Quadratic Programming Application in Block-cave Mining

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Abstract

Block-cave mining with high rate of production and low operation costs seems to be one of the best options among underground mining methods and also a good alternative for deep open pit mines. Production scheduling is one of the critical steps in the block-caving design process. Optimal production schedule can add significant value to the block-cave mining project. Extraction of one drawpoint can change the production profile of its adjacent drawpoints. Therefore, block-cave mining operations can be complicated and behave as a non-linear phenomenon. So, production scheduling for this kind of operations with lots of involving dynamic parameters could be a big size problem in non-linear environment. A linear programming formulation may result in high levels of horizontal mixing between drawpoints. This paper uses mixed integer quadratic programming to model production scheduling in block-cave mining in order to reduce unexpected horizontal mixing, and as a result dilution during the life of the mine.

1. Introduction

Among underground mining methods, the operation cost of block caving is comparable to open pit mining which makes it attractive as an alternative for deep ore bodies. But the operation for block caving can be complicated because many constraints are involved and the material movement which directly affects the production can behave as a non-linear phenomenon. Extracting from a drawpoint can change the grade or tonnage of other drawpoints in its neighborhood. Maintaining a uniform extraction from drawpoints can reduce the unexpected movements of material. This will improve the production schedule, reduce the probability of horizontal mixing and as a result the dilution. In this research, production schedule for a block cave mining operation is modeled using mathematical programming. The model is mixed integer quadratic programming in which the objective function is quadratic and the constraints are linear. The proposed model is then tested for a real case block cave mine operation.

2. Summary of literature review

Operations research as a strong tool has been vastly used for optimizing production scheduling in mining projects. Some models have been proposed for block caving. Models are defined based on Linear Programming (Guest, Van Hout, & Von Johannides, 2000; Hannweg & Van Hout, 2001; Winkler, 1996), Mixed-Integer Linear Programming (Alonso-Ayuso, et al., 2014; Chanda, 1990; Epstein, et al., 2012; Guest, et al., 2000; Parkinson, 2012; Pourrahimian, 2013; Rahal, 2008; Rahal, Dudley, & Hout, 2008; Rahal, Smith, Van Hout, & Von Johannides, 2003; Rubio, 2002; Rubio & Diering, 2004; Smoljanovic, Rubio, & Morales, 2011; Song,

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1989; Weintraub, Pereira, & Schultz, 2008; Winkler, 1996), and Quadratic Programming (Diering, 2012; Rubio & Diering, 2004). In the case of block-cave scheduling, a linear programming (LP) formulation will always seek to take the maximum tons from the highest value drawpoints and the least tons from the lower-valued drawpoints (Diering, 2012). As a result, this kind of scheduling may result in high levels of horizontal mixing between drawpoints because draw columns have different heights. Table 1 summarizes the advantages and disadvantages of methodologies examined in previous studies (Khodayari & Pourrahimian, 2015b).

Table 1. Advantages and disadvantages of applied mathematical methodologies in block caving

Methodology	**Features**	
LP	*Advantage*	• LP method has been used most extensively • It can provide a mathematically provable optimum schedule
	Disadvantage	• Straight LP lacks the flexibility to directly model complex underground operations which require integer decision variables • Mine scheduling is too complex to model using LP and the only possible approach is to use some combination of theoretical and heuristic methods to ensure a good, if not optimal schedule
MILP	*Advantage*	• Computational ease in solving a MIP problem (and MILP) is dependent upon the formulation structure • MILP could be used to provide a series of schedules which are marginally inferior to a provable optimum • MILP is superior to simulation when used to generate sub-optimal schedules, because the gap between the MILP feasible solution and the relaxed LP solution provides a measure of solution quality • MILP can provide a mathematically provable optimum schedule
	Disadvantage	• It is often difficult to optimize large production systems using the branch-and-bound search method • The block-caving process is non-linear (the tons which you mine in later periods will depend on the tons mined in earlier periods), so it would not be appropriate to use LP for production scheduling in block caving
QP	*Advantage*	• Since the block-caving process is non-linear, QP could be an appropriate option to model it • It can find solutions in the interior of the solution space, which results in an even height of extraction as well as lower horizontal mixing between drawpoints
	Disadvantage	• Solving this kind of problem could be a challenge.

Khodayari and Pourrahimian (2015b) presented a comprehensive review of operations research in block-caving production scheduling, and summarized authors' attempts to develop methodologies to optimize production scheduling in block caving.

3. Mathematical formulation

In this paper, the production scheduling problem for a block-cave mining operations is modelled using mixed-integer quadratic programming (MIQP). IBM/CPLEX is used to model and solve the optimization problem. The model's related indices, variables, and parameters are discussed in this section.

Notation

- *Indices*

$t \in \{1,...,T\}$ Index for scheduling periods

$n \in \{1,...,N\}$ Index for drawpoints

m Index for a drawpoint belonging to the set S^n

- *Sets*

S^n For each drawpoint *n*, there is a set S^n defining the predecessor drawpoints that must be started prior to extraction of drawpoint *n*

- *Variables*

$DP_n^t \in [0,1]$ Depletion Percentage which is the portion of draw column *n* which has been extracted till period *t* (continuous variable)

$X_n^t \in [0,1]$ Continuous decision variable that represents the portion of draw column *n* which is extracted in period *t*

$Y1_n^t \in \{0,1\}$ Binary variable which determines whether drawpoint *n* in period *t* is active ($Y1_n^t = 1$) or not ($Y1_n^t = 0$)

$Y2_n^t \in \{0,1\}$ Binary variable which determines whether drawpoint *n* till period *t* (periods 1, 2,..,t) has started its extraction ($Y2_n^t = 1$) or not ($Y2_n^t = 0$)

- *Parameters*

$ext\,ton_n^t$ Optimum tonnage of extraction for the drawpoint *n* at period *t* based on the solution of the production scheduling problem (based on problem optimization, it is an output of model)

$\exp ton_n^t$ Objective tonnage of extraction for the drawpoint *n* at period *t* based on the production goals (input)

g_n Average copper grade of draw column associated with drawpoint *n*

ton_n Ore tonnage of draw column associated with drawpoint *n*

$M_{\min}$ Minimum mining capacity based on the capacity of mining equipment

$M_{\max}$ Maximum mining capacity based on the capacity of mining equipment

$G_{\min}$ Minimum production grade

$G_{\max}$	Maximum production grade
$ActMin$	Minimum number of active drawpoints in each period
$ActMax$	Maximum number of active drawpoints in each period
M	An arbitrary big number

3.1 Objective function

Production goals determine the required tonnage of extraction in a mining project. There are always some constraints that control the goals. In this research, the optimization problem is looking for the best solution to reduce the gap between the expected production and the optimal production considering the related constraints. The objective function, Eq. (1), is going to minimize the deviation of the drawpoint extraction and the expected extraction for each drawpoint in each period of production:

$$Minimize \ \sum_{t=1}^{T}\sum_{n=1}^{N}(ext\,ton_n^t - \exp ton_n^t)^2 = \sum_{t=1}^{T}\sum_{n=1}^{N}\left[(ton_n \times X_n^t) - \exp ton_n^t\right]^2 \tag{1}$$

Extraction from drawpoints while having a uniform extraction surface is one of the most important concerns in block-cave mining. It will reduce the dilution, which can be improved by solving this optimization problem.

3.2 Constraints

There are many geotechnical, operational, and economic constraints related to mining projects, which limit the whole system in achieving the operational and strategic plans. This research will try to make sure that related constraints are considered so that the model's results can be applicable in real case block-cave mining.

Binary variables

Two sets of binary variables are used to be able to define the related constraints in the model, Y1 and Y2. Each drawpoint has both variables (Y1 and Y2) per each period.

The first set of binary variable (Y1) determines whether drawpoint *n* is active in period *t* or not; if any extraction from drawpoint *n* at period *t* occurs, it means that the drawpoint is active ($X_n^t > 0$) then $Y1_n^t = 1$ and if there is no any extraction ($X_n^t = 0$) it means that the drawpoint is not active then $Y1_n^t = 0$. To formulate this concept, equations (2) and (3) are used.

$$\forall t \in T \ \& \ n \in N \rightarrow \quad Y1_n^t - Mx_n^t \le 0 \tag{2}$$

$$\forall t \in T \ \& \ n \in N \rightarrow \quad x_n^t - Y1_n^t \le 0 \tag{3}$$

The second set of binary variable (Y2) determines whether the depletion percentage of drawpoint *n* in period *t* is 0 or not. Depletion percentage (DP) is the summation of the *X* values for drawpoint *n* from period 1 to period *t* based on the draw rate curve.

$$\forall n \in N \rightarrow \quad DP_n^t = \sum_{t=1}^{t} x_n^t \tag{4}$$

If the depletion percentage is 0 ($DP_n^t = 0$) then $Y2_n^t = 0$; otherwise $Y2_n^t = 1$. Two equations are defined for this set:

$$\forall t \in T \ \& \ n \in N \rightarrow \quad DP_n^t - Y2_n^t \le 0 \tag{5}$$

$$\forall t \in T \;\&\; n \in N \rightarrow \quad Y2_n^t - (M \times DP_n^t) \le 0 \tag{6}$$

Mining capacity

This constraint considers the total production (extraction from all drawpoints) for each period of time. It is determined based on the equipment and the scale of the mining operations. It helps to make sure that the system is working in an optimal capacity.

$$\forall t \in T \rightarrow \quad M_{\min} \le \sum_{n=1}^{N} ton_n \times X_n^t \le M_{\max} \tag{7}$$

Average grade of production

The average grade of the extracted material should be in an acceptable range. This constraint helps to have a uniform extraction of the ore during the mine life and can be determined based on processing plant requirements. Equations (8) and (9) control this constraint.

$$\forall t \in T \rightarrow \quad G_{\min} \times (\sum_{n=1}^{N} ton_n \times X_n^t) \le \sum_{n=1}^{N} g_n \times ton_n \times X_n^t \tag{8}$$

$$\forall t \in T \rightarrow \quad \sum_{n=1}^{N} g_n \times ton_n \times X_n^t \le G_{\max} \times (\sum_{n=1}^{N} ton_n \times X_n^t) \tag{9}$$

Reserve

The best height of draw (BHOD) is calculated before applying the mathematical model. This constraint ensures that the fractions of draw columns that are extracted over the scheduling periods are going to sum up to 1, which means all the material within the draw column, based on the BHOD, is going to be extracted.

$$\forall n \in N \rightarrow \quad \sum_{t=1}^{T} X_n^t = 1 \tag{10}$$

Number of allowable active drawpoints

This constraint controls minimum and maximum number of active drawpoints at each period of time.

$$\forall t \in T \rightarrow \quad ActMin \le \sum_{n=1}^{N} Y1_n^t \le ActMax \tag{11}$$

Development direction and mining precedence

Two of the key steps in block-caving operation scheduling are development direction and drawpoints' precedence determination. Extraction of each drawpoint can be started if the predecessor drawpoints have been started before. The precedence constraint is defined by Eq (12):

$$\forall n \in N \;\&\; t \in T \;\&\; m \in S^n \rightarrow \quad Y2_n^t \le Y2_m^t \tag{12}$$

Eq. (12) ensures that all drawpoints belonging to relevant set, S^n , are started prior to the extraction of drawpoint *n*. This set is defined based on the method presented by Khodayari and Pourrahimian (2015a).

Continuous mining

Extraction from each drawpoint must be continuous. Eq. (13) ensures that if extraction from a drawpoint starts in a period, at least a portion of the draw column associated with the drawpoint is extracted based on draw rate constraint until all of the material within that drawpoint has been extracted. It should be noted that Eq. (13) works interactively with equations (2), (3), (4), (5), and (6).

$$\forall n \in N \,\&\, t \in T \rightarrow \qquad Y1_n^t + DP_n^{t-1} \geq Y1_n^{t-1} \qquad (13)$$

4. Computational experience

A real case data for a copper block-cave mine operation is implemented to test the MIQP model. Mine development has been finished and the life of the mine is 10 years. The mine has been designed and the production is going to be based on 102 drawpoints. Figure 1 shows the plan view of drawpoints and the advancement direction determined by the methodology presented by Khodayari and Pourrahimian (2015a). Based on the reserve estimation, total tonnage is 13.45 million tonne with the average weighted grade of %1.33 of Cu. The scheduling parameters are presented in Table 2.

The optimization model contains 5100 variables in which the first 1020 variables are continuous and the rest are binary variables. The objective function is going to minimize the difference between an initial tonnage of extraction and the tonnage of extraction which is based on the production schedule.

The resulted tonnage and grade of production during the life of mine shows that the MIQP model tries to produce an even amount during the life of the mine while satisfying the mining capacity and grade constraints (Fig. 2).

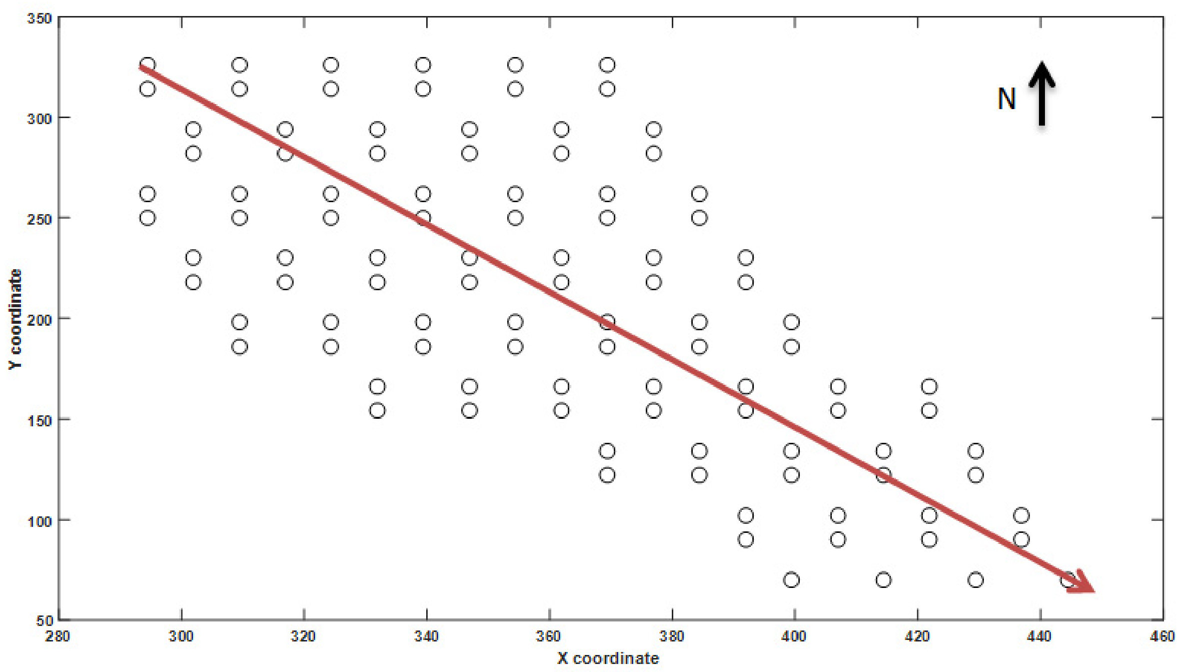

Fig. 1. Plan view of the drawpoints and determined advancement direction

Table 2. Scheduling parameters

Parameter	Description	Value	unit
T	Mine Life	10	Year
G_{min}	Minimum grade	1.1	%
G_{max}	Maximum grade	1.5	%
M_{min}	Minimum mining capacity	0.5	Mt
M_{max}	Maximum mining capacity	1.4	Mt
$ActMin$	Minimum number of active drawpoints	0	-
$ActMax$	Maximum number of active drawpoints	45	-
$EGap$	MIP gap	5	%

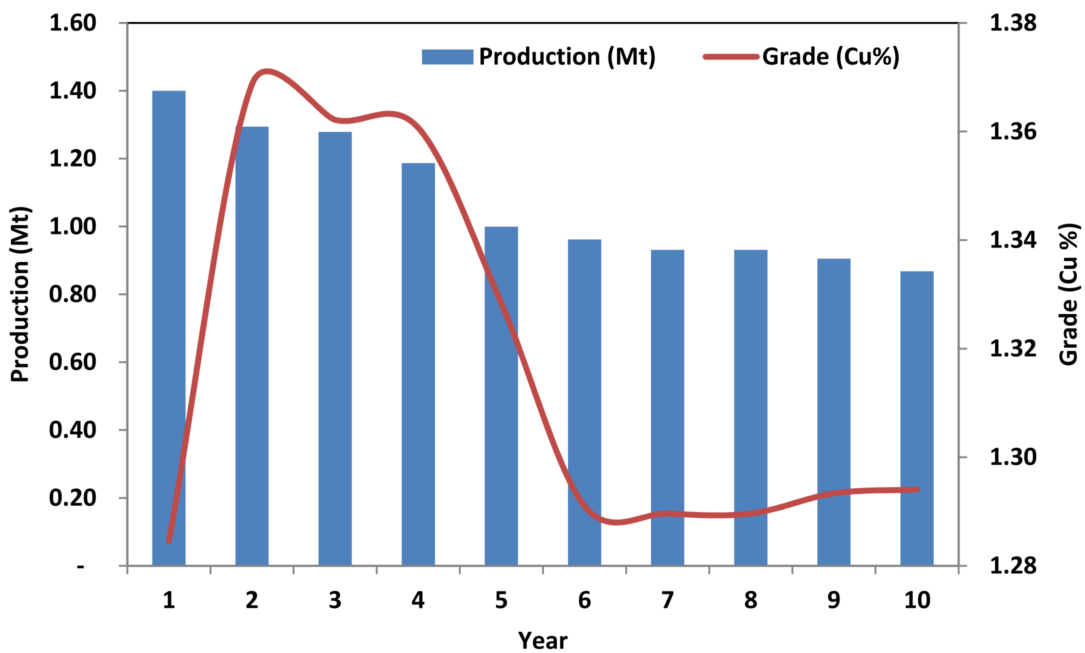

Fig. 2. Total production and average grade during the life of mine

Number of active drawpoints during the life of mine shows that the model is following the related constraints (Fig. 3). Figure 4 shows the trend and how the active drawpoints affect their adjacent drawpoints based on the defined advancement direction and precedence between drawpoints.

Figure 5 shows that the profile of extraction resulted from MIQP model is uniform. It is shown that the extraction starts from West at year one and then expands to the centre of the mine at the second year. The extraction continues to the eastern area in years 4 and 5. There is not that much changes for the last two years because extraction from almost all drawpoints has already been started. It can be seen that the MIQP model can generate a practical profile with low probability of horizontal mixing.

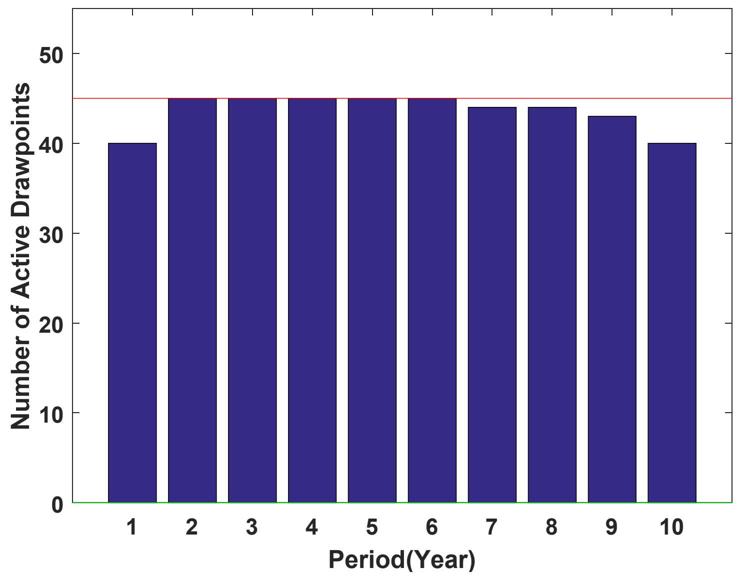

Fig. 3. Number of active drawpoints during the life of mine (the red and green lines are the defined upper and lower bounds, respectively)

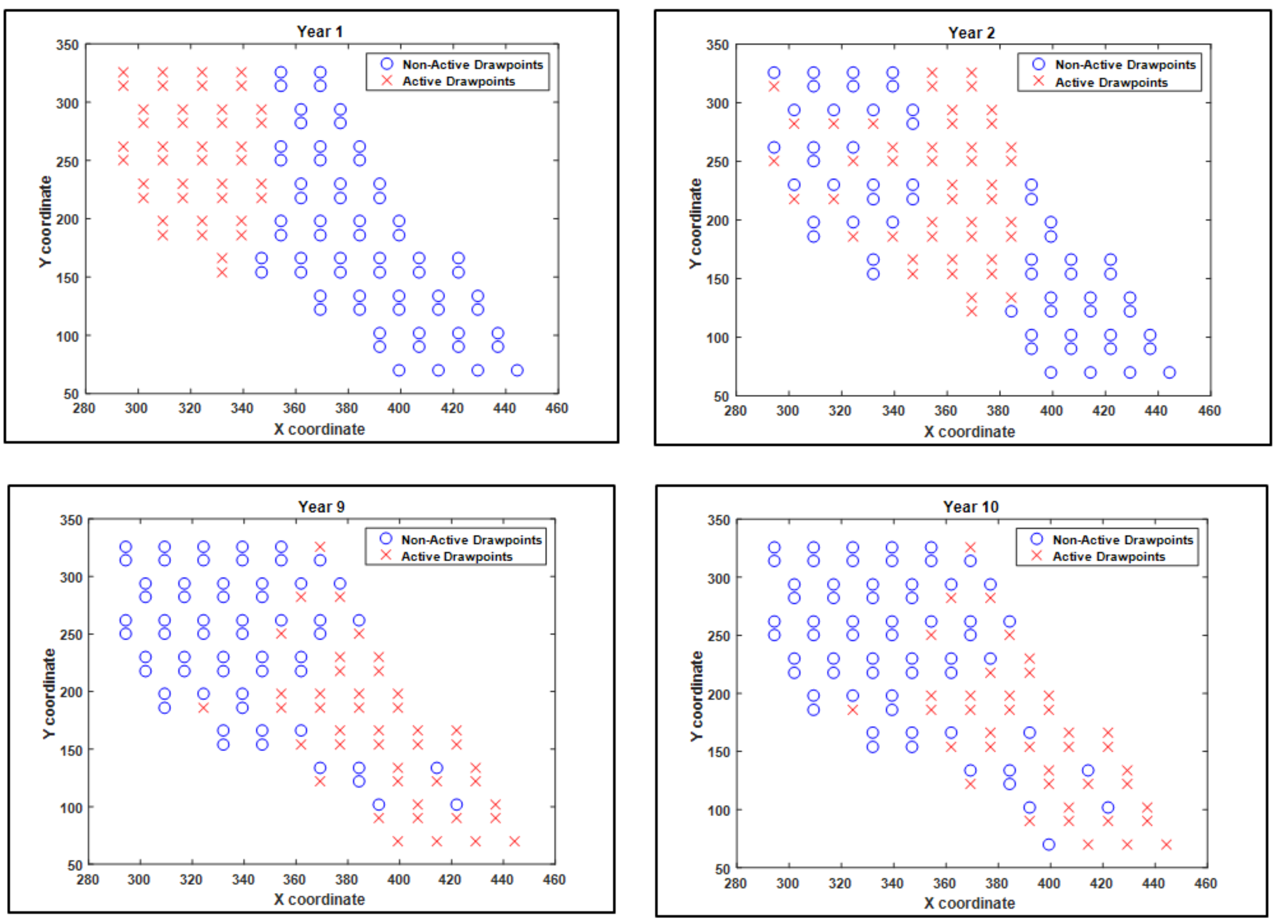

Fig. 4. Number of active drawpoints during the life of mine (years 1,2, 9, and 10)

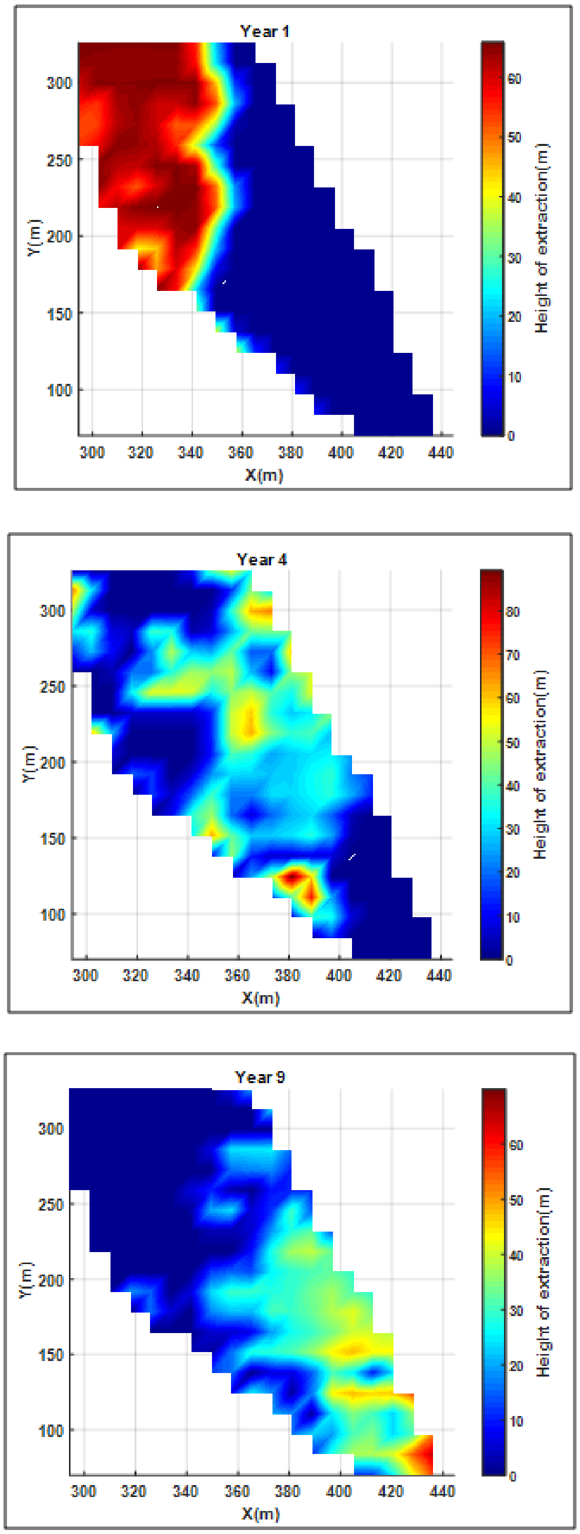

Fig. 5. Generated height of extraction during the life of the mine for different years

5. Discussion

In this research, mixed-integer quadratic programming (MIQP) as a non-linear methodology was used to model production scheduling in block-cave mining operation. The objective function was minimization of the difference between the extraction from drawpoints and an initial tonnage to generate a uniform extraction profile. The uniform extraction profile can reduce the horizontal movements and as a result the dilution. Although the solution time for the MIQP model is longer than the MILP models, the MIQP model extracts from the drawpoints smoothly with a very low fluctuation of tonnage and grade during the life of the mine. This will generate uniform extraction profile with lower dilution.

6. References

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