among other models because in the first periods it forces the model to deplete the material from the drawpoints with maximum draw rate. The US strategy has the higher NPV after the SD strategy. It can be seen that after period 11, the US strategy has higher cash flow than SD because there is no ramp-down periods for the US strategy. The lowest NPV belongs to UD strategy.

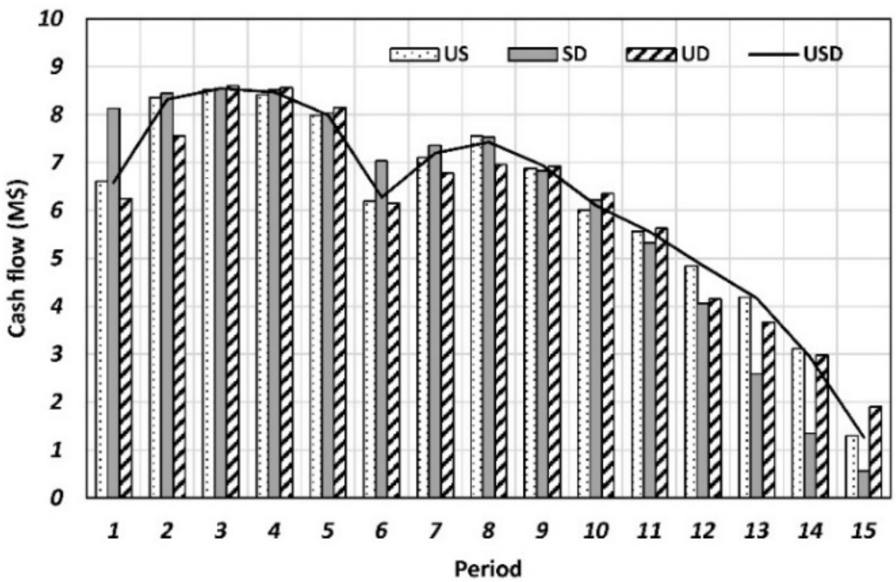


Fig. 12. Comparison of cash flow for all draw rate strategies

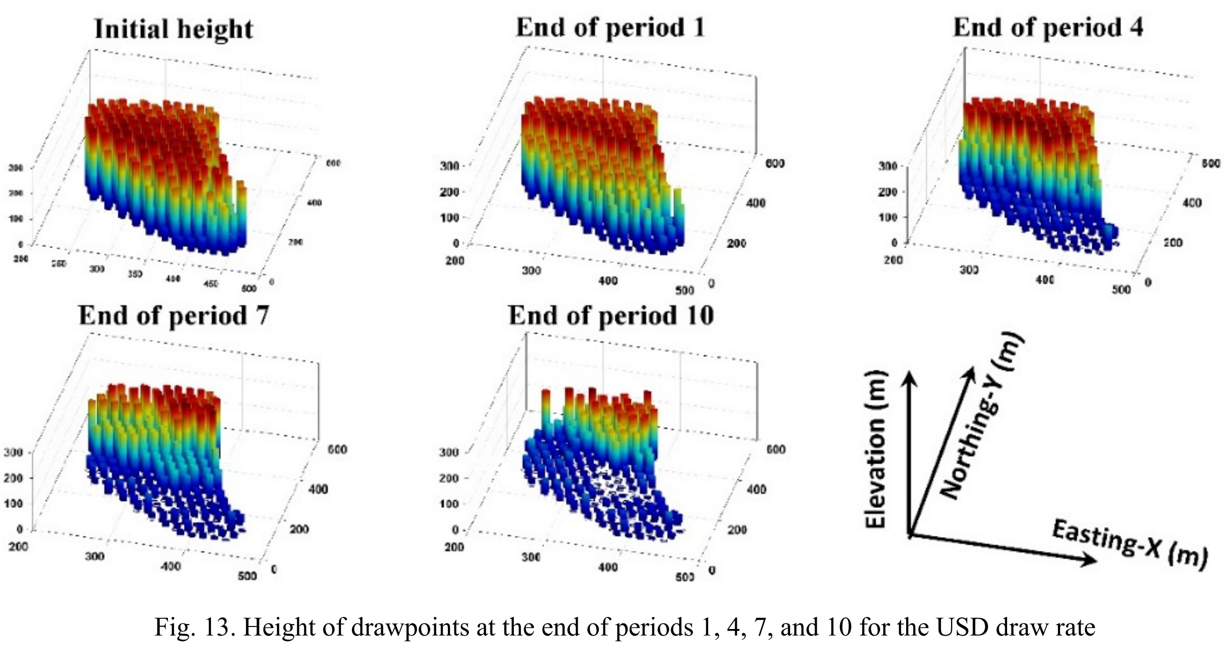


Fig. 13. Height of drawpoints at the end of periods 1, 4, 7, and 10 for the USD draw rate

6. Conclusion

This paper presented a mathematical draw control system for block-cave production scheduling optimization based on PRC. MILP formulation for block-cave production scheduling was developed, implemented, and tested in the CPLEX/IBM environment. The formulation maximizes the NPV subject to defined operational constraints. To manage drawpoint production, the PRC which limits production based on the amount of material that has been drawn previously was established. This means that production depends on the cumulative tonnes mined from a drawpoint.

The PRC was classified in four alternative general forms to be modeled and practiced by all mines according to their draw requirements. Among these four models, the USD model is a comprehensive model which can produce other models by changing depletion boundaries. Consequently, drawing pattern and dilution are controlled by using the introduced mathematical model. The surface displacement is controlled by using the defined draw rate in all drawpoints during the life of the mine.

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