Linearized Stockpile Modeling for Long-Term Open-Pit Production Planning

Mohammad Tabesh and Hooman Askari-Nasab
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

Long-term open-pit production planning is a complicated process that includes deciding on the order of extraction of blocks and their destinations in order to satisfy various technical constraints. Moreover, stockpiles can be considered to as buffers of material for future use or sources of high or low-grade ore for controlling blending requirements. In this paper, we present a mathematical formulation that uses aggregated units for making mining, processing and stockpiling decisions while respecting various mining and processing constraints. First, we propose a non-linear model that estimates stockpile grade and controls the head grade of material sent to the processing plants. Next, we use the idea of piecewise linearization to modify the model to be able to solve it with mixed integer linear programming solvers. Afterwards, we show how our model compares against other linear stockpiling models in the literature. Finally, the model is tested on a small dataset to evaluate the performance of the model and show the errors introduced by linearization.

1. Introduction

Open-pit mining is the most common and the oldest method of mining valuable material from the ground. It has attracted many researchers to study various aspects of the operation among which short- to long-term mine planning has been the center of applications of operations research techniques. Various heuristic, meta-heuristic and mathematical programming techniques have been implemented to improve the operation. Long-term plans usually deal with larger units of production and decide on the sequence of extraction of material and their destination. Short-term plans, on the other hand, deal with smaller units and make more detailed decisions. Grade blending and stockpiling is another important aspect present in short- to long-term mine planning. The main two approaches to this is to include the stockpiling decisions while developing a long-term term plan or to develop a long-term plant with no stockpiles and postponing the stockpiling decisions to medium- and short-term plans.

Since Johnson (1969) introduced mathematical programming and in particular linear programming to the mine planning research area many scholars have used the same principles to address the production planning problem in open pit mines. However, no globally recognized solution technique has been obtained yet due to size and complexity of the problem. In addition to the size obstacles, adding stockpiles to the model introduce a non-linear term which make finding the solution even harder. Numerous studies have been published around the long-term open pit

This paper, in its entirety, is submitted for publication in International Journal of Mining Science and Technology.

production planning problem. Osanloo et al. (Osanloo et al., 2008) and Newman et al. (Newman et al., 2010) provide comprehensive reviews of the literature on this subject to 2010.

Since 2010, Moreno et al. (Moreno et al., 2010) develop a multistep algorithm to solve large-scale LTOPP problems with one capacity constraint. They solve the LP-relaxation of the problem using a critical multiplier procedure and use the results in a TopoSort procedure to obtain good feasible solutions. Bienstock and Zuckerberg (Bienstock and Zuckerberg, 2010) propose a new decomposition based solution method to solve the LP-relaxations of large instances of the LTOPP. They implement and test their algorithm on instances of more than hundred thousand blocks and obtain an LP-relaxation solution in their predefined time threshold. Similarly, Chicoisne et al. (Chicoisne et al., 2012) use decomposition techniques to obtain the LP-relaxation of the LTOPP and use a heuristic technique to find an integer feasible solution to the problem. Cullenbine et al. (Cullenbine et al., 2011) use a sliding time window approach to decompose the LTOPP into smaller problem and solve it. Lambert and Newman (Lambert and Newman, 2014) propose a hybrid solution technique where they obtain an initial feasible solution to the LTOPP and use a modified Lagrangian relaxation to obtain near-optimal solutions. They use two different heuristics for obtaining the initial solution and propose a preprocessing step to set some of the variables to fixed numbers before looking for solutions. Finally, Lamghari et al. (Lamghari et al., 2015) propose a hybrid linear programming and neighborhood search method where they solve an LP for every period using a normalized value for blocks and improve the solution by searching the neighborhood. Using their proposed hybrid method, they report obtaining near-optimal solutions for models with up to 100 thousand blocks in short time periods. However, all these efforts are focused on tackling the size of the problem and not including stockpiles in the planning process.

Bley et al. (2012) model the LTOPP with stockpiling by adding the non-linear constraints and proposing a problem-specific solution method. Moreover, they offer a stronger formulation in which they track the flow of material from aggregates to stockpile and plant. Gholamnejad and Kasmaee (Gholamnejad and Kasmaee, 2012) develop a goal programming model to satisfy the blending requirements of an Iron ore processing plant by blending material from two high grade and low grade stockpiles. Their model provides the optimal reclamation schedule by dividing the stockpiles into blocks and assigning grade values to each block. However, their proposed model does not include decisions on extracting material from the mine and stockpiling and solely focuses on reclamation decisions.

Waqar Ali Asad and Dimitrakopoulos (2012) consider stockpile while determining the cutoff grade in presence of uncertainty. They use the grade-tonnage curves instead of planning units and model the stockpile by using grade ranges and tonnages of material in each range. Ramazan and Dimitrakopoulos (2013) propose a production scheduling model with uncertain supply that includes stockpiling. Their model uses a predetermined constant grade for reclaiming material from the stockpile and allow blocks into the stockpile based on the probability of block grade being within the acceptable range for the stockpile. However, the authors do not compare the actual grade of material in the stockpile to the predefined grade.

Smith and Wicks (2014) propose an MIP for medium-term production planning with stockpiling in a copper mine. The authors divide ore into different categories based on low and high grade and recovery of the main two elements and define a stockpile for rehandling low-grade ore when needed. However, they avoid nonlinearity by not keeping track of elements grades going to and reclaimed from the stockpile. Mousavi et al. (2016) also consider stockpiling with a predetermined grade and use a non-exact approach deal with the problem. They compare their results against solutions obtained via exact method and show how close to the optimum solution their solutions are. However, they do not study the errors caused by assuming a fixed reclamation grade for the stockpile and their largest case-study has 2,500 blocks which is a relatively small number.

Kumar and Chatterjee (2017) propose and apply a mathematical formulation for production scheduling with stockpiling in a coal mine. Their formulation follows the same approach and assumes a fixed predetermined reclamation grade for the stockpile and show that the observed element head grades are within the required boundaries. Finally, Moreno et al. (2017) classify the production scheduling and stockpiling models in the literature and propose a new modeling approach. Moreover, they provide extensive computational results for the models they studied and developed. We will discuss their proposed model in the next chapter in more details.

In this paper, we present a mixed integer linear programming model for long-term multi-destination open-pit production planning problem that uses two sets of aggregated units for making mining and processing decisions and incorporates the blending constraints and stockpiling. The model is a continuation of our work on long-term open-pit production planning started by aggregating blocks into larger mining-cuts in (2011), formulating an MILP for optimizing the production schedule in (2014) and adding stockpiles to the formulation in (2015).

2. Mathematical Formulations

As mentioned earlier, various LTOPP mathematical models have been proposed in the literature. In this paper, we tried to model stockpiling while avoiding the non-linear constraints by benefiting from the piecewise linearization technique. In order to model the stockpile element grade we introduce multiple stockpiles with different acceptable grades to be able to assign fixed reclamation grades to each stockpile. These input grade ranges, as well as reclamation grades, are determined based on histograms of grades to be representative of data prior to solving the model. Moreover, we decrease the problem size by using two different sets of aggregates to make mining and processing decisions: bench-phases (intersections of benches and pushbacks) are used to make mining scheduling decisions and mining-cuts are used to make destination decisions. The mining cuts are generated through a clustering algorithm that not only accounts for the similarities but also respects the size and shape constraints. The clustering algorithm mentioned is a variation of hierarchical agglomerative clustering and is thoroughly explained in (2013). We use the clustering algorithm to create processing units within the boundaries of bench-phases. Therefore, the bench-phases are divided into smaller units with similar rock-type and grade which are the basis for making processing and stockpiling decisions. The mathematical formulation and notations are an improved and tailored versions of the model presented in (2015).

- Sets

S^m	For each bench-phase m, there is a set of bench-phases (S^m) that have to be extracted prior to extracting bench-phase m to respect slope and precedence constraints
U^m	Each bench-phase m is divided into a set of clusters. U^m is the set of clusters that are contained in bench-phase m

- Indices

$d \in \{1,...,D\}$	Index for material destinations
$m \in \{1,...,M\}$	Index for bench-phases
$p \in \{1,...,P\}$	Index for clusters
$c \in \{1,...,C\}$	Index for processing plants

$e \in \{1,...,E\}$	Index for elements
$t \in \{1,...,T\}$	Index for scheduling periods

- Parameters

D	Number of material destinations (including processing plants and waste dumps)
M	Total number of bench-phases
P	Total number of clusters
E	Number of elements in the block model
T	Number of scheduling periods
$\overline{MC}^t$	Upper bound on the mining capacity in period t
$\underline{MC}^t$	Lower bound on the mining capacity in period t
$\overline{PC}_c^t$	Maximum tonnage allowed to be sent to plant c in period t
$\underline{PC}_c^t$	Minimum tonnage allowed to be sent to plant c in period t
$\overline{G}_c^{t,e}$	Upper limit on the allowable average grade of element e at processing plant c in period t
$\underline{G}_c^{t,e}$	Lower limit on the allowable average grade of element e at processing plant c in period t
s_m	Number of predecessors of bench-phase m (members of S^m)
o_m	Total ore tonnage in bench-phase m
w_m	Total waste tonnage in bench-phase m
o_p	Total waste tonnage in cluster p
w_p	Total waste tonnage in cluster p
c_m^t	Unit discounted cost of mining material from bench-phase m in period t
$r_{p,c}^t$	Unit discounted revenue of sending material from processing unit p to processing destination c in period t minus the processing costs
$r_c^{t,e}$	Unit discounted revenue of processing one unit of element e from stockpile in processing destination c in period t minus the processing and rehandling costs
g_p^e	Average grade of element e in cluster p

- Decision Variables

$y_m^t \in [0,1]$	Continous decision variable representing the portion of bench-phase m extracted in period t
$x_{p,c}^t \in [0,1]$	Continous decision variable representing the portion of ore tonnage in cluster p extracted in period t and sent to processing plant c

$b_m^t \in \{0,1\}$	Binary decision variable indicating if all the predecessors of bench-phase m are completely extracted by or in period t
f_c^t	Continous decision variable representing the tonnage reclaimed from the stockpile and sent to processing plant c in period t
$G^{t,e}$	Continous decision variable representing the reclamation grade of element e in period t

2.1. Non-linear Model

We first present the original LTOPP with stockpiling mathematical model with non-linear stockpile grade calculation. The model is a multi-destination LTOPP which uses two different sets of units for making mining and processing decisions. Two sets of variables are defined for bench-phases: $y_m^t \in [0,1]$ is the portion of bench-phase extracted in each period and $b_m^t \in \{0,1\}$ is the binary variable to control the precedence. Since the number of bench-phases is less than number of blocks and clusters, controlling the precedence with bench-phases results in less binary variables and less resource consumption for solving the model. Moreover, using bench-phases as mining units is the common practice in the mining industry. However, making material destination decisions requires more accurate units with distinction between ore and waste. This is achieved by dividing every bench-phase into smaller units using the clustering algorithm.

- Objective Function

$$\max \sum_{t=1}^{T}\left(\sum_{p=1}^{P}\sum_{c=1}^{C}(r_{p,c}^{t} \times o_p \times x_{p,c}^{t}) - \sum_{m=1}^{M}(c_m^t \times (o_m + w_m) \times y_m^t) + \sum_{e=1}^{E}\sum_{c=1}^{C}\left(f_c^t \times G^{t,e} \times r_c^{t,e}\right)\right) \tag{1}$$

- Constraints

$$\underline{MC}^t \le \sum_{m=1}^{M}\left((o_m + w_m) \times y_m^t\right) \le \overline{MC}^t \qquad \forall t \in \{1,...,T\} \tag{2}$$

$$\sum_{p \in U^m}\sum_{d=1}^{D}\left(o_p \times x_{p,d}^t\right) \le (o_m + w_m) \times y_m^t \qquad \forall t \in \{1,...,T\}, \forall m \in \{1,...,M\} \tag{3}$$

$$\underline{PC}_c^t \le \sum_{p=1}^{P}\left(o_p \times x_{p,c}^t\right) + f_c^t \le \overline{PC}_c^t \qquad \forall t \in \{1,...,T\}, \forall c \in \{1,...,C\} \tag{4}$$

$$\underline{G}_c^{t,e} \le \frac{\sum_{p=1}^{P}\left(o_p \times g_p^e \times x_{p,c}^t\right) + f_c^t \times G^{t,e}}{\sum_{p=1}^{P}\left(o_p \times x_{p,c}^t\right) + f_c^t} \le \overline{G}_c^{t,e} \qquad \forall t \in \{1,...,T\}, \forall c \in \{1,...,C\}, \forall e \in \{1,...,E\} \tag{5}$$

$$G^{t,e} = \frac{\sum_{p=1}^{P}\left(o_p \times g_p^e \times x_{p,c'}^t\right) - \sum_{t'=1}^{t-1}\sum_{c=1}^{C} f_c^{t'} \times G^{t',e}}{\sum_{p=1}^{P}\left(o_p \times x_{p,c'}^t\right) + \sum_{t'=1}^{t-1}\sum_{c=1}^{C} f_c^{t'}} \qquad \forall t \in \{1,...,T\}, \forall e \in \{1,...,E\} \tag{6}$$

$$\sum_{t'=1}^{t}\sum_{c=1}^{C} f_c^{t'} \le \sum_{t'=1}^{t-1}\sum_{p=1}^{P}\left(o_p \times x_{p,c'}^{t'}\right) \qquad \forall t \in \{2,...,T\} \tag{7}$$

$$\sum_{t=1}^{T} y_m^t = 1 \qquad \forall m \in \{1,...,M\} \tag{8}$$

$$\sum_{i=1}^{t} y_m^i \le b_m^t \qquad \forall m \in \{1,...,M\}, \forall t \in \{1,...,T\} \tag{9}$$

$$s_m \times b_m^t \le \sum_{i \in S^m} \sum_{j=1}^{t} y_i^j \qquad \forall m \in \{1,...,M\}, \forall t \in \{1,...,T\} \tag{10}$$

$$b_m^t \le b_m^{t+1} \qquad \forall m \in \{1,...,M\}, \forall t \in \{1,...,T-1\} \tag{11}$$

The objective function (equation (1)) is summation of discounted revenue made from sending material to the processing plants directly from the mine and reclaiming from the stockpile minus the total cost of mining material from the ground. Equations (2) and (4) are responsible for controlling the minimum and maximum extraction and processing capacity in each period. Equation (3) controls the relation between the tonnage mined from each bench-phase and the tonnage processed from the clusters within that bench-phase. Note that the difference between the tonnage extracted and the tonnage processed is the waste extracted and sent to the waste dump. However, if we have a waste dump with an extra haulage cost the dump can be defined as a destination with negative revenue. Equation (5) controls the average head grade of material sent to processing plants in each period by averaging the grade of material sent directly from the mine with the average grade of material reclaimed from the stockpile. However, to avoid non-linearity the equations are rearranged before putting into matrix format. Equation (8) ensures that all the material within the ultimate pit is extracted during mine life, and Equations (9) to (11) are the precedence control constraints with the binary variables.

The stockpile is added as a destination with the index of c' to control the tonnage and grades of material sent to the stockpile using the same constraints as for the processing plants. $G^{t,e}$ is the reclamation grade of element e in period t and is calculated using equation (6). Note that the objective function and equation (6) are non-linear equations. Equation (7) ensures that the summation of tonnages reclaimed from stockpile from the first period to the current period does not exceed the summation of tonnages sent to the stockpile by the current period. The constraint is defined using the cumulative sent and reclaimed tonnages to avoid introducing extra variables for keeping track of the stockpile inventory.

2.2. Linearized Model

In order to have a linear LTOPP model with stockpiling, we assume that there are multiple stockpiles with tight ranges for the acceptable element grades. Therefore, we can assign an average reclamation grade and the corresponding reclamation revenue to each stockpile. The more stockpiles defined the smaller error is introduced into the model. However, more stockpiles sacrifices the complete blending assumption present in most stockpiling scenarios. Therefore, making reasonable assumptions regarding the number of stockpiles to define and the acceptable element grade ranges is crucial to obtaining reasonable results.

In order to create the linear LTOPP model with stockpiling we define S stockpiles and add S destinations for materials to the list of processing plants. $G_s^{e,t}$ is the predetermined average reclamation grade of element e from stockpile s in period t, and $r_{s,c}^t$ is the unit discounted revenue of reclaiming material from stockpile s with the average grade and processing them in plant c in period t minus the processing and rehandling costs. $\underline{G}_d^{e,t}$ and $\overline{G}_d^{e,t}$ are the lower and upper bounds on the acceptable average element grade e for stockpile destination $d = C + s$.

$f^t_{s,c} \geq 0$ is the set of variables representing the tonnage of material reclaimed from stockpile s in period t and sent to processing destination c. Now we can rewrite the model by replacing the objective function with equation (12) and equations (4) to (7) with equations (13) to (16) respectively. Moreover, we add a constraint on the element content of the material sent to stockpile and the element content of material reclaimed from the stockpile while avoiding non-linear grade control. Equation (17) ensures that reclaiming from the stockpile is stopped if the predetermined average grade is significantly higher than the actual grade of material sent to the stockpile. For example, if 100 tonnes of ore with average element grade of 5% is sent to the stockpile and the reclamation grade is assumed to be 10%, the model will limit the reclamation to 50 tonnes.

$$\max \sum_{t=1}^{T}\left(\sum_{p=1}^{P}\sum_{c=1}^{C}(r^t_{p,c}\times o_p \times x^t_{p,c}) - \sum_{m=1}^{M}(c^t_m \times \left(o_m + w_m\right)\times y^t_m) + \sum_{s=1}^{S}\sum_{c=1}^{C}\left(f^t_{s,c}\times r^t_{s,c}\right)\right) \tag{12}$$

$$\underline{PC}^t_c \leq \sum_{p=1}^{P}\left(o_p \times x^t_{p,c}\right) + \sum_{s=1}^{S} f^t_{s,c} \leq \overline{PC}^t_c \qquad \forall t \in \{1,...,T\}, \forall c \in \{1,...,C\} \tag{13}$$

$$\underline{G}^{t,e}_c \leq \frac{\sum_{p=1}^{P}\left(o_p \times g^e_p \times x^t_{p,c}\right) + \sum_{s=1}^{S}\left(f^t_{s,c}\times G^{t,e}_s\right)}{\sum_{p=1}^{P}\left(o_p \times x^t_{p,c}\right) + \sum_{s=1}^{S} f^t_{s,c}} \leq \overline{G}^{t,e}_c \qquad \forall t \in \{1,...,T\}, \forall c \in \{1,...,C\}, \forall e \in \{1,...,E\} \tag{14}$$

$$\underline{G}^{t,e}_d \leq \frac{\sum_{p=1}^{P}\left(o_p \times g^e_p \times x^t_{p,d}\right)}{\sum_{p=1}^{P}\left(o_p \times x^t_{p,d}\right)} \leq \overline{G}^{t,e}_d \qquad \forall t \in \{1,...,T\}, \forall e \in \{1,...,E\}, \forall d \in SC \middle| d = C + s \tag{15}$$

$$\sum_{i=1}^{t}\sum_{c=1}^{C} f^i_{s,c} \leq \sum_{i=1}^{t-1}\sum_{p=1}^{P}\left(o_p \times x^i_{p,d}\right) \qquad \forall s \in \{1,...,S\}, \forall t \in \{2,...,T\}, \forall d \in SC \middle| d = C + s \tag{16}$$

$$\sum_{i=1}^{t}\sum_{c=1}^{C} G^{t,e}_s \times f^i_{s,c} \leq \sum_{i=1}^{t-1}\sum_{p=1}^{P}\left(o_p \times g^e_p \times x^i_{p,d}\right) \qquad \forall s \in \{1,...,S\}, \forall t \in \{2,...,T\}, \forall e \in \{1,...,E\}, \forall d \in SC \middle| d = C + s \tag{17}$$

2.3. Comparison with other models

As mentioned earlier, we have developed a mathematical model to simultaneously make decisions on the extraction of material from the mine, destination of material extracted and the stockpiling scenarios in long-term horizon. The LTOPP has been widely studied and modeled using various approaches, however, the LTOPP with stockpiling option has not been as widely studied. Moreno et al. (2017) present and compare the most common modeling approaches and propose a new model for linear modeling of stockpile. In this section, we show how our proposed model compares against some of the studied approaches taken towards modeling the stockpiles.

In order to compare the models, we need to modify and simplify our model depending on the model we are comparing against. First simplification is that we only need one processing destination and one element of interest. Thus, we remove indices $e \in \{1,...,E\}$, assuming that $E = 1$, from all variables and parameters. It is an important change especially for processing operations that are sensitive to the grade of deleterious elements in the ore. Next, we need to use blocks for making mining and processing decisions. Therefore, both variables $x^t_{p,c} \in [0,1]$ and

$y_m^t \in [0,1]$ should be defined for blocks as $x_{b,c}^t \in [0,1]$ and $y_b^t \in [0,1]$. Furthermore, in all cases the blocks are assumed to be completely ore or waste. Therefore, we will replace o_b and $o_b + w_b$ with W_b as used in (2017). Another modification required is to replace grade of blocks with metal contents. It is an easy modification since the grade is a known parameter and we can always multiply the tonnage of a block by the grade to obtain the metal content $M_b = W_b \times g_b$. Now, we will compare our model against the linear L-bound, K-bucket and L-Average models in (2017). The L-bound and L-Average models use only one stockpile, and therefore, we will set $S = 1$ in our model to limit our model to one stockpile. Moreover, since there is only one processing plant and one stockpile we limit $c \in \{1,...,C\}$ to $c \in \{1,2\}$ where destination 1 is the processing plant and destination 2 is the stockpile. The L-bound model uses a lower bound for the grade of blocks sent to the stockpile and assumes a fixed reclamation grade for the stockpile. As pointed out by (2017), this model is too conservative and can be improved by controlling the average grade of material sent to the stockpile instead of individual block grades. However, in order to obtain the same result with our model we need to replace equation (15) with equation (18) and use a large upper bound on the average input grade for the stockpile.

$$x_{b,2}^t = 0 \qquad \begin{array}{l} \forall t \in \{2,...,T\}, \\ \forall b \in \{1,...,B\} \big| g_b < \underline{G}_2^t \end{array} \tag{18}$$

Following the same approach as for the L-bound model we can obtain the K-bucket model by defining $S = K$ stockpiles and replacing equation (15) with equation (19). However, in order to obtain the L-Average model we do not need to add or modify any constraints. We can obtain the L-Average model by limiting our model to one processing destination and one stockpile and changing to notations.

$$x_{b,1+s}^t = 0 \qquad \begin{array}{l} \forall t \in \{2,...,T\}, \forall s \in \{1,...,S\}, \\ \forall b \in \{1,...,B\} \big| g_b < \underline{G}_{1+s}^t \end{array} \tag{19}$$

3. Case Study

In order to evaluate the proposed model and quantify the error introduced by linearizing the stockpile grade calculation we implemented the model on an iron-ore block model. The dataset contains 430 million tonnes of material in the final pit discretized into 19,561 blocks. The goal is to obtain a long-term plan over 20 years of production with three years of pre-stripping. The dataset is divided into four pushbacks using the parameterization approach which results in 40 bench-phases. Afterwards, the hierarchical clustering with shape control technique from Tabesh and Askari-Nasab (2013) is implemented on the dataset resulting in 1870 mining-cuts.

The processing plant capacity is seven million tonnes per year starting from year four of the operation. The mining capacity is 32 million tonnes at the beginning of the operation and is gradually decreased to eight million tonnes towards the end of the mine life. The block model contains seven different rock types (three ore and four waste) and tracks three different elements. Iron content is tracked via mass percent of magnetic weight (MWT) and deleterious elements Sulfur (S) and Phosphor (P) are tracked in mass percent units. The processing plant specifications require ore with a minimum MWT of 78% and maximum P and S content of 0.14% and 1.7% respectively.

We formulated the models in Matlab and solved them to optimality using Gurobi (2017) optimization engine. The clustering algorithm takes 5 seconds to group blocks into clusters and all the models are solved to optimality in less than 15 seconds.

3.1. Original Schedule

We first run the MILP without any constraints of the head grade to adjust the mining capacity for an acceptable schedule. We will use the same settings for the next scenarios to show how the stockpile can help the mine planner and we will also show the errors introduced by using a linear stockpile model. The original schedule with no head grade constraint is presented in Figure 1. The head grade for P and MWT is presented in Figure 2. The plant Sulfur head grade constraint is not binding for any of the scenarios and is omitted from graphs to make them easier to read. It can be seen that MWT and P requirements are not met specially in the earlier periods where access to high quality ore is limited. The generated schedule results in NPV of 2,615 million dollars.

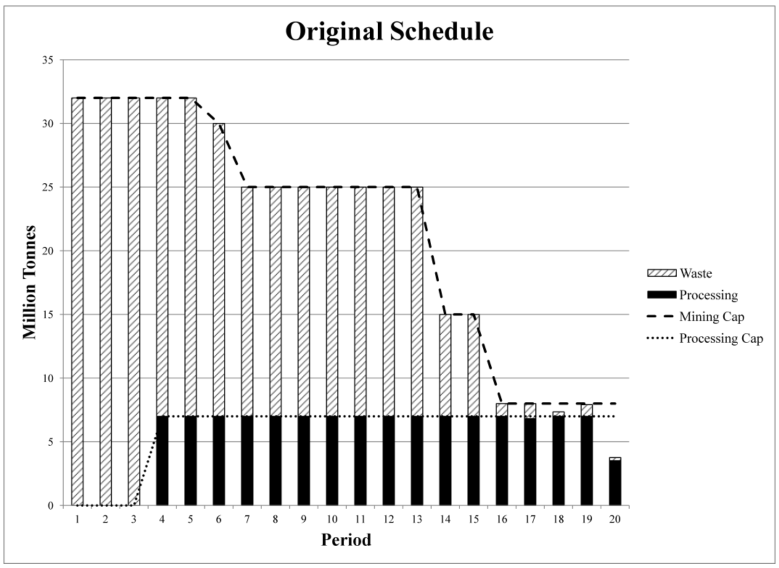

Figure 1.Original Schedule

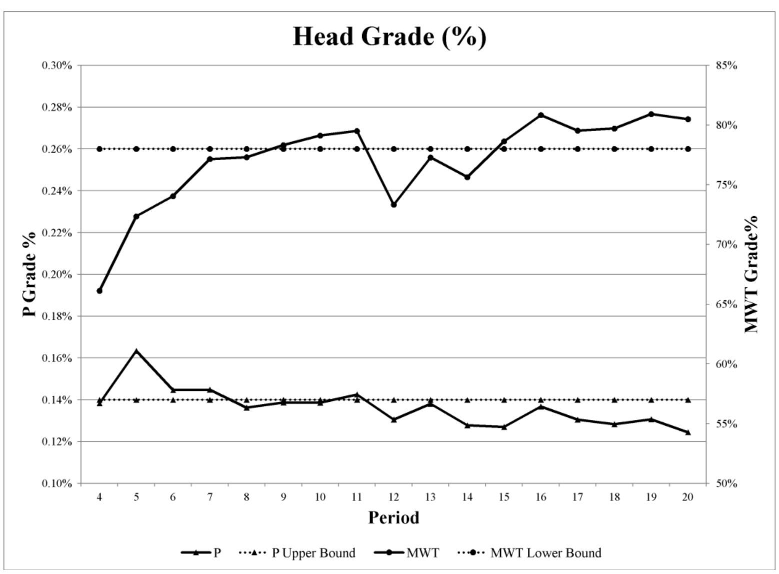

Figure 2.Head Grade in Original Schedule

3.2. Head Grade Constraints

Now we will add the head grade constraints for the three elements and run the MILP again. The schedule presented in Figure 3 shows that plant is not fully utilized in most years. Moreover, since there is no ore extracted in period four since the grade of ore is not acceptable for the plant. The generated NPV is also dropped to 2,109 million dollars which is a 23% decrease.

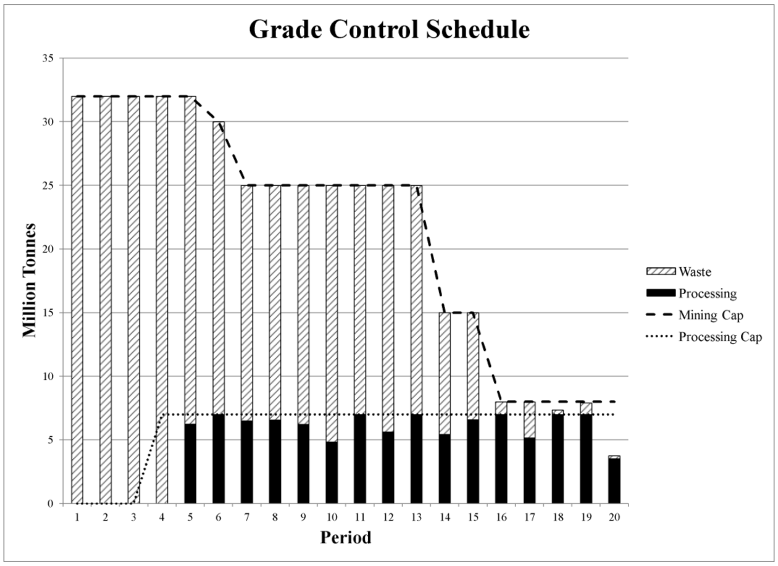

Figure 3.Grade Control Schedule

3.3. Single Stockpile

Now we will add a stockpile to help the operation balance the head grade. The stockpile is defined with average input grade ranges and reclamation grades presented in Table 1. Since the relationship between the elements of interest is not linear the tonnage of material within the ranges varies by changing the stockpile ranges. Figure 4 shows the range of mining-cut grades allowed to the stockpile. The reclamation grade is calculated based on the average grade of ore within the acceptable stockpile input grade. The revenue of reclaiming material from the stockpile and sending to plant is calculated based on the MWT reclamation grade and a rehandling cost of \$0.5/tonne. It is worth noting that the mining fleet require for reclamation is assumed to be independent of the mining fleet. As can be seen in Figure 5 the plant is fully utilized in all years except than year four where there is not enough high quality ore to feed the plant. However, the ore is stored in stockpile and reclaimed in later years while being mixed with higher quality ore and fed to the plant. The resulted NPV is 2,291 million dollars which is 9% higher than the no stockpile scenario. Figure 6 shows the difference between the actual grade of material in stockpile and the predetermined reclamation grade. It is obvious that the optimizer tried to send material with lower MWT and higher P grade to the stockpile and take advantage of the better reclamation grade to increase the NPV while respecting the head grade constraint. The average absolute error in grade is 11.6% and the total material reclaimed from the stockpile over the mine life is 12 million tonnes.

Table 1.Single Stockpile Parameters

Stockpile	Element	$\underline{G}_d^{t,e}$ (%)	$\overline{G}_d^{t,e}$ (%)	$G_s^{t,e}$ (%)
1	P	0.10	0.15	0.13
	S	1.00	2.00	1.59
	MWT	70.00	80.00	76.55

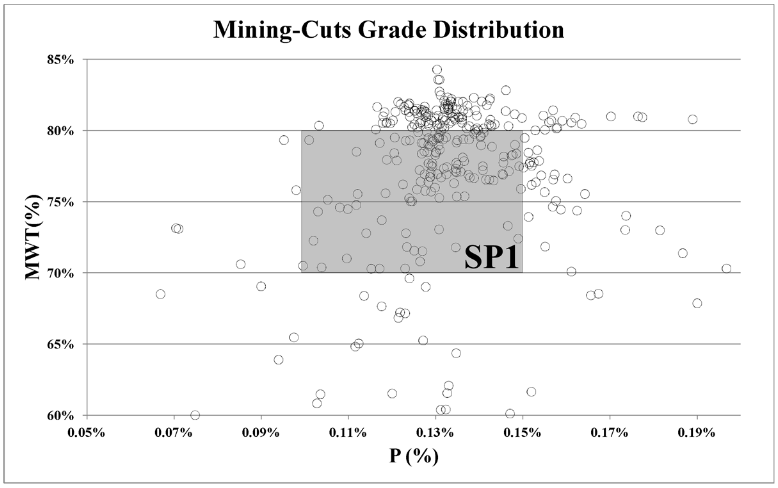

Figure 4.Single Stockpile Range

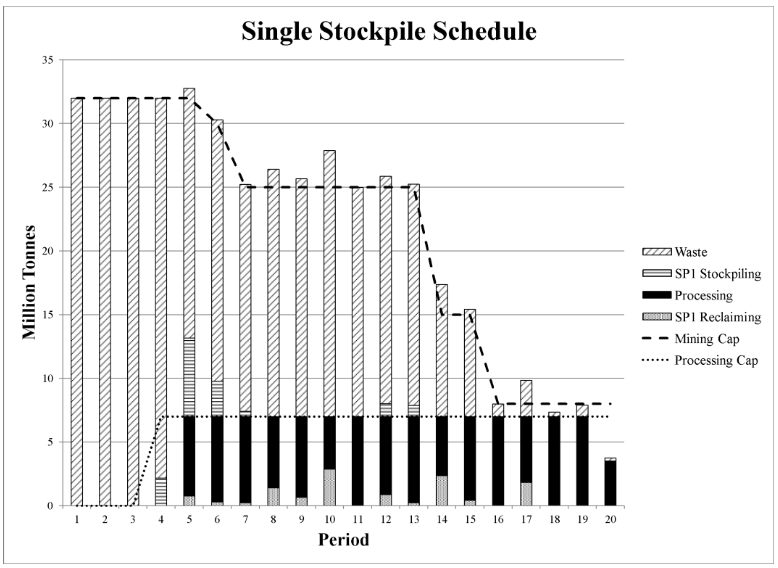

Figure 5.Single Stockpile Schedule

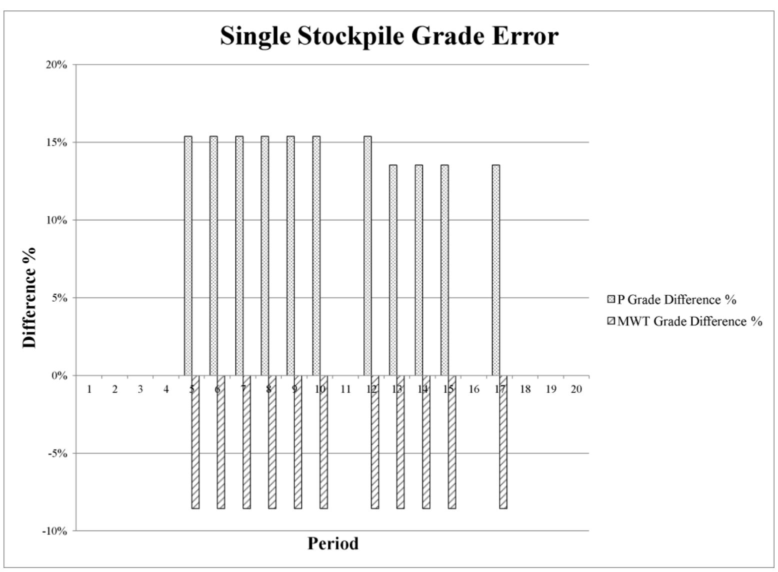

Figure 6.Single Stockpile Grade Error

3.4. Double Stockpile

Now we will add another stockpile to tighten the ranges of the acceptable grades and decrease the errors with the parameters provided in Table 2. The range of mining-cuts that are within the acceptable ranges for the stockpiles in shown in Figure 7. You can see how the difference between grade ranges affects the mining-cut ranges in Figure 4 and Figure 7.

Table 2.Double Stockpile Parameters

Stockpile	Element	$\underline{G}_d^{t,e}$ (%)	$\overline{G}_d^{t,e}$ (%)	$G_s^{t,e}$ (%)
1	P	0.10	0.13	0.12
	S	1.00	2.00	1.59
	MWT	70.00	75.00	72.42
2	P	0.13	0.15	0.14
	S	1.00	2.00	1.59
	MWT	75.00	80.00	79.49

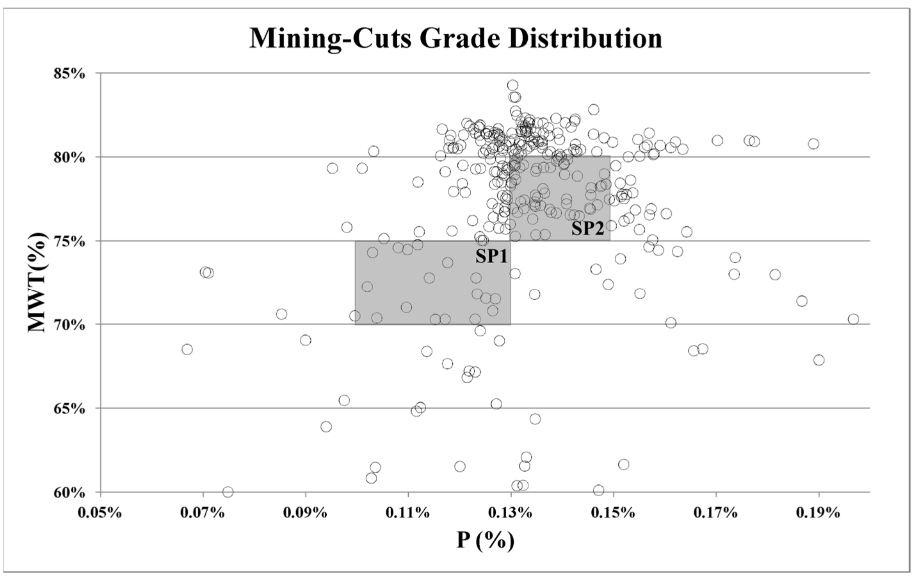

Figure 7.Double Stockpile Range

Figure 8 shows the production schedule with two stockpiles that results in an NPV of 2,234 million dollars. This is 5.5% higher than the no stockpile scenario and 2.6% lower than the single stockpile scenario. Note that no extra cost has been considered for building and maintaining the stockpiles and the 2.6% difference is due to decreasing the error introduced by predetermined stockpile grade assumption.

Figure 9 shows the difference between the actual grade of material in stockpile and the predetermined reclamation grade. The average absolute error in grade is 6.5% and the total material reclaimed from the stockpiles over the mine life is 8 million tonnes.

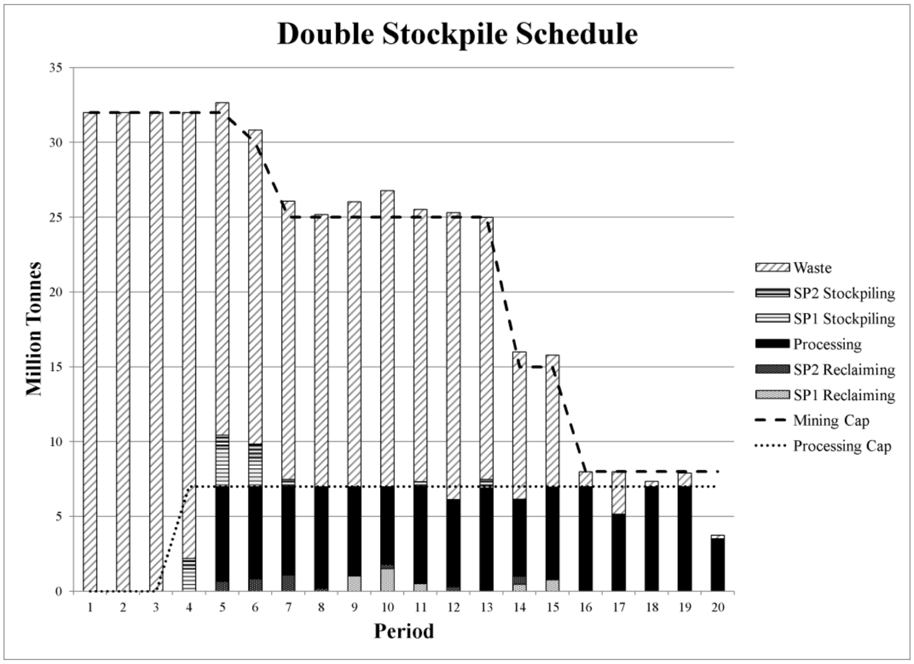

Figure 8.Double Stockpile Schedule

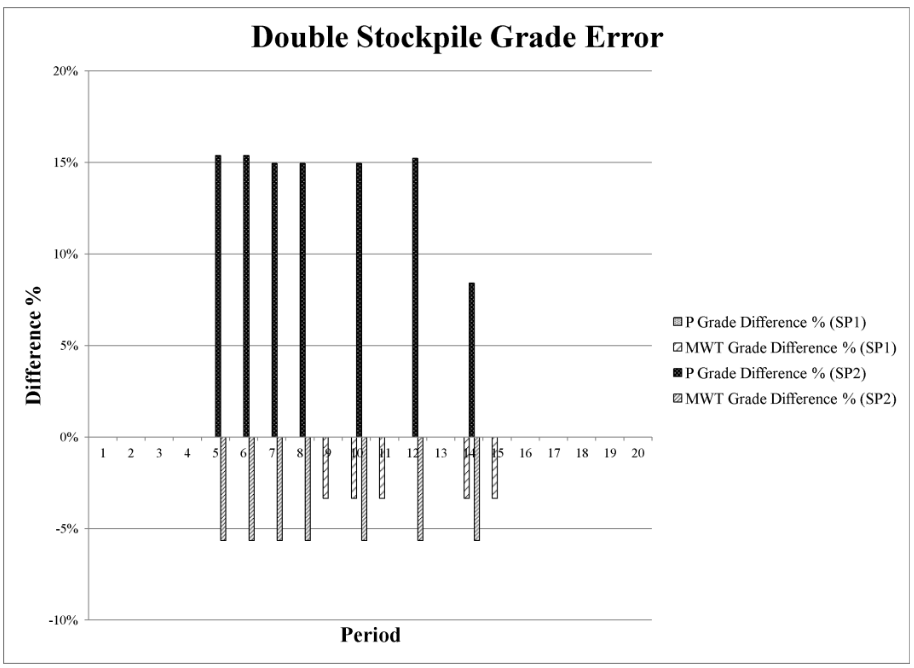

Figure 9.Double Stockpile Grade Error

3.5. Triple Stockpile

We further increase the number of stockpiles to three to study the effects of tightening the grade range for input to stockpiles. The three stockpile parameters are provided in Table 3. The area covered by stockpile input ranges shrinks compared to less number of stockpiles as shown in Figure 10. Using three stockpiles results in an NPV of 2,155 million dollar which is 3.4% less than the double stockpile scenario and 2.3% higher than no stockpile scenario. The average absolute error in grade is 3% and the total material reclaimed from the stockpiles over the mine life is 6 million tonnes.

Table 3.Triple Stockpile Parameters

Stockpile	Element	$\underline{G}_d^{t,e}$ (%)	$\overline{G}_d^{t,e}$ (%)	$G_s^{t,e}$ (%)
1	P	0.10	0.11	0.10
	S	1.00	2.00	1.59
	MWT	70.00	74.00	71.83
2	P	0.11	0.13	0.12
	S	1.00	2.00	1.59
	MWT	74.00	78.00	76.47
3	P	0.13	0.15	0.14
	S	1.00	2.00	1.59
	MWT	78.00	82.00	80.34

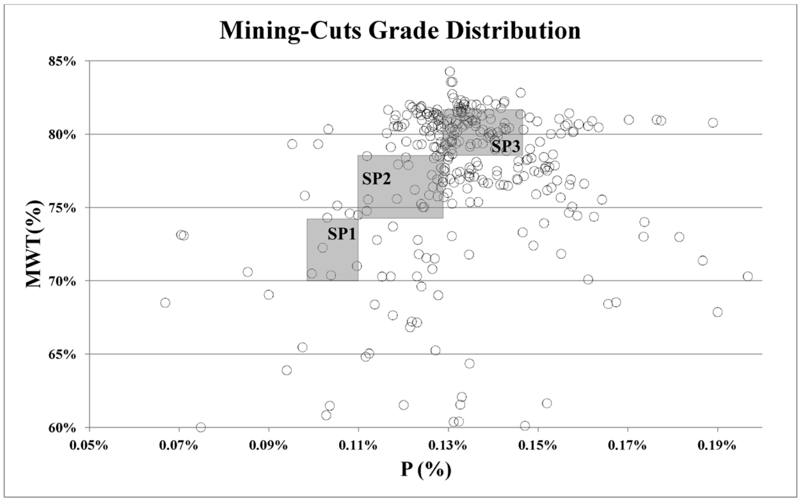

Figure 10.Triple Stockpile Range

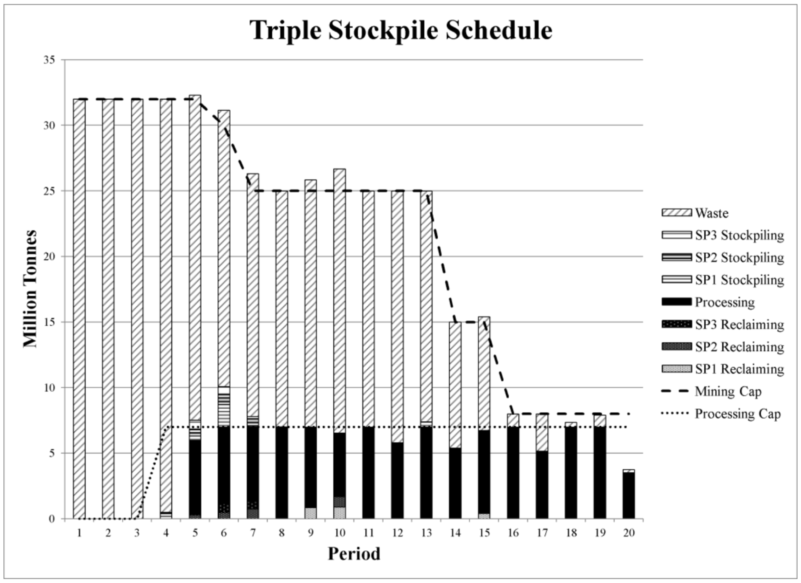

Figure 11.Triple Stockpile Schedule

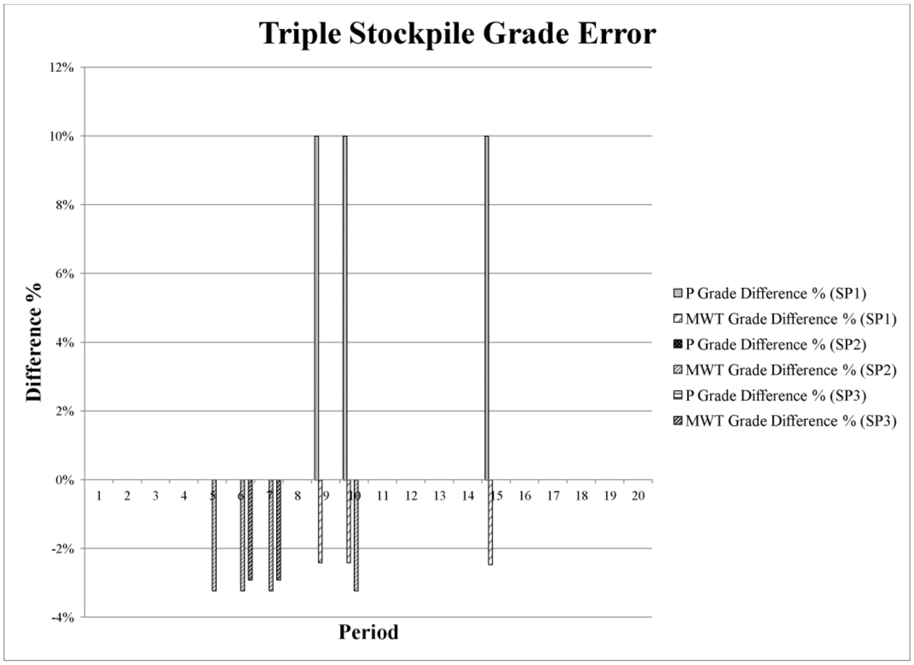

Figure 12.Triple Stockpile Grade Error

3.6. Quadruple Stockpile

In order to avoid changing the number of mining-cuts that are allowed to be sent to the stockpiles and obtain a solution comparable to single stockpile scenario, we created a scenario with four stockpiles in which the defined ranges cover the same area, as can be seen by comparing Figure 13 against Figure 4. Since we are controlling two elements at the time, we need four stockpiles to cover the same range. The four stockpile parameters are provided in Table 3. Using four stockpiles results in an NPV of 2,331 million dollar which is 1.7% higher than the single stockpile scenario and 10.5% higher than no stockpile scenario. The average absolute error in grade is 5.1% and the total material reclaimed from the stockpiles over the mine life is 12.1 million tonnes.

Table 4.Quadruple Stockpile Parameters

Stockpile	Element	$\underline{G}_d^{t,e}$ (%)	$\overline{G}_d^{t,e}$ (%)	$G_s^{t,e}$ (%)
1	P	0.10	0.13	0.12
	S	1.00	2.00	1.59
	MWT	75.00	80.00	77.75
2	P	0.13	0.15	0.14
	S	1.00	2.00	1.59
	MWT	75.00	80.00	77.75
3	P	0.10	0.13	0.12
	S	1.00	2.00	1.59
	MWT	70.00	75.00	72.24
4	P	0.13	0.15	0.14
	S	1.00	2.00	1.59
	MWT	70.00	75.00	72.24

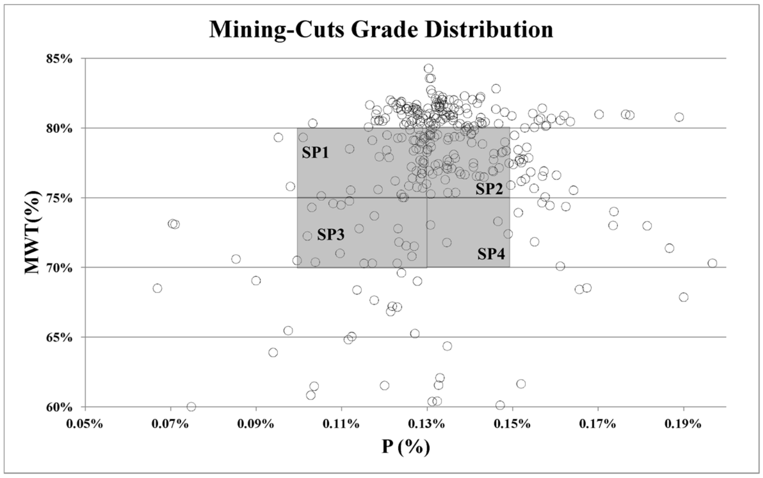

Figure 13. Quadruple Stockpile Range

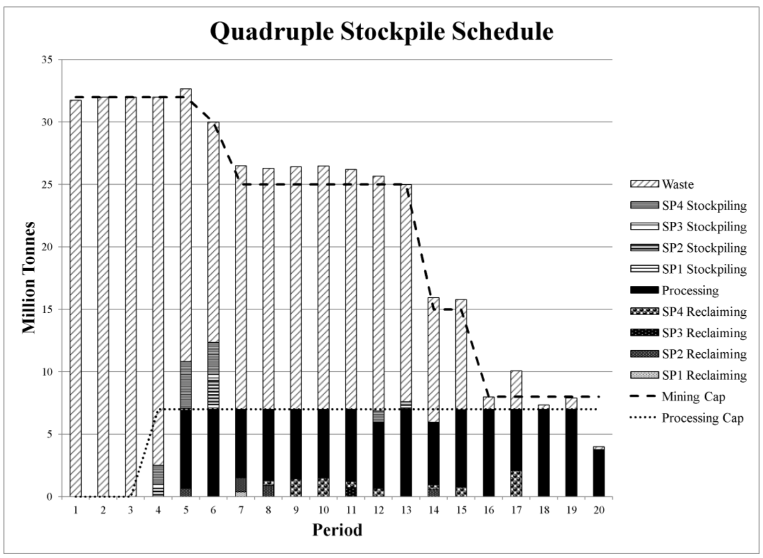

Figure 14. Quadruple Stockpile Schedule

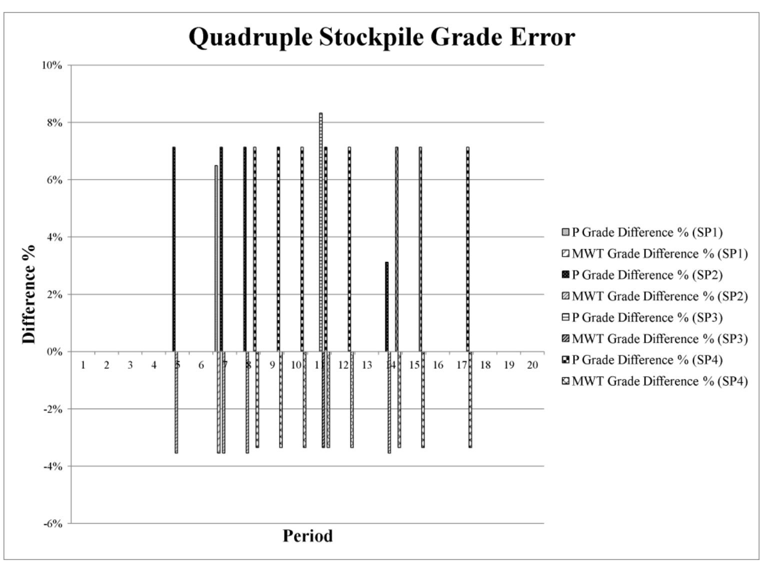

Figure 15. Quadruple Stockpile Grade Error

3.7. Summary of the results

We tested our proposed model on multiple scenarios in an Iron ore mine. We started by obtaining an acceptable schedule for the operation without restricting the grade of material sent to the processing plant. As expected, this scenario resulted in the highest NPV and we use this as the benchmark for calculating difference percentages for other scenarios.

As can be seen in Table 5, the no stockpile scenario results in the lowest NPV and even fails to feed the plant properly (Figure 3). Therefore, we added stockpiles to feed the plant and increase the NPV. However, there is a linearization error associated with each stockpile which we are trying to quantity. Thus, we devised four different stockpiling scenarios. Scenarios with one, two and three stockpiles follow the same trend where tightening the ranges of stockpile inputs and increasing the number of stockpiles decreases the resulted NPV, the total tonnage of material reclaimed from the stockpiles and the average grade estimation error. As shown in the previous sections, controlling multiple element grades for the stockpiles input plays a significant role in the way we define our stockpiles and estimate the reclamation error. Finally, we tested a scenario where four stockpiles covered the same grade range as the single stockpile and showed that the average grade estimation error can be decrease without decreasing the usability of the stockpiles.

Table 5.Summary of the results

Scenario	NPV ($M)	Diff (%)	CPU Time (s)	Reclaimed Tonnage (MT)	Average Grade Error (%)
No limit	2615	0	0.99	-	-
Grade Control	2108	-19%	2.17	-	-
Single SP	2291	-12%	4.17	12.0	11.6
Double SP	2234	-14%	8.43	8.2	6.5
Triple SP	2155	-18%	8.82	5.7	3.0
Quadruple SP	2231	-15%	15.86	12.0	5.1

4. Conclusion and future work

In this paper, we presented continuation of our work on LTOPP by including stockpiling in the long-term plan formulation. We presented the original non-linear model and showed how we can benefit from piecewise linearization and develop a linear model. We compared our linear model against variety of models proposed in the literature and showed the differences and similarities. In the third section, we presented a case study to show the effects of adding stockpiles to long-term production plans and the effects of defining stockpile grade ranges when dealing with linearized grade estimations in the MILP. We started by showing how having constraints on the input grade of material into the processing plant affects the production schedule and how adding stockpiles can help to overcome the shortcomings. Afterwards, we presented multiple stockpiling scenarios to study the errors introduced by linearly estimation of stockpile output grades.

We first defined one stockpile with control over the average grade of material sent to the stockpile. We showed that having one stockpile in place can prevent shortfalls in plant feed. However, the linear estimation of output grade incurred an 11.5% error. We then increased the number of stockpiles by tightening the ranges of acceptable materials for each stockpile and showed how it can decrease the error. Moreover, we illustrated how controlling multiple element grades increases the complexity of stockpile estimations and limits the tonnages of material that can go into the stockpiles. Finally, we presented a case where four stockpiles cover the same grade range as the single stockpile and showed that it is possible to decrease the estimation error and allow the same tonnage of material into the stockpile. However, deciding on the number of stockpile and their grade ranges will heavily depends on the characteristics of deposit and operation due to homogenous reclamation and other important stockpiling assumptions. Moreover, our work highlighted an important shortcoming in the studies of stockpiling in open-pit mines where most approaches assume a single element of interest and ignore the errors and effects of other elements.

5. References

[1] Bienstock, D. and Zuckerberg, M. (2010). Solving LP Relaxations of Large-Scale Precedence Constrained Problems. in *Integer Programming and Combinatorial Optimization SE - 1*, *Lecture Notes in Computer Science*, F. Eisenbrand and F. B. Shepherd, Eds., 6080 ed, Springer Berlin Heidelberg DA - 2010/01/01, pp. 1-14.

[2] Bley, A., Boland, N., Froyland, G., and Zuckerberg, M. (2012). Solving mixed integer nonlinear programming problems for mine production planning with stockpiling. University of New South Wales, Technical Report, paper Document Number,

[3] Chicoisne, R., Espinoza, D., Goycoolea, M., Moreno, E., and Rubio, E. (2012). A New Algorithm for the Open-Pit Mine Production Scheduling Problem. *Operations Research*, *60* (3), 517-528.

[4] Cullenbine, C., Wood, R., and Newman, A. (2011). A sliding time window heuristic for open pit mine block sequencing. *Optimization Letters*, *5* (3), 365-377.

[5] Gholamnejad, J. and Kasmaee, S. (2012). Optimum blending of iron ore from Choghart stockpiles by using goal programming. *Journal of Central South University*, *19* (4), 1081-1085.

[6] Gurobi (2017). Gurobi Optimizer. Ver. 6.5, Houston.

[7] Johnson, T. B. (1969). *Optimum Production Scheduling*. in Proceedings of 8th International Symposium on Computers and Operations Research, Salt Lake City, Utah, pp. 539-562.

[8] Kumar, A. and Chatterjee, S. (2017). Open-pit coal mine production sequencing incorporating grade blending and stockpiling options: An application from an Indian mine. *Engineering Optimization*, *49* (5), 762-776.

[9] Lambert, W. B. and Newman, A. M. (2014). Tailored Lagrangian Relaxation for the open pit block sequencing problem. *Annals of Operations Research*, *222* (1), 419-438.

[10] Lamghari, A., Dimitrakopoulos, R., and Ferland, J. A. (2015). A hybrid method based on linear programming and variable neighborhood descent for scheduling production in open-pit mines. *Journal of Global Optimization*, *63* (3), 555-582.

[11] Moreno, E., Espinoza, D., and Goycoolea, M. (2010). Large-scale multi-period precedence constrained knapsack problem: A mining application. *Electronic Notes in Discrete Mathematics*, *36* 407-414.

[12] Moreno, E., Rezakhah, M., Newman, A., and Ferreira, F. (2017). Linear models for stockpiling in open-pit mine production scheduling problems. *European Journal of Operational Research*, *260* (1), 212-221.

[13] Mousavi, A., Kozan, E., and Liu, S. Q. (2016). Open-pit block sequencing optimization: A mathematical model and solution technique. *Engineering Optimization*, *48* (11), 1932-1950.

[14] Newman, A. M., Rubio, E., Caro, R., Weintraub, A., and Eurek, K. (2010). A Review of Operations Research in Mine Planning. *Interfaces*, *40* (3), 222-245.

[15] Osanloo, M., Gholamnejad, J., and Karimi, B. (2008). Long-term Open Pit Mine Production Planning: A Review of Models and Algorithms. *International Journal of Mining, Reclamation and Environment*, *22* (1), 3 - 35.

[16] Ramazan, S. and Dimitrakopoulos, R. (2013). Production scheduling with uncertain supply: a new solution to the open pit mining problem. *Optimization and Engineering*, *14* (2), 361-380.

[17] Smith, M. L. and Wicks, S. J. (2014). Medium-Term Production Scheduling of the Lumwana Mining Complex. *Interfaces*, *44* (2), 176-194.

[18] Tabesh, M. and Askari-Nasab, H. (2011). Two-stage clustering algorithm for block aggregation in open pit mines. *Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology*, *120* (3), 158-169.

[19] Tabesh, M. and Askari-Nasab, H. (2013). Automatic Creation of Mining Polygons using Hierarchical Clustering Techniques. *Journal of Mining Science*, *49* (3), 426-439.

[20] Tabesh, M., Askari-Nasab, H., and Peroni, R. (2015). A Comprehensive Approach to Strategic Open Pit Mine Planning with Stockpile Consideration. in *Application of Computers and Operations Research in the Mineral Industry (APCOM 2015)*. Fairbanks, AK

[21] Tabesh, M., Mieth, C., and Askari-Nasab, H. (2014). A Multi-Step Approach To Long-Term Open-Pit Production Planning. *International Journal of Mining and Mineral Engineering*, *5* (4), 273-298.

[22] Waqar Ali Asad, M. and Dimitrakopoulos, R. (2012). Optimal production scale of open pit mining operations with uncertain metal supply and long-term stockpiles. *Resources Policy*, *37* (1), 81-89.